ABSTRACT

We deal in this paper with a problem arising when several kinds of traffic are offered to a circuit switched network. It can be considered that telephone networks may be used for both telephone calls and data transfers. The mean holding times for these two kinds of traffic will be very different. It is thus necessary to develop a method for designing such networks. We propose here a method which allows firstly to calculate the mean and variance of overflowing traffics when Poisson traffics are offered, and secondly to determine the blocking probability of a last choice trunk group when several peaky traffics, for each class of users, are offered. Finally, numerical results are given to validate the method, and dimensioning considerations are discussed.

1- INTRODUCTION

This study is focused on the evaluation of a circuit switched network to which are offered several classes of traffic. It is not strictly speaking a study concerning integration of services but we are concerned with the utilization of a telephone network in which several classes of customers are introduced. In fact, it is more and more usual to utilize a telephone network to carry telematic communications: terminal to terminal or terminal to computer connections. These kinds of communication have duration characteristics very different from those of classical telