TRAFFIC ENGINEERING IN DIFFERENTIATED RATE PERIOD

Tito CERASOLI - Alberto MORENO (*) - Paulo BORGES

Empresa Brasileira de Telecomunicações (EMBRATEL)
(*) Telecomunicações do Rio de Janeiro (TELERJ)

ABSTRACT

The Brazilian Telecommunications System has been using since 1980 a 50% charge reduction in long distance calls between 8 p.m. and 8 a.m. This has resulted in an expressive concentration of interest in calls between 8 p.m. and 9 p.m. which led to call congestions. This scene brings about a problem to the traffic engineering in Brazil. Which grade of service level should be used in trunk groups with busy hour localized in differentiated rate period? This study was then developed to provide a support in a decision about a grade of service that would assure a good service to the subscribers and at the same time make possible the economic investment in the plant.

1. INTRODUCTION

The economic analysis is based on the comparison between revenue loss due to a non increase in the trunk group versus the increasing cost, in circuit per circuit basis, during the equipment life time scope.

The revenue loss calculation must consider the seasonal traffic variations around the annual representative traffic value (VRA).

Loss value determination which is fundamental in this calculation is obtained with simulation support and will be described in next chapter.

In the increasing costs only the equipment costs (transmission and switching) are computed considering the manpower costs as marginal, not changing the results.

The study consists of the following steps:

<table>
<thead>
<tr>
<th>CIRCUITS INVESTMENT</th>
<th>ANNUAL REVENUE</th>
<th>ANNUAL ADDITIONAL REVENUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>PR(N)</td>
<td></td>
</tr>
<tr>
<td>N+1</td>
<td>CJ</td>
<td>PR(N+1)</td>
</tr>
<tr>
<td>N+X</td>
<td>CJ</td>
<td>PR(N+X)</td>
</tr>
</tbody>
</table>

Fig. 1 - Additional Revenue Calculation Flow

N - number of existent circuits.
CJ - costs of one circuit (transmission+switching).
PR - Annual revenue loss.
GR - Annual additional revenue.

This means that the investment of one additional circuit to N existing will imply an additional revenue GR(N+1) that can be a good economic business decision or not, based on the return rate (TIR) of this investment, calculated in the equipment life time scope.

\[ \text{GR} = \text{PR}(N+1) - \text{PR}(N) \]

![Fig. 2 - Cash Flow Diagram.](image)

Where T means life time.

The analysis is repeated with the addition of one circuit at a time until the TIR becomes less than a minimum payment level (attractivity rate - TA), when the last additional circuit will not be economically a good business.

![Fig. 3 - TIR Comportment](image)

Where n means economic circuit number.
2. THEORETICAL DEVELOPMENT

2.1 Valuation of Losses

To evaluate the call congestion, necessary to an economical valuation, it was firstly attempted a formulation on an analytical Markovian model. This idea was put aside, since the simulation that had to be made to attend the analyzed situation within this kind of model, could mask the results, and invalid the study.

It was then chosen a model which was improved as the study developed. At its final configuration, the simulated system corresponds to the following diagram.

Calls can be proceeded from a primary source, if it is a first attempt, or from a secondary source, if it is a repeated call. The calls which find idle circuits are completed successfully and occupy these circuits during a random time interval.

If the circuits are congested or the call can't be completed, there are two possible situations: the call abandonment or its repetition due to the customers retrial (perseverance), after a random time interval.

The calls which return to the system due to the customers perseverance occupy a limited number of positions in the secondary sources to make new attempts. However, this number can be considered so greater as we want.

Along the simulation development the following hypothesis were considered:

- full availability.
- sequencial search of idle circuits.
- exponentially distributed time interval between consecutive primary calls.
- constant time interval to repeat a call that returns due to congestion.
- exponentially distributed time interval to repeat a call that returns because the system is not OK.
- the ratio of calls that returns to the system due to congestion or because they cannot be completed by the system is constant (f1 e f2 respectively).
- the well suceeded calls occupy the circuits during a time interval equal to the sum of a constant (time of switching) and a randomic value exponentially distributed (conversation interval time).
- the calls that don't find congestion but are not completed by the system occupy the circuits during a constant time of switching.

The main variables involved in the simulation process are the following:

- TTC<sub>i</sub> - instant time when the (i+1)th primary call arrives in the system.
- TS<sub>i</sub> - service time of the (i+1)th call that seized the circuit (primary or repeated).
- TTR<sub>i</sub> - instant time when the (i+1)th repeated call arrives in the system.
- KC - number of congestions.
- KR - number of repeated calls.
- KO - number of primary calls generated.
- TOC - time interval during which all circuits are busy.
- CONG - ratio between the number of congestions and the number of primary or secondary offered calls (lost of calls).
- CONGT - ratio between the total seizure time (TOC) and the total observation interval (time congestion).

