AN APPROXIMATION FOR EVALUATING THE QUALITY OF SERVICE OF A TELEPHONE NETWORK IN FAILURE CONDITIONS

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ABSTRACT

An approximate method for computing the end-to-end blockings of a network in failure conditions is presented. The application of the method to a network dimensioning problem is also described. The tests performed on real networks for evaluating the influence of the approximations show that the proposed method is accurate enough for engineering the network, taking reliability aspects into account.

1. INTRODUCTION

In the design of circuit switched communication networks, reliability and survivability considerations are of paramount importance. The constraints of an optimisation procedure should regard not only the grade of service in normal condition but also the network behaviour in abnormal states, i.e. overloads and failures. In this paper the failure situations only are considered. But the method presented can also be extended to evaluate overloaded networks.

According to the most recent trends, in normal condition the end-to-end blocking probabilities are used for characterizing the performance of the network. The same quantities can be adopted to represent the quality of service in any failure conditions.

The computation of end-to-end blockings for a real network, using simulations or accurate analytical models is a big task even for fast computers. So these methods, which can be used efficiently for evaluating the network performance in normal condition, are not suitable in the design phase or when it is necessary to examine a great number of failure configurations.

This paper presents a new approximate method for evaluating the quality of service of a network in failure states. The suggestion is to modify the set of parameters characterizing the performance of the network in normal condition, according to the circuit break-downs.

This method has been adopted within the project COST 201, a cooperation of 10 European countries with the aim of developing computer based planning procedures to optimize the dimensioning of telecommunication networks [1, 2, 3].

2. THE EQUIVALENT TRUNK GROUP (ETG) APPROACH

The problem is to compute the end-to-end blockings in a network for many failure states.

The network behaviour in no failure condition can be evaluated precisely through simulation or analytic techniques. Considering any failure as a "small" perturbation of the normal state of the network, the end-to-end blockings in failure condition can be deduced in an approximate way. The approximation is based on the idea of taking advantage of the knowledge of the network behaviour in no failure condition, in order to evaluate the blockings in a failure state.

More precisely, the traffic carried by a route in normal condition is supposed to be carried by an equivalent number of circuits. When a failure is considered, the equivalent number of circuits is reduced as a function of the reduced sizes of the trunk groups of the route hit by the failure. Summing up the equivalent number of circuits of the routes used in each traffic relation, an equivalent trunk group is obtained. The loss probability of this trunk group can be used as an approximation of the end-to-end blocking.

Let us introduce the following notations:

$A_r$ - traffic offered to relation $r$ (Poisson)
$N_{to}$ - number of circuits of trunk group $t$ in no failure condition
$N_{tf}$ - number of circuits of trunk group $t$ in failure $f$
$Y_{ko}$ - traffic carried by route $k$ in no failure condition
$q_{ko}$ - number of circuits equivalent to route $k$ in no failure condition
$q_{kf}$ - number of circuits equivalent to route $k$ in failure $f$
$Q_{ro}$ - number of circuits equivalent to relation $r$ in no failure condition
$Q_{rf}$ - number of circuit equivalent to relation $r$ in failure $f$
$B_{ro}$ - end-to-end blocking of relation $r$ in no failure condition
Fr f — end-to-end blocking of relation r in failure f

In order to show the use of ETG approach, the network of Fig. 1a is considered. The traffic offered to relation A-B is carried by the 3 routes shown in Fig. 1b.

<table>
<thead>
<tr>
<th>route</th>
<th>trunk groups</th>
<th>carried traffic circuits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>y10, q10</td>
</tr>
<tr>
<td>2</td>
<td>2 4</td>
<td>y20, q20</td>
</tr>
<tr>
<td>3</td>
<td>3 5 4</td>
<td>y30, q30</td>
</tr>
</tbody>
</table>

Fig. 1 - An example of the ETG approach

As an example, let us assume that failure f involves trunk groups 2 and 4. Being g(·) the relationship between the actual circuits and the equivalent ones, we assume:

\[ q_{1f} = q_{10} \]
\[ q_{2f} = g(q_{20}, N_{20}, N_{2f}, N_{40}, N_{4f}) \]
\[ q_{3f} = g(q_{30}, N_{40}, N_{4f}) \]

In failure f, the relation A-B has the following number of equivalent circuits:

\[ Q_{ABf} = q_{1f} + q_{2f} + q_{3f} \]

and the end-to-end blocking is simply expressed by the Erlang B formula:

\[ B_{ABf} = E_{Q_{ABf}}(A_{AB}) \]

3. COMPUTATION OF EQUIVALENT CIRCUITS IN NO FAILURE CONDITION

In no failure condition, the traffics carried by the routes can be computed by analytical models, such those presented in [4], [5], or by simulation.

