ABSTRACT

This paper deals with systems with interdependent traffic sources which are suitable to increase the channel efficiency and the admissible offered traffic by dynamic one-way channel allocation. The number of one-way channels is assumed to be equal or less than the number of subscribers. A limited number of storage places may be provided for reducing the loss of speech samples. Such systems may, e.g., be suitable for overload protection in PABX-systems. The state probabilities and the characteristic traffic values are calculated exactly by means of closed form solutions and iterative methods. Results for these models are presented in examples and diagrams.

1. INTRODUCTION

In order to increase the channel efficiency in communication systems, some modern multichannel TDM-systems admit a number of sources to share a smaller number of channels by voice-activated switching. Publications about the analysis of such systems assure that there doesn't exist any dependence between two subscribers. It is, however, well known that certain interdependencies between the two traffic sources of a connection can be recognized concerning their talking and listening periods (active and passive periods).

In this paper models and analyzing methods for two systems are presented which take into account this particular interdependence of a pair of traffic sources. After the description of the considered system configuration in section 2, a model for the talking activities of a pair of sources is derived in section 3. The equations of steady state and some characteristic values for model No. 1 - systems without any waiting places - are given in section 4 and those of model No. 2 - systems with a limited number of waiting places in section 5. Finally, both models are extended to the more general case of a variable number of conversations being in progress as a function of time in section 6. A conclusion follows in section 7.

2. SYSTEM CONFIGURATIONS

In the sequel, TDM-switching systems are considered which admit dynamic one-way channel allocation. During each conversation three different microstates can be defined taking into account the number of active sources. The first model deals with a given number of conversations being in progress, each conversation concerning the connection of a pair of sources. An extension of this model leads to systems in which more than two sources may take part in a connection. The number of one-way channels is assumed to be equal or less than the number of subscribers. In the latter case, it is possible that speech samples are lost.

The second model deals with a system which enables the storage of speech samples for the fact that the number of channels is not sufficient for immediate transmission of all speech samples. If the number of waiting places is, however, too small a loss of speech samples is possible again. The whole system comprising channels as well as waiting places is operating as a delay-loss system in the usual sense with FIFO service discipline.

3. MODELS FOR THE TALKING ACTIVITIES

Many investigations of systems with dynamic channel allocation are based upon the description of speech and pause intervals of one speaker only given by a binomial distribution /1/-/5/. During a telephone connection between two subscribers four different states are possible at any time considering the number of speakers just being active. In the early sixties P.T. Brady has carried out measurements of 16 telephone conversations during 137.4 minutes using special speech detectors with variable thresholds between signals representing speech and pause intervals. His analyses /6/ lead to the definition of transition probabilities between the states mentioned above which are shown in Table 1.

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Neither</th>
<th>A</th>
<th>B</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neither</td>
<td></td>
<td>0.98940</td>
<td>0.00529</td>
<td>0.00530</td>
<td>0.00001</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>0.00387</td>
<td>0.99486</td>
<td>0.00001</td>
<td>0.00126</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>0.00367</td>
<td>0.0</td>
<td>0.99510</td>
<td>0.00123</td>
</tr>
<tr>
<td>Both</td>
<td></td>
<td>0.00005</td>
<td>0.00885</td>
<td>0.01015</td>
<td>0.98095</td>
</tr>
</tbody>
</table>

The values of Table 1 admit a certain modification of these transition probabilities. In this paper only three states are considered, in which no, one or two speech sources are active. The corresponding transition probabilities are denoted by q(i,j) with 0≤i≤2, 0≤j≤2 and q(0,2)=q(2,0)=0. Considering connections between n≥2 speech sources transition probabilities can be formally defined by q(i,j) with 0≤i≤n, 0≤j≤n and q(1,j)=0 if |j|+1 or |j|-1, respectively.