The first and second primary calls seize the idle circuits during service times respectively equal to 65 and 70 seconds. The third primary call arrives at TTC<sub>2</sub>=30 sec., finds all circuits busy and returns three times at instants TTR<sub>1</sub> respectively equals to 40, 50 and 90 sec., when it finds an idle circuit and seize it for 20 sec.

In TTC<sub>3</sub>=60 sec. the fourth primary call arrives, doesn't find an idle circuit and returns once in TTR<sub>2</sub>=80 sec., when it seize an idle circuit for 40 sec.

Finally, the fifth primary calls arrive at TTC<sub>4</sub>=130 sec. and seize the
idle circuit.

In accordance with the above conditions we can now write:

\[
\begin{align*}
KO &= 5 \\
KR &= 4 \\
KC &= 4 \\
\text{TOC} &= (65-10)+(110-90)= 75 \text{ seconds}
\end{align*}
\]

The call congestion and time congestion can be estimated as follows:

\[
\begin{align*}
\text{CONG} &= \frac{KC}{KO+KR}=\frac{4}{4+5}=0.444 \\
\text{CONGT} &= \frac{\text{TOC}}{\text{observation interval}} \\
\text{CONGT} &= \frac{75}{130}=0.577
\end{align*}
\]

Naturally the precision of the above valuation improves as the observation time of the simulation or the number of generated calls grows.

During the development of the simulation it was necessary to study the following questions to achieve good results within an acceptable precision, with a minimum computational effort.

a - how to generate randomic numbers?

b - how many simulated calls would guarantee the wanted precision?

c - how to guarantee that the precision was achieved with the least number of simulated calls?

With these answers:

a - To generate randomic numbers uniformly distributed it was used the congruential method. The routine RANDU, developed by IBM, and presented in [1] was attached to the computer program, codified in PL/I.

b - To settle the number of calls that had to be simulated it was applied the following theorem presented in [3].

Theorem - for a process tending asymptotically to a Markovian process or quasi-Markovian, the relative reliable interval, with significance level of 95%, is given by.

\[
\Delta p = 2 \sqrt{\frac{\lambda}{B}}
\]

\[\text{where} \] 

\[
\begin{align*}
\Delta p &= \text{reliability interval to call congestion} \\
P &= \text{call congestion} \\
B &= \text{number of times that the congested event was observed} \\
\lambda &= \frac{S}{P}. \text{VAR}(p), \text{where} \\
S &= \text{number of simulated calls} \\
\text{VAR}(p) &= \text{variance of p}
\end{align*}
\]

c - The variance reduction technique employed was the anti-thetic variable method. So during the processing of a simulation to a certain number of calls \( S \) we worked in two phases. In the first to \( S/2 \) calls we applied a set of uniform randomics and in the second \( S/2 \) calls we applied the set of complementary randomics. The result applied was taken as the average results of the two mentioned phases [4].

2.2 Daily Revenue Loss Calculation (PRDIA)

Considering that the daily revenue loss was only computed for the busy hour revenue loss, without considering the other hours, the following diagram can be used:

\[\text{Fig. 6 - System Call Flow}\]

\[C_D - \text{Busy hour basic calls.} \]

\[C_T - \text{Busy hour total calls.} \]

\[B_C - \text{Busy hour lost call rate.} \]

\[OK - \text{Busy hour completed call rate.} \]

\[f_1 - \text{Busy hour probability of retrial for blocked calls loss.} \]

\[f_2 - \text{Busy hour probability of retrial for non completed calls due to line busy or absent subscriber.} \]

\[
\begin{align*}
\text{RP} &= \frac{C_{B-C}(1-f_1)+C_{(1-OK)}(1-f_2)}{TC \cdot DM} \\
\text{RA} &= C_{T}(1-B_C) \cdot OK \cdot TC \cdot DM
\end{align*}
\]