Let \( Y_k \) be the traffic carried by the first k routes of relation r:

\[ Y_k = \sum_{i=1}^{k} y_{i0} \]

A trunk group carries the traffic \( Y_k \) if it has a number of circuits \( D_k \) satisfying the expression:

\[ E_{D_k}(A_T) = 1 - Y_k/A_T \]

From the set \( D_k \) \((k = 1, 2, \ldots)\) the number of circuits \( q_{k0} \) equivalent to the routes are easily computed:

\[ q_{k0} = D_k - D_{k-1} \]

with \( D_0 = 0 \).

As the routes are assumed to be each other independent, in no failure condition the relation r has a number of equivalent circuits equal to the sum of equivalent circuits of its routes:

\[ Q_{ro} = \sum_{k} q_{k0} \]

The end-to-end blocking of relation r in no failure condition can be expressed using the Erlang B formula:

\[ B_{ro} = E_{Q_{ro}}(A_r) \]

This formula gives the exact blocking, because the equivalent circuits of the routes are determined in such a way as they carry the actual traffics.

In case of failure, the same expressions are used:

\[ Q_{rf} = \sum_{k} q_{kf} \]  \hspace{1cm} (1)
\[ B_{rf} = E_{Q_{rf}}(A_T) \]  \hspace{1cm} (2)

4. COMPUTATION OF EQUIVALENT CIRCUITS IN FAILURE CONDITIONS

The crucial point in the ETG (Equivalent Trunk Group) approach is the relationship between the actual circuits and the equivalent ones in case of failure.

In order to answer to this problem an investigation on a single route is performed. The following assumptions are made:

i) there is only one trunk group in the route
ii) the offered traffics are not changed by the failures
iii) the offered traffics are Poisson distributed

The route k under examination offers a traffic stream \( A_{ko} \) to the trunk group t, which is globally offered a traffic \( A_{to} \). The parameters \( 0 < \beta < 1 \) is the ratio of the two traffics:

\[ \beta_k = \frac{A_{ko}}{A_{to}} \]  \hspace{1cm} (3)

The trunk group t has \( N_k \) circuits in normal condition. As all traffics are pure chance, the blocking of the trunk group t and the blocking of the route k are equal and given by the Erlang formula:

\[ B_{ko} = E_{N_k}(A_{to}) \]

The number of circuits \( q_{k0} \) equivalent to route k in no failure condition is given by the
expression:

\[ B_{ko} = E_{qko} (A_{ko}) \]

Let us assume that the failure \( f \) reduces the size of trunk group \( t \) according to the factor \( \delta_{tf} (0 < \delta_{tf} < 1) \):

\[ \delta_{tf} = \frac{N_{tf}}{N_{to}} \]

The blocking of trunk group \( t \) and route \( k \) in failure \( f \) is:

\[ B_{kf} = E_{qtf} (A_{to}) \]

This blocking probability corresponds to a number of equivalent circuits given by:

\[ B_{kf} = E_{qkf} (A_{ko}) \]

So, for having the right blocking, in failure \( f \) the reduction in terms of equivalent circuits must be:

\[ Y_{kf} = \frac{q_{kf}}{q_{ko}} \]

For evaluating the relationship between the actual circuits and the equivalent ones in case of failure, it is sufficient to compute the values of \( Y_{kf} \) as a function of \( \delta_{tf} \) in various situations. Some numerical results, obtained varying the values of parameters \( N_{to}, B_{ko}, \) and \( A_{kt} \), are shown in Fig. 2.