© IAC, 1985

ITC 11, Minoru Akiyama (Editor)
Elsevier Science Publishers B.V. (North-Holland)

OL ON MODELING AND PERFORMANCE OF SYSTEMS WITH INTERDEPENDENT TRAFFIC SOURCES AND DYNAMIC ONE-WAY CHANNEL ALLOCATION

Holger DAHMS

Institute for Electronic Systems and Switchings, University of Dortmund
Dortmund, FRG

2.1A-1-1
4. SYSTEMS WITHOUT WAITING PLACES

The models of these systems are based on the transition probabilities according to section 3. Transition rates with respect to the input process and the terminating process (referring to new calls and terminating conversations) are negligible in the sequel because these transition rates are much smaller than the transition probabilities caused by speech activities. Therefore at first a model for a given number of conversations being in progress is investigated taking into account this particular dependence of each pair of traffic sources.

4.1 Steady states and their probabilities

In the case of \( x \) connections all possible steady states can be described by a vector with three parameters \( a_i \) (\( i = 0,1,2 \)) indicating the number of conversations with \( i \) active sources, respectively. A state in which both sources are inactive at \( a_0 \) pairs, one source is active at \( a_1 \) pairs and both sources are active at \( a_2 \) pairs be denoted by \( \{a_0, a_1, a_2\} \) and the probability of this state by \( p(a_0, a_1, a_2) \). The parameters \( a_i \) are defined such that

\[
2 \sum_{i=0}^{2} a_i = x \quad (1a)
\]

and

\[
y = \sum_{i=0}^{2} ia_i \quad (1b)
\]

where \( y \) is the number of occupied timeslots. A change of this number can occur by random events at discrete times defined by the TDM frames. Consequently this stochastic process can be described by a homogenous Markov chain.

4.1.1 One pair of sources

Fig. 1 shows the steady state transition diagram of the Markov chain for an example of one pair of traffic sources.

\[
\begin{align*}
q(0,0) & \quad 2q(0,1) \quad q(1,1) \quad q(1,2) \quad q(2,2) \\
1,0 & \quad 0,1 & \quad 0,1 & \quad 0,1
\end{align*}
\]

Fig. 1 Markov chain for a pair of sources

For this case the following equations of state are obtained:

\[
2q(2,1)p(0,0,1) = q(1,2)p(0,1,0) \quad (2a)
\]

\[
(q(1,2) + q(1,1))p(0,1,0) = 2q(0,1)p(1,0,0) + 2q(2,1)p(0,0,1) \quad (2b)
\]

\[
q(1,0)p(0,1,0) = 2q(0,1)p(1,0,0) + 2q(2,1)p(0,0,1) \quad (2c)
\]

with the normalizing condition

\[
p(1,0,0) + p(0,1,0) + p(0,0,1) = 1 \quad (2d)
\]

The solution of this set of equations is

\[
p(1,0,0) = \frac{1}{1 + 2q(0,1) + q(1,0)q(1,2)} \quad (3a)
\]

\[
p(0,1,0) = \frac{2q(0,1)}{q(1,0)} p(1,0,0) \quad (3b)
\]

\[
p(0,0,1) = \frac{q(0,1)q(1,2)}{q(1,0)q(2,1)} p(1,0,0) \quad (3c)
\]

4.1.2 Several pairs of sources

If \( x \) pairs of traffic sources have established a connection, the set of eqs. (3a)–(3c) are valid for each of the \( x \) pairs. The combination of possible active subscribers belonging to these \( x \) pairs leads to the states shown in the state transition diagram in Fig. 2 later on. Their probabilities, however, can be calculated exactly even now in the following way.

