

TOPOLOGIES AND ROUTING FOR 2-REGULAR RING NETWORKS

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ABSTRACT

A ring network is a directed graph where each node (which can be a station, a computer, a processor, a memory unit, an interface..) has one inlink and one outlink and the links connect the nodes into a cycle. Due to its simple structure, easy implementation and expandability, the ring network is one of the most popular topologies for local computer networks. However, a ring network is also known to be unreliable and to have long delay. To be more specific, define the diameter of a ring network as the maximum distance over all pairs of nodes. Then a large diameter signifies the existence of pairs of nodes which can be connected to each other only through many other intermediate nodes and hence have long delay. A ring network of size n has diameter $n-1$ which is longest possible. In addition, a common measure of reliability is the connectivity of a network, which is defined to be the largest k such that every node has a directed path to every other node after the removal of any k nodes. The connectivity of a ring network is one, which means that the failure of one node can disrupt the whole network.

One way to shorten the diameter and increase the connectivity is to add links to a ring network. Two general principles are usually followed in adding links. The first is to add as few links as possible, since links can be costly and more links per node increase the control cost at the node. The second is to add links evenly to the nodes so that we can use standardized nodes to build the network. The simplest addition under these two principles is to add one inlink and one outlink to each node. We call such a network a 2-regular ring network. Note that the existence of a second outlink at a node makes the routing algorithm no longer trivial.

There are two general classes of 2-regular ring networks depending on whether the network is node-symmetric or not. The class of node-symmetric networks, also called double-loop networks, can be characterized by two parameters (s_1, s_2) such that the $2n$ links are $i \rightarrow i+s_1$, $i \rightarrow i+s_2 \pmod{n}$, $i = 0, 1, \dots, n-1$. Since the network must be a ring, any (s_1, s_2) network is equivalent to a $(1, s)$ network for some $2 \leq s \leq n-1$. The $(1, n-1)$ network was proposed by Liu^[1] and is called the distributed double loop computer network.. The $(1, n-2)$ network was proposed by Grnarov, Kleinrock and Gerla^[2] and is called the daisy-chain loop. Wang and Coppersmith,^[3] and Raghavandra and Gerla,^[4] studied the

problem of selecting s to minimize diameter (RGA also called a $(1, s)$ network a forward loop backward hop network). Du, Hsu and Hwang studied a 2-regular ring network, called a doubly linked ring network, which is not node-symmetric.

It is easily shown that a $(1, s)$ network has connectivity two. Furthermore, since a $(1, s)$ network is node-symmetric, the routing algorithms stored at each node are mathematically equivalent. Therefore we need only discuss the routing algorithm at node 0. One algorithm which assures shortest-path routing requires the storage of $n/2$ numbers such that if the destination is one of those numbers, the link $0 \rightarrow 1$ is used; otherwise, the link $0 \rightarrow s$ is used. A simpler self-routing algorithm uses the link $0 \rightarrow 1$ if the destination lies between $[1, s-1]$; otherwise the link $0 \rightarrow s$ is used. This self-routing algorithm guarantees that the path length never exceeds $s+n/s$.

The problem of selecting s to minimize the diameter is a difficult mathematical problem. Wang and Coppersmith proposed the selection of $s=n^{1/2}$ which yields a diameter of value $2n^{1/2}$. They also gave $(3n)^{1/2}-2$ as a lower bound for the diameter. Recently Hwang and Xu showed that essentially for every n , the diameter $(3n)^{1/2}+2n^{1/4}$ can be achieved.

The doubly linked ring network proposed by Du, Hsu and Hwang also has connectivity two but has a much shorter diameter $\lceil \log_2 n \rceil$. We give a self-routing algorithm which assures that the path is not longer than $\lceil \log_2 n \rceil$ and a variation of it which assures shortest-path routing.

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