Where:

\[R_P - \text{Busy hour revenue loss.} \]

\[RA - \text{Busy hour revenue.} \]

\[TC - \text{Mean time conversation.} \]

\[DM - \text{Average revenue per minute.} \]

\[\text{PRD} - \text{Revenue loss rate.} \]

\[
\text{PRD} = \frac{B_C(1-f_1)+(1-B_C)(1-OK)(1-f_2)}{RA(1-B_C) \cdot OK}
\]
Small variations in the involved factors do not affect the reliability of this relation.
If the revenue loss were directly calculated, the result wouldn't have the same reliability as the one with the relation applied to a separated revenue calculation, described below:

\[ RA = A_C \times (\% \text{ TRAF.CONV.}) \times 60 \times \text{DM} \]

\[ A_C = \frac{A_0 (1-B_T)}{(1-B_T,f_1)} \]

\[ A_0 - \text{Busy hour offered traffic} \]
\[ B_T - \text{Time congestion} \]
\[ \% \text{ TRAF.CONV.} - \text{conversation traffic percentage} \]
\[ \% \text{ TRAF.CONV.} = \frac{\text{OK.TC}}{TX+\text{OK.TC}} \]
\[ T_X - \text{Mean switching time.} \]
\[ \text{RP(DIA)} \times \text{RP.HMM} = \text{PRD.RA.HMM} \]

\[
\left[ \frac{(1-f_1) B_C + (1-B_C)(1-OK)(1-f_2)}{(1-B_C) \times (1-B_T,f_2) (TX+OK.TC)} \right] \times A_0 (L-B_C) \times \text{OK.TC} \times 60 \times \text{DM}
\]

2.3 Annual Revenue Loss Calculation
The daily revenue loss during the year must be considered in this calculation.
This was made considering the mean frequency distribution of the traffic daily values in relation to the annual representative traffic values (VRA) for the network in Brazil, as showed in the following table, where NU means the number of days in which the traffic is greater than the VRA.

Table 1 - Daily traffic values distribution

<table>
<thead>
<tr>
<th>% VRA</th>
<th>NU</th>
<th>% VRA</th>
<th>NU</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>1</td>
<td>90</td>
<td>10</td>
</tr>
<tr>
<td>120</td>
<td>1</td>
<td>89</td>
<td>10</td>
</tr>
<tr>
<td>115</td>
<td>3</td>
<td>87</td>
<td>8</td>
</tr>
<tr>
<td>110</td>
<td>7</td>
<td>86</td>
<td>8</td>
</tr>
<tr>
<td>105</td>
<td>22</td>
<td>85</td>
<td>6</td>
</tr>
<tr>
<td>100</td>
<td>9</td>
<td>84</td>
<td>5</td>
</tr>
<tr>
<td>98</td>
<td>10</td>
<td>83</td>
<td>6</td>
</tr>
<tr>
<td>97</td>
<td>11</td>
<td>82</td>
<td>4</td>
</tr>
<tr>
<td>96</td>
<td>12</td>
<td>81</td>
<td>4</td>
</tr>
<tr>
<td>95</td>
<td>13</td>
<td>80</td>
<td>5</td>
</tr>
<tr>
<td>94</td>
<td>13</td>
<td>75</td>
<td>13</td>
</tr>
<tr>
<td>93</td>
<td>13</td>
<td>70</td>
<td>9</td>
</tr>
<tr>
<td>92</td>
<td>12</td>
<td>60</td>
<td>7</td>
</tr>
<tr>
<td>91</td>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For each traffic distribution value, all necessary parameters to revenue loss calculation are determined. When all values accumulated we have the annual revenue loss.

2.4 Annual Additional Revenue Calculation (GR)
This will be obtained from the difference between adjacents circuits revenue loss.

\[ N - \text{PR}_{\text{ANO}}(N) \]
\[ N+1 - \text{PR}_{\text{ANO}}(N+1) - \text{PR}_{\text{ANO}}(N) = \text{PR}_{\text{ANO}}(N) - \text{PR}_{\text{ANO}}(N+1) \]
\[ N+X - \text{PR}_{\text{ANO}}(N+X) - \text{PR}_{\text{ANO}}(N+X) = \text{PR}_{\text{ANO}}(N+X) - \text{PR}_{\text{ANO}}(N+X) \]
2.5 Return Rate Calculation (TIR)

The economic viability in the circuit will be decided by the Return Rate (TIR), conjugating the additional circuit cost and the additional revenue with this additional circuit. The calculation considers the circuit life time scope.

\[
TIR = \frac{T}{\Sigma \frac{F_X}{(1+i)^t}} = 0
\]

\(F_X\) - cash flow component.

\(T\) - life time scope.

3. RESULTS

3.1 Input Data

The following input data were used in the study.