Under the condition that the pairs of sources do not depend upon each other the probabilities that the microstates \( \{a_0, a_1, a_2\} \) occur at the same time at these \( x \) pairs are given by

\[
p(x,0,0) = p^x(1,0,0) \quad (4a)
\]

\[
p(0,x,0) = p^x(0,1,0) \quad (4b)
\]

\[
p(0,0,x) = p^x(0,0,1) \quad (4c)
\]

Taking into account all possible combinations which may form the state \( \{a_0, a_1, a_2\} \) leads to the formula

\[
p(a_0, a_1, a_2) = \left( \begin{array}{c}
\frac{2q(0,1)}{q(1,0)} a_1 \\
q(0,1)q(1,2) a_2
\end{array} \right) \quad bc
\]

where

\[
b = \left( x \right) \left( \begin{array}{c}
\left( a_0 \right) \\
\left( a_1 \right) \\
\left( a_2 \right)
\end{array} \right)
\]

By inserting eqs. (3a)–(3c) and eq. (1) in eq. (5) it can easily be shown that eq. (5) represents a multinomial distribution

\[
p(a_0, a_1, a_2) = \frac{x!}{a_0!a_1!a_2!} p(1,0,0)^{a_0} p(0,1,0)^{a_1} p(0,0,1)^{a_2} \quad (6)
\]

4.1.3 Steady state transition diagram

The state diagram in Fig. 2 shows the microstates for an example with \( x=3 \) connections.
The number of microstates \( X(x) \) at \( x \) connections can be calculated by a sum of natural numbers.

\[
X(x) = \sum_{i=1}^{x+1} \binom{x+1}{i} = \binom{x+2}{2}
\]  

(8)

Summing over \( x \) from \( x=0 \) to \( x=n \) yields the total number \( X \) of microstates which are possible at up to \( n \) active pairs of sources:

\[
X = \sum_{x=0}^{n} X(x) = \sum_{x=0}^{n} \binom{x+2}{2} = \binom{n+3}{3}
\]  

(9)

The numbers calculated in eqs. (8) and (9) are special cases of the so-called figured numbers, known as "triangle numbers" and "tetrahedron numbers" /7/.

For the microstates the following state equation can be obtained with the aid of Fig. 2:

\[
p(a_0, a_1, a_2) = p(a_0, a_1+1, a_2-1) (a_1+1) q(1,2) + 2a_2 q(2,1) = \sum_{i=0}^{a_0} \binom{a_0+1}{i} \binom{a_1+1}{a_2-1} (a_1+1) q(1,2)
\]

(10)

Inserting eq. (5) in eq. (10) it can easily be shown that this state equation is the solution of this general state equation. The steady state probabilities of the considered system are therefore given by the multinomial distribution of eq. (5).

4.1.4 Modified transition probabilities

In horizontal groups, Fig. 2 shows several parallel microstates with the same number of \( y \) occupied one-way channels but without any direct transitions. The microstates of a horizontal group can be combined to a macrostate \( \pi(y) \) comprising all microstates corresponding to the same value of \( y \). The probability \( p(y) \) for \( y \) timeslots is given by

\[
z(x,y) = \binom{x+2}{y+1}
\]

(11)

\[
p(y) = \sum_{i=1}^{x+1} \binom{x+1}{i} \binom{i}{y+1} \binom{(2x-y)/2}{y+1}
\]

(12)

\( z(x,y) \) is the greatest possible number of parallel microstates in the case of \( x \) pairs of sources and \( y \) occupied timeslots. \( [a] \) be the greatest integer number less or equal \( a \). \( p(y) \) is equal to the probability of the above mentioned states given by eq. (5). The parameters \( a_0, a_1, \) and \( a_2 \) can be expressed as a function of \( x \) and \( y \) in the following manner with \( 1 \leq i \leq z(x,y) \):

\[
a_0 = x-y+1 \quad a_0 = i
\]

(12a)

\[
a_1 = y-1 \quad i = 2(x-i)-1 \quad y \geq x
\]

(12b)

\[
a_2 = i \quad a_2 = y+x
\]

(12c)

The symmetry of the arrangement of the microstates of Fig. 2 can be seen in eqs. (12a) and (12b), too, if \( y \) is replaced in eq. (12b) by \( y=2x-y \) for \( y \geq x \). Eqs. (5), (11), (12) and the abbreviations

