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PROCESSOR LOAD MODELLING

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A TWO LEVEL MODEL

In a communication system a processor is part of the secondary resources engaged in administrating the primary resources carrying the subscriber generated traffic. In a well designed system the secondary system operation will not distort the desired behavior of the primary system in any noticable way.

Let the primary resource net be an open Jackson net without feedbacks. Node j in the net is assumed to be generating tasks according to a Poisson process with rate η_j and $\tilde{u}_j$ required instructions per task. Let there be m processors where processor i has a capacity e_i [instructions/sec]. The total arriving intensity to processor i is then

$$\lambda_i = \sum_{j \in I_i} \eta_j \tag{1}$$

I_i is the set of nodes feeding processor i.

We now model each processor as an M/G/1-model.

THE EFFECT OF DEPENDENT INPUT

Each node in the above Jackson network has a Poisson input and output process for primary system customers, but these processes are not independent. To study this dependence let the primary system be one M/M/∞-system with arrival rate $\lambda/2$ and service rate μ. The secondary system is a one server delay system with n e d service time of mean $1/\gamma$. Tasks to the secondary system are generated when customers enter or leave the primary system. Under stationary conditions let

p(i,j) = prob(there are I=i customers in the primary system and J=j tasks in the secondary) (2)

We find (p(i,j) = 0 for i,j<0):

$$-[\lambda/2+i\mu+\Delta(j)\gamma]p(i,j)+\lambda p(i-1,j-1)/2 + (i+1)\mu p(i+1,j-1)+\gamma p(i,j+1) = 0 \tag{3}$$

where $\Delta(j)=1$ if $j>0$, otherwise $\Delta(j)=0$.

We introduce the generating function

$$P(x,y) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} p(i,j)x^i y^j \tag{4}$$

and find

$$\mu(y-x)y\frac{\partial P(x,y)}{\partial x} = [(\lambda/2+\gamma-\lambda xy/2)y-\gamma]P(x,y) + \gamma(1-y)P(x,0) \tag{5}$$

P(x,0) has to be determined. We take the derivative of (5) with respect to y and set y=1.

$$\lambda E(J)-\mu E(I \cdot J)=\lim_{x \to 1} \frac{1}{x-1}[(\gamma-\lambda x/2-\lambda/2)e^{\frac{\lambda}{2\mu}(x-1)} - \gamma P(x,0)] \tag{6}$$

The left side of (6) must be finite which requires the pole for x=1 in the right side to be balanced by zeroes in the bracketted expression. This readily determines P(x,0) to be

$$P(x,0) = (1-\frac{\lambda}{\gamma})e^{-\frac{\lambda}{2\mu}(1-x)} \tag{7}$$

Having determined P(x,0), (5) can be solved using an approach similar to Lagrange's method for ordinary differential equations.

$$P(x,y) = (\gamma-\lambda)\cdot e^{-\frac{\lambda}{2\mu} + y\frac{\lambda}{2\mu}(1-y) + \frac{\lambda}{2\mu}yx} \cdot \left\{\frac{1}{-\lambda y^2/2-\lambda y/2+\gamma} - (1-y)\sum_{i=1}^{\infty}\frac{[\frac{\lambda}{2\mu}(1-y)]^i}{i!} \cdot \frac{(x-y)^i}{-\lambda y^3/2+(i\mu+\lambda/2+\gamma)y-\gamma}\right\} \tag{8}$$

(With x=1 and $\mu \to \infty$ we get the known p g f for number of jobs in a bulk arrival system with R=2 jobs in each bulk.)

The expected number of tasks in the secondary system is found to be

$$E(J) = \frac{3\lambda/2}{\gamma-\lambda} \tag{9}$$

This equals the expected number of jobs in the above mentioned bulk arrival system.

CONCLUSIONS

The above results suggest that the task generation process from a node with a substantial number of primary resources and arrival rate η should be one Poisson process with rate η. A task should be a lumping of subtasks at node arrival and departure instances.

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