- \(T_C\) - Mean time conversation - 300 sec.
- \(T_X\) - Mean time switching - 30 sec.
- \(OK\) - Completed calls rate - 61.72%
- \(DM\) - Mean revenue per minute - Cr$180

Circuit Costs (Switching + Transmission)
- Local-Toll Ticket Circuit
  - Circuit Costs (Switching + Transmission) - Cr$7 000 000 (US$6 700)
- Toll-Toll Circuit
  - Circuit Costs (Switching + Transmission) - Cr$4 760 000

3.2 Simulation Results

The chosen example was:
- traffic from primary calls
- offered traffic, including repeated calls (supposing there is not congestion)

Some typical results obtained from the simulation process for the above parameters are following presented:

<table>
<thead>
<tr>
<th>A</th>
<th>N</th>
<th>Call Congestion Time Congestion</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>18</td>
<td>.4884  .3579</td>
</tr>
<tr>
<td>19</td>
<td>.4341  .3125</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>.3809  .2609</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>40</td>
<td>.0415  .0250</td>
</tr>
<tr>
<td>41</td>
<td>.0275  .0166</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>.0159  .0105</td>
<td></td>
</tr>
</tbody>
</table>

A - offered traffic
N - number of circuits

These results correspond to a 20 000 calls simulation. The relative precision of the call congestion can be evaluated from the theorem already stated.

\[
\frac{\lambda}{p} = 2 \sqrt{\frac{\lambda}{B}} = 4 \sqrt{\frac{1}{S_p \text{ level of 95\%}}}
\]

For a call congestion .4884 (N=18 and A=20) the relative precision of 20 000 calls is:

\[
\frac{\lambda}{p} = 4 \sqrt{\frac{1}{20000 \times 0.04884}} = 4.03\%
\]

3.3 Economic Study Example

The chosen example was 100 Erlangs offered to 99 and 100 circuits, leading to the following results:

<table>
<thead>
<tr>
<th>A</th>
<th>N</th>
<th>BC</th>
<th>BT</th>
<th>PR (C/$)</th>
<th>N</th>
<th>BC</th>
<th>BT</th>
<th>PR (C/$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1</td>
<td>176</td>
<td>184</td>
<td>.7877</td>
<td>1</td>
<td>194</td>
<td>200</td>
<td>.7877</td>
</tr>
<tr>
<td>120</td>
<td>2</td>
<td>256</td>
<td>274</td>
<td>.8453</td>
<td>2</td>
<td>284</td>
<td>290</td>
<td>.8453</td>
</tr>
<tr>
<td>140</td>
<td>3</td>
<td>342</td>
<td>360</td>
<td>.8994</td>
<td>3</td>
<td>372</td>
<td>380</td>
<td>.8994</td>
</tr>
<tr>
<td>160</td>
<td>4</td>
<td>432</td>
<td>450</td>
<td>.9533</td>
<td>4</td>
<td>462</td>
<td>480</td>
<td>.9533</td>
</tr>
<tr>
<td>180</td>
<td>5</td>
<td>528</td>
<td>550</td>
<td>.9971</td>
<td>5</td>
<td>558</td>
<td>570</td>
<td>.9971</td>
</tr>
<tr>
<td>200</td>
<td>6</td>
<td>624</td>
<td>650</td>
<td>1.0410</td>
<td>6</td>
<td>654</td>
<td>680</td>
<td>1.0410</td>
</tr>
<tr>
<td>220</td>
<td>7</td>
<td>720</td>
<td>750</td>
<td>1.0849</td>
<td>7</td>
<td>750</td>
<td>780</td>
<td>1.0849</td>
</tr>
<tr>
<td>240</td>
<td>8</td>
<td>816</td>
<td>850</td>
<td>1.1288</td>
<td>8</td>
<td>856</td>
<td>890</td>
<td>1.1288</td>
</tr>
<tr>
<td>260</td>
<td>9</td>
<td>912</td>
<td>950</td>
<td>1.1733</td>
<td>9</td>
<td>952</td>
<td>990</td>
<td>1.1733</td>
</tr>
<tr>
<td>280</td>
<td>10</td>
<td>1008</td>
<td>1050</td>
<td>1.2178</td>
<td>10</td>
<td>1052</td>
<td>1100</td>
<td>1.2178</td>
</tr>
<tr>
<td>300</td>
<td>11</td>
<td>1104</td>
<td>1150</td>
<td>1.2622</td>
<td>11</td>
<td>1152</td>
<td>1200</td>
<td>1.2622</td>
</tr>
<tr>
<td>320</td>
<td>12</td>
<td>1200</td>
<td>1250</td>
<td>1.3070</td>
<td>12</td>
<td>1252</td>
<td>1300</td>
<td>1.3070</td>
</tr>
<tr>
<td>340</td>
<td>13</td>
<td>1300</td>
<td>1350</td>
<td>1.3515</td>
<td>13</td>
<td>1352</td>
<td>1400</td>
<td>1.3515</td>
</tr>
<tr>
<td>360</td>
<td>14</td>
<td>1408</td>
<td>1460</td>
<td>1.3962</td>
<td>14</td>
<td>1462</td>
<td>1510</td>
<td>1.3962</td>
</tr>
<tr>
<td>380</td>
<td>15</td>
<td>1512</td>
<td>1570</td>
<td>1.4410</td>
<td>15</td>
<td>1572</td>
<td>1630</td>
<td>1.4410</td>
</tr>
</tbody>
</table>