\( A_1 = q(0,1)/q(1,0) \), \( A_2 = q(1,2)/2q(2,1) \) (13)

yield

\[
p(y) = \binom{x+1}{y+1} \sum_{i=0}^{x+1} \binom{y+1}{i} \binom{(2x-y)/2}{y+1} p(i) q_{y+1,y} \binom{y+1}{i} \binom{(2x-y)/2}{y+1}
\]

(14)

with \( Y = \{ y, y \leq x \}

(15)

(15a)

\[
p_{y+1,y} = \binom{y+1}{i} \binom{(2x-y)/2}{y+1} \binom{y+1}{i} \binom{(2x-y)/2}{y+1} p(i) q_{y+1,y}
\]

(15b)

The new transition probabilities \( q_{y+1,y} \) and \( q_{y,y+1} \) can be calculated by the probabilities \( p_1(y), q_1(y,y+1) \) and \( q_4(y+1,y) \).

\[
s(x,y) = \binom{x}{y+1} \sum_{i=0}^{x} \binom{x}{i} \binom{y+1}{i} \binom{(2x-y)/2}{y+1} p(i) q_1(y,y+1)
\]

(16a)

\[
p_1(y) = \sum_{i=0}^{x} \binom{x}{i} \binom{y+1}{i} \binom{(2x-y)/2}{y+1} p(i) q_1(y,y+1)
\]

(16b)

\[
q_{y,y+1} = \binom{y+1}{i} \binom{(2x-y)/2}{y+1} \binom{y+1}{i} \binom{(2x-y)/2}{y+1} p(i) q_1(y,y+1)
\]

(17a)

\[
p_{y+1,y} = \binom{y+1}{i} \binom{(2x-y)/2}{y+1} \binom{y+1}{i} \binom{(2x-y)/2}{y+1} p(i) q_1(y+1,y)
\]

(17b)

Eqs. (15a) and (16b) yield the probabilities for these transitions between the states with \( y \) and \( y+1 \) active sources as the number of occupied timeslots is equivalent to the number of active talkers. These equations are therefore the fundamental formulae for the second model, too, which is described in section 5.

4.1.5 Influence of the transmission probabilities concerning one pair of talkers

In section 4.1.4 a one-dimensional process has been obtained with transitions only to neighboring states characterizing the number of talkers being active. This model is based on the transition probabilities of Table 1 taking into account the interdependence between the two talkers of each pair of sources.

The new transition probabilities \( q_{y+1,y} \) and \( q_{y,y+1} \) can be calculated by the probabilities \( p_1(y), q_1(y,y+1) \) and \( q_4(y+1,y) \).

\[
s(x,y) = \binom{x}{y+1} \sum_{i=0}^{x} \binom{x}{i} \binom{y+1}{i} \binom{(2x-y)/2}{y+1} p(i) q_1(y,y+1)
\]

(16a)

\[
p_1(y) = \sum_{i=0}^{x} \binom{x}{i} \binom{y+1}{i} \binom{(2x-y)/2}{y+1} p(i) q_1(y,y+1)
\]

(16b)

\[
q_{y,y+1} = \binom{y+1}{i} \binom{(2x-y)/2}{y+1} \binom{y+1}{i} \binom{(2x-y)/2}{y+1} p(i) q_1(y,y+1)
\]

(17a)

\[
p_{y+1,y} = \binom{y+1}{i} \binom{(2x-y)/2}{y+1} \binom{y+1}{i} \binom{(2x-y)/2}{y+1} p(i) q_1(y+1,y)
\]

(17b)
As the values of Table 1 represent measurements of talkers from the English-speaking world, the influence of a variation of A1 and A2 given by eq. (13) upon the steady state probabilities \( p(y) \) is shown in Fig. 4 (dashed and dashed dotted lines). In addition the curve for a binomial distribution is plotted as a dotted line (curve No. 6) with the same mean as the multinomial distribution shown in curve No. 1 as a solid line.

4.2 Cutout fraction

The preceding models are based on the fact that there are always enough timeslots being allocatable by activ talkers. If \( x \) pairs of sources have established a connection maximally \( 2x \) timeslots are required to prevent a loss of speech samples. In the sequel, only \( n<2x \) channels be available, so that a loss of some samples may arise. The probability for \( k \) talkers being active is given by eq. (14). The cutout fraction \( \text{cof} \) can be defined by the ratio of the number of samples getting lost and the total number of samples arriving:

\[
\text{cof} = \frac{\sum_{k=n+1}^{2x} (k-n)p(k)}{\sum_{i=0}^{2x} ip(i)} \quad (16)
\]

4.3 Transmission advantage

In conventional TDM-systems the ratio of the number \( 2x \) of busy sources to the number \( n \) of timeslots provided for these busy sources is \( 2x/n=1 \). Systems with voice-activated switching and one-way channel allocation yield a so called transmission advantage \( \text{tra} \), which is given by

\[
\text{tra} = 2x/n^*(\text{cof}) - 1 \quad (17)
\]

where \( n^*(\text{cof}) \) means the dynamic number of timeslots required to maintain the cutout fraction below a desired threshold.

Listening tests /1/ have shown that cutout fractions of 0.5 percent up to 2 percent are perceptible. Fig. 5 shows the values of \( \text{tra} \) for a maximal cutout fraction of 0.1, 0.5 and 2 percent (solid lines) in comparison with results based on a binomial distribution with the same mean known from literature /3/ (dashed lines). Taking into account the interdependence between two talkers the transmission advantage exceeds the values based on a binomial distribution about 10 to 30 percent.

4.4 Several n-tupel of sources

An extension of eq. (5) yields formulae which enable the calculation of the steady state probabilities for \( m \) connections each between an n-tupel of sources /9/. These results can, however, not be presented here for lack of space.

5. SYSTEMS WITH WAITING PLACES

For systems with waiting places for speech samples, the service discipline applied can be described as follows:

1. As long as not all timeslots are occupied \((\text{and all waiting places are idle})\), incoming samples can be directly allocated to the timeslots of a frame.
2. If there are more samples than available one-way channels in a frame, samples of all talkers are transferred into a buffer and the \( n \) timeslots of the frame are allocated to the first \( n \) samples of the buffer.
3. Henceforth only samples are transmitted which have been taken from waiting places until the buffer is empty again.
4. As long as the buffer is not empty again the special case may arise that traffic sources will occupy even more than one timeslot within a frame.
5. The total system comprising channels as well as waiting places is operating as a delay-loss system in the usual sense with FIFO-service discipline.

5.1 Steady state transition diagram

States of this model can be denoted by three parameters:

1. the number of active sources \( k \) \((0 \leq k \leq 2x)\),
2. the number of allocated channels \( y \) \((0 \leq y \leq n)\),
3. the number of occupied waiting places \( j \), \((0 \leq j \leq s)\).

Here \( x \) means the number of pairs of sources, \( n \) and \( s \) the maximal numbers of timeslots and waiting places, respectively. Based on the model of a Markov chain according Fig. 3 a steady state diagram can be derived which is shown schematically in Fig. 6. From Fig. 6 it can be seen that three areas can be distinguished in this state diagram which are denoted as area I, area II and area III. Fig. 7a to 7c show states of each area with their possible transitions in more detail. In the state diagram, described in Figs. 6 and 7a to 7c, transitions between non-neighboring states can occur (in contrary to the state diagram shown in Fig. 1 in which only transitions to neighboring states are possible).