The curves of Fig. 2 can be approximated in a simple way by the function:

\[ \frac{q_{kf}}{q_{ko}} = \left( \frac{N_{tf}}{N_{to}} \right)^{\lambda} \]  

\[ (4) \]

In Tab. 1 are shown the values of exponent \( \lambda \) giving the correct results for \( \delta_{tf} = 0.5 \).

<table>
<thead>
<tr>
<th>( N_{to} )</th>
<th>10</th>
<th>10</th>
<th>10</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_{ko} )</td>
<td>0.01</td>
<td>0.01</td>
<td>0.20</td>
<td>0.01</td>
</tr>
<tr>
<td>( A_{kt} )</td>
<td>0.1</td>
<td>0.5</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>1.36</td>
<td>1.11</td>
<td>1.21</td>
<td>1.38</td>
</tr>
</tbody>
</table>

Tab. 1 - Best values for exponent \( \lambda (\delta_{tf} = 0.5) \)

When more than one trunk group fails in a route the expression (4) is extended in the following way:

\[ \frac{q_{kf}}{q_{ko}} = \min_{t,k} \left( \frac{N_{tf}}{N_{to}} \right)^{\lambda} \]  

\[ (5) \]

This means that the effect of a failure involving more than one trunk group in a route is assumed to be equivalent to a failure on the trunk group having the greatest percentage of circuit reduction.

5. THE VALUE OF EXPONENT \( \lambda \)

As it is shown in Tab. 1 the coefficient \( \lambda \) varies according to a set of parameters, the most important of which is the ratio \( A_{kt} \) between the traffic offered to the route and the total traffic offered to the trunk groups.

Moreover the expression (4) is not the only approximation implied in the ETG method. Other approximations regards the indipendence among the traffic relations, the indipendence among the routes, the expression (5) used to evaluate the effect of a failure involving more than one trunk group in a route,....

In order to test if a good choice of the exponent \( \lambda \) can improve the ETG approach with regards to all the assumptions implied in the ETG method, an investigation on some networks is performed.

With a program based on the algorithms presented in [5], the end-to-end blockings \( B_{rf} \) of all relation \( r \) in all failure \( f \) are computed. These blockings are attributed to \( Q_{tf} \) equivalent circuits such as

\[ B_{rf} = E_{Q_{tf}} (A_{r}) \]

From (1) and (5), we obtain that the right value of exponent \( \lambda \) for relation \( r \) in failure \( f \) is given by

\[ \sum_{k} q_{ko} \cdot \min_{t} \left( \frac{N_{tf}}{N_{to}} \right)^{\lambda_{rf}} = Q_{rf} \]  

In this way a set of good \( \lambda \) values corresponding to different situations are obtained.
It must be noticed that not for all the blockings it is possible to obtain the right values, choosing a suitable \( \lambda \). This happens because, in some failure, some trunk groups have nevertheless blocking values different from the normal ones. The reason is that the traffic pattern is changed, against the assumption ii) in section 4.

From the numerical results obtained on some test networks it appears that the "optimal" \( \lambda_{opt} \) are rather stable with respect to the failures. The variations of \( \lambda_{opt} \) among the relations are stronger.

An attempt of correlating the \( \lambda \) values with the coefficients \( \beta \) introduced in section 4 is carried out. To this aim, the definition of \( \beta \), given for a route composed by a single trunk group, is generalized to a traffic relation, according to the following formula:

\[
\beta_r = \sum_{k \in r} \beta_k \min \left( \frac{A_{kt}}{\sum_{k \in r} \beta_k} \right)
\]

where \( \beta_k \) is given by (3).

\( \beta_r \) represents for relation \( r \) the degree of sharing the trunk groups with the other relations. The greater is \( \beta_r \), the more independent is the relation and the interference from extraneous traffics should be less. Decreasing values of \( \lambda_{opt} \) should correspond to increasing values of \( \beta_r \). This is true only as a mean tendency; the single values of \( \lambda_{opt} \) are rather scattered and sometimes it happens that \( \lambda_{opt1} < \lambda_{opt2} \) corresponds to \( \beta_1 > \beta_2 \).