TOTAL: 56557421 M 51936e

4.1B-5-5
The additional annual revenue with one additional circuit to 99 existing will be Cr$s 138 056 and the investment Cr$s 7 000 000 (Local-Toll circuit) or Cr$s 4 760 000 (Toll-Toll circuit).
The return rate of this investment, in 13 years, will be:
Local-Toll circuit - 12.21%
Toll-Toll circuit - 21.6%
The additional annual revenue with one additional circuit to 99 existing will be Cr$s 138 056 and the investment Cr$s 7 000 000 (Local-Toll circuit) or Cr$s 4 760 000 (Toll-Toll circuit).
The return rate of this investment, in 13 years, will be:
Local-Toll circuit - 12.21%
Toll-Toll circuit - 21.6%
The adjacent circuits analysis have the following results:

<table>
<thead>
<tr>
<th>REVENUE (Cr$)</th>
<th>TIR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRAF</td>
<td>CIRC</td>
</tr>
<tr>
<td>100</td>
<td>99</td>
</tr>
<tr>
<td>100</td>
<td>7.6</td>
</tr>
<tr>
<td>101</td>
<td>7.0</td>
</tr>
<tr>
<td>102</td>
<td>6.4</td>
</tr>
<tr>
<td>103</td>
<td>5.8</td>
</tr>
<tr>
<td>104</td>
<td>5.3</td>
</tr>
<tr>
<td>100</td>
<td>105</td>
</tr>
</tbody>
</table>

Considering 12% as a minimum payment level (Attractivity Rate), the minimum loss for 100 Erlangs offered traffic will be:
Local-Toll Group Trunk - 7.0%
Toll-Toll Group Trunk - 5.3%

3.4 Overall Results
The exemplified procedure in 3.3 was made for 10 Erlangs, with the following results:

<table>
<thead>
<tr>
<th>VRA</th>
<th>Local-Toll Group Trunk</th>
<th>Toll-Toll Group Trunk</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>12.0%</td>
<td>8.4%</td>
</tr>
<tr>
<td>100</td>
<td>7.0%</td>
<td>5.3%</td>
</tr>
</tbody>
</table>

4. MAXIMUM LIMIT LOSS
The loss values obtained consider only economic aspects and not the service quality.
The service quality consideration was introduced according to the CCITT 541 recommendation (white Book – VI).
"It is experience of Administrations that an acceptable automatic service on a final circuit group cannot be maintained if the traffic loading on the group exceeds a level corresponding to a calculated Erlang grade of service of 10%.

Beyond this traffic loading, and especially owing to the cumulative effect of repeat attempt calls, the service rapidly deteriorates."
Conjugating this recommendation with the economic analysis results, we have the following table:

<table>
<thead>
<tr>
<th>VRA</th>
<th>Local-Toll Group Trunk</th>
<th>Toll-Toll Group Trunk</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>10%</td>
<td>8.4%</td>
</tr>
</tbody>
</table>

5. CONSIDERATIONS
The obtained results cannot be assumed as definitive. The study development must continue under many different approaches summarized below:

A - Improvement of the Data Collection
The used process must be object of an intense analysis through a more comprehensive measurement process.

B - Model Validity Check
The assumed hypotheses must be tested through a statistic process to be made through a specific experiment.

C - Simulation Model
It will occasionally be necessary to change some hypothesis to improve the model. In this item it would be logical to mention the subscriber behaviour when submitted to a hard congestion situation.
It is a reasonable supposition that the retrial probability (perseverance) is not constant with direct variations in relation to the trunk group congestion. This fact brings about an important problem to studied - the call loss control limit determination that saves the system from congestions above certain limits without economic considerations.

D - Investment Costs
The investment costs should consider a broader possibility set, for example, the investment only in switching equipments if transmission equipments are available or vice-versa.

E - Conclusion
This study considered only economic aspects and doesn't intend to obtained definitive dimensioning values, showing that trunk groups with greater traffic capacity must be privileged.
Nowadays the trunk groups with differentiated busy hour rate in the Brazilian network are dimensioned with 10% loss, specially owing to investment limitations. The study will continue, including traffic demands in normal rate hours, better costs investment analysis and more considerations regarding the service quality.

REFERENCES