5.2 The equations of state

As can be seen in Fig. 6 the states of area I and III are characterized by the fact that no waiting places are occupied \((j=0)\), those of area II by the fact that all timeslots are occupied \((y=n)\).
Therefore the state probabilities of area I and III be denoted by $p_0(k,y)$ and those of area II by $p_n(k,j)$. The values $k$, $y$, $j$ be again the number of active sources, occupied timeslots and occupied waiting places, respectively. With the aid of Figs. 7a–7c and eqs. (15a) and (15b) the following equations of state are obtained for the system considered:

**Area I:**

\[ p_0(0,0)q_0 = q_1, \quad p_0(1,1)q_0 = q_1, \quad \sum_{i=1}^{n} p_0(0,i) = 1 \]  

\[ p_0(k,k)q_k = q_{k+1}, \quad \sum_{i=k+1}^{n} p_0(k,i) = 1 \]  

\[ p_0(n,n)q_n = q_{n+1}, \quad \sum_{i=n+1}^{n} p_0(n,i) = 1 \]

**Area III:**

\[ p_0(k,y)q_0 = q_1, \quad p_0(0,y)q_0 = q_1, \quad \sum_{i=1}^{n} p_0(0,i) = 1 \]  

\[ p_0(k,k)q_k = q_{k+1}, \quad \sum_{i=k+1}^{n} p_0(k,i) = 1 \]  

\[ p_0(n,n)q_n = q_{n+1}, \quad \sum_{i=n+1}^{n} p_0(n,i) = 1 \]

**Area II:**

\[ p_n(0,0)q_0 = q_1, \quad p_n(1,1)q_0 = q_1, \quad \sum_{i=1}^{n} p_n(0,i) = 1 \]  

\[ p_n(k,k)q_k = q_{k+1}, \quad \sum_{i=k+1}^{n} p_n(k,i) = 1 \]  

\[ p_n(n,n)q_n = q_{n+1}, \quad \sum_{i=n+1}^{n} p_n(n,i) = 1 \]

The set of eqs. (21a)–(21l) can, e.g., be solved iteratively according to the successive overrelaxation (SOR) method /8/.
5.3 Characteristic values

5.3.1 Overflow probability and cutout fraction

A limitation of the number of waiting places may cause an overflow and a loss of several speech samples. The probability for an overflow of i samples denoted by pso(i) is obtained as

\[ pso(i) = \sum_{k=n+1}^{2x} \{ pn(k-1,s-k+n+1)q_{k+1,k} + pn(k,s-k+n+1)q_{k,k} \}, \]

with \( 0 < i \leq 2x-n \)

The cutout fraction denoted by cofw is the ratio of the means of the number of lost samples and the total number of samples arriving. It holds

\[ cofw = \frac{\sum_{i=1}^{2x-n} ipso(i)}{\sum_{i=1}^{2x} ipso(i)}. \]

5.3.2 Delay time

For calculating the delay time of single speech samples the system is regarded at the time t. The state \( \{ k,n,j \} \) with \( 0 \leq k \leq 2x \) and \( 0 \leq j \leq 2x \) is characterized by the following attributes:

1. k speech samples arrive from k active talkers
2. n samples out of the buffer are transmitted
3. j waiting places are occupied

A stored burst of k samples will be delayed by the time \( td(k,j) \) which is just the mean of the delay-time of one speech sample.

\[ td(k,j) = \left\lceil \frac{j-k}{2} \right\rceil + 1 \]

The mean delay-time \( tdm \) of all speech samples arriving in a frame is then given by

\[ tdm = \sum_{k=0}^{2x} \sum_{j=1}^{2x} pn(k,j)k \cdot td(k,j). \]

The mean delay time with respect to waiting samples be denoted by \( tdm_w \) and the mean delay time with respect to arbitrary speech samples arriving at the system by \( tdm_{aw} \). It holds

\[ tdm_w = \sum_{j=1}^{2x} \sum_{k=1}^{2x} pn(k,j)k \cdot td(k,j) \]

\[ tdm_{aw} = \sum_{j=1}^{2x} \sum_{k=1}^{2x} pn(k,j)k \cdot td(k,j) \]