A linear regression based on the numerical results of some small and rather meshed test networks has led to the following expression:

\[
\lambda = 1.87 - 0.87 \beta_r
\]

The point \((\beta_r = 1, \lambda = 1)\) is considered as fixed because in this case, the relation \( r \) is completely independent from the other ones.

In large networks the computation or the storage of coefficients \( \beta_k \) required by (6) can represent a problem. This can be avoided if the exponent \( \lambda \) is assumed to be constant for all relations. From the numerical results of the same test networks, we obtain

\[
\lambda = \text{mean of mean } \lambda_{opt} = 1.25
\]

In Tab. 2 some parameters which can be used to characterize the quality of service of a network in failure conditions are shown. The approximate models based on (6) and (7) are compared with the exact one. The results regard one of the networks used for computing the (6) and (7).

The agreement between the approximate models and the exact one seems to be rather good. Model (6) is more accurate of model (7), because the former tries to take into account the peculiarities of each traffic relation.

<table>
<thead>
<tr>
<th>QUALITY OF SERVICE</th>
<th>EXACT MODEL</th>
<th>MODEL (6)</th>
<th>MODEL (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean of max ( B_r )</td>
<td>0.276</td>
<td>0.295</td>
<td>0.302</td>
</tr>
<tr>
<td>max of max ( B_r )</td>
<td>0.595</td>
<td>0.578</td>
<td>0.532</td>
</tr>
<tr>
<td>mean of ( B_r )</td>
<td>0.812</td>
<td>0.801</td>
<td>0.798</td>
</tr>
</tbody>
</table>

Tab. 2 - Evaluation of models based on formulae (6) and (7) with respect to Quality of Service parameters.

The results would have been less close to the exact model if a network not used for deducing (6) and (7) were tested. The constants appearing in (6) and (7) depend, indeed, by the characteristics of the network. An attempt of correlating the \( \lambda \) values with the general features of the network (mean number of traffic relation using a trunk group, mean number of trunk groups involved in a failure and so on) is actually under study.

6. AN APPLICATION OF ETC APPROACH TO NETWORK DIMENSIONING

In order to show how the ETC approach can be used in planning procedures let us consider the following problem:

The switching network has been dimensioned in no failure condition, the trunk groups have been routed on the transmission media, we want to know how many circuits should be protected by a stand-by network in order to meet a quality of service constraint. The constraint regards the maximum end-to-end blocking allowed in failure states.

But, in any failure, this constraint can be satisfied imposing different degree of protection to the various trunk groups. The best we can do from the cost point of view, is to minimize the total number of circuits to be protected in each failure situation.

Adopting the ETC approximation this problem can be approached in the following way.

To guarantee, in failure condition, an end-to-end blocking not greater than a threshold \( B \), the number of equivalent circuits of relation \( r \) must be greater than the value \( \bar{Q}_r \) given by:

\[
\bar{Q}_r = E_{Q_r} (A_r)
\]

So the constraint on the blocking is expressed in terms of equivalent circuits. If these circuits are not sufficient, it is necessary to increase their number with an amount:

\[
\Delta Q_r = \bar{Q}_r - Q_r
\]

In general this can be done in different ways, depending from the subdivision of needed capacity among the various routes used by the relation:

\[
\Delta Q_r = \sum_k \Delta Q_{rk}
\]
In order to increase the traffic carrying capacity of a route, it is necessary to increase the number of operating circuits of failed trunk groups, adding some stand-by capacity. Being $N_{rf}$ the stand-by capacity of trunk group $t$ in failure $f$, from (5) we have:

$$\Delta q_{kf} = q_{ko} \min_{t \in k} \left( \frac{N_{rf} + \Delta N_{rf}}{N_{to}} \right)^\lambda - q_{kf}$$  \tag{9}

The (8) and (9) represents a constraint in our problem which is difficult to handle because of the minimum function. An approximate solution can be obtained assuming that the stand-by capacities of all failed trunk groups used in route $k$ are such as to reach exactly the value $\Delta q_{kf}$ of the equivalent circuits. This is sensible because, according to ETG approach, it is useless to put more capacity in one trunk group than in the others belonging to the same route.

Following this simplification, from (9) we have:

$$\Delta N_{tf} = N_{to} \cdot \left( \frac{q_{kf} + \Delta q_{kf}}{q_{ko}} \right)^{1/\lambda} - N_{tf}$$  \tag{10}

and setting

$$\alpha_{kt} = \frac{N_{to}}{q_{ko}} \cdot \left( \frac{q_{kf} + \Delta q_{kf}}{q_{ko}} \right)^{1/\lambda}$$  \tag{11}

$$x_k = q_{kf} + \Delta q_{kf}$$  \tag{12}

we obtain

$$\Delta N_{tf} = \alpha_{kt} x_k^{1/\lambda} - N_{tf}$$  \tag{13}

The quantity to be minimized is

$$\Sigma \Delta N_{tf} = \Sigma \Sigma \alpha_{kt} x_k^{1/\lambda}$$

t

k
d

neglecting the term $N_{tf}$ which is constant and summing the trunk groups route by route.

Set

$$\tilde{\alpha}_k = \max_{t \in k} \alpha_{kt}$$  \tag{14}

the problem becomes, in mathematical term:

$$\min_k \tilde{\alpha}_k x_k^{1/\lambda}$$  \tag{15}

subject to

$$\Sigma x_k = \Delta Q_{rf} + \Sigma q_{kf}$$  \tag{16}

$$x_k \geq q_{kf}$$  \tag{17}

$$x_k \leq q_{ko}$$  \tag{18}

The (15) implies the minimization of stand-by requirements. The variables of the problem are the equivalent capacities $x_k$ of the routes. The (16) expresses the quality-of-service constraint. The (17) and (18) give the boundaries of the $x_k$'s.

The problem has always a feasible solution provided that the end-to-end blockings in no failure condition are less than the allowed blocking $B$ in failure.

As it is $\lambda > 1$, the curves $\tilde{\alpha}_k x_k^{1/\lambda}$ never intersect each other. The solution of the problem (15) + (18) is obtained by ordering the values $\tilde{\alpha}_k$. The algorithm is the following:

1. set $X = \Sigma q_{kf}$

2. for each route $k$ without failed trunk group, set $\tilde{\alpha}_k = \infty$

3. while $X < \Delta Q_{rf} + \Sigma q_{kf}$, choose

$$k = \min_k \tilde{\alpha}_k \text{ and set } q = \min_k \left\{ \Delta Q_{rf} + \Sigma q_{kf} - X, q_{ko} - q_{kf} \right\}$$

$$\Delta q_{kf} = q$$

$$x = X + q$$

$$\alpha_{kt} = \infty$$

At the end of the algorithm the quantities $\Delta q_{kf}$ are known and the stand-by capacities are obtained through (10).

The solution adopted to computed the stand-by capacities requires essentially the independence of the routes. If the same trunk group is used in more than one route, these are no longer independent. Two situations are distinguished:

CASE I: more than one route is in failure, the same trunk group is common to all the failed routes and no other trunk groups fails. As every route in failure uses the same failed trunk group they can be aggregated together to constitute a fictitious route $k$ with an equivalent number of circuits equal to the sum of equivalent circuits of the failed routes using $t$. This fictitious route $k$ can be treated as a real one according to the proposed procedure.

CASE II: more than one route is in failure, some trunk groups are common to the failed routes, some others not. The proposed solution is to adopt the simplified approach, neglecting the fact that some "anomalous" trunk groups are used in more than one route and to set the stand-by capacities of these trunk groups at the maximum number computed for each route using them.

More details can be found in [6].

So far the traffic relations have been considered one at a time. But, what happens if a trunk group, used in more than one relation, requires different numbers of stand-by circuits for protecting the different relations? The suggested solution is to take the maximum of stand-by requirements. This approach is in line with ETG approximation, which assumes a fixed sharing of the trunk group capacity among the traffic relations using it.
7. CONCLUSIONS

The ETG approach has been tested on real networks (up to 600 switching nodes) and it results to be accurate enough in network dimensioning problems. As the proposed method requires reduced computer times, it can be considered an useful tool for the design of large networks under reliability constraints.

ACKNOWLEDGMENTS

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REFERENCES