5.3.3 Mean probability of a delay

The mean probability \( W \) that an arriving sample has to wait until it will be transmitted can be calculated in the following way where \( w(j,k) \) is the probability that an arbitrary sample out of a block of k samples has to wait. For \( j \geq k \): all new samples have to wait, \( w(j,k)=1 \),

\[ w(j,k) = \begin{cases} 1 & \text{if } j \geq k \\ 0 & \text{if } j < k \end{cases} \]

The cutout fraction with respect to waiting samples be denoted by \( cofw \), the cutout fraction with respect to arbitrary speech samples arriving at the system by \( cof_{aw} \). It holds

\[ cofw = \sum_{x=0}^{X_{max}} \sum_{y=0}^{Y_{max}} q(x)p(y)[x,y]. \]

6. VARIABLE NUMBER OF CONVERSATIONS IN PROGRESS

The models of section 4 and 5 deal with the case of a fixed number of connections. The transition rates concerning the input process and the terminating process, however, are not taken into account up to now. In the sequel the steady state probabilities calculated by the formulae in section 4 and 5 will be weighted with the probabilities \( q(x) \) that \( x \) connections are established.

\[ q(x) = \sum_{k=0}^{x} \sum_{j=0}^{y} k p(k)q(x) \]

For systems with waiting places the cutout fraction \( cof_{aw} \) is derived from eqs. (25) and (26) by multiplying the overflow probabilities \( pso(i) \) and the state probabilities with \( q(x) \):

\[ cof_{aw} = \sum_{x=0}^{X_{max}} \sum_{y=0}^{Y_{max}} q(x). \]

For systems with waiting places the cutout fraction \( cof_{aw} \) is derived from eqs. (25) and (26) by multiplying the overflow probabilities \( pso(i) \) and the state probabilities with \( q(x) \):
The expected advantage concerning an increased admissible offered traffic for systems with dynamic one-way channel allocation compared to common systems is investigated by calculating the following values:

1. the maximal offered traffic \( A = f(B, x_{\text{max}}) \) according to Erlang's loss formula,
2. the number \( n < x_{\text{max}} \) of one-way channels leading to a cutout fraction less than a given threshold with the same offered traffic \( A \),
3. the maximal offered traffic \( A_m = f(B, m) \) with \( m = n/2 \).

The enhancement of the permissible offered traffic can therefore be described by means of the ratio \( A / A_m \) which is shown in Fig. 8 for the systems with waiting places (dashed lines) and without waiting places (solid lines) as a function of the number of channels, the loss probability \( B \) and the cutout fraction \( c_{\text{off}} \) or \( c_{\text{off}}^w \).

As to be seen in Fig. 8 the admissible offered traffic may be about 5 times greater than in usual systems. The corresponding mean delay time can be calculated by eqs. (29) and (30) by multiplying the steady state probabilities in these formulae with \( q(x) \), again. The values for the mean delay times are less than 20 TDM frames for this special example with \( s = 10x_{\text{max}} \) waiting places.

7. CONCLUSION

The first model considered in this paper leads to closed form solutions not only for pairs but also for \( n \)-tupels of interdependent sources if the transition probabilities are known, e.g., from measurements. Based on these solutions the reduction of the state transition diagram yields the second model for systems with waiting places which can be calculated exactly by means of iterative methods. The investigations have shown that dynamic one-way channel allocation leads to higher channel efficiency which is even greater than obtained in a model assuming independence of all traffic sources. These results lead to the idea that one-way channel allocation may be a suitable way for overload protection, too. In PABX-systems, e.g., the external traffic can be switched in the usual way, whereas the internal traffic (where a slightly decreased speech quality may be acceptable) could be switched by one-way channel allocation. Such service disciplines seem to be suitable for applications to systems with possible overload situations.

REFERENCES

/7/ Flachsmeyer, J., Kombinatorik, VEB Deutscher Verlag der Wissenschaften, Berlin 1969
/8/ Flachsmeyer, J., Kombinatorik, VEB Deutscher Verlag der Wissenschaften, Berlin 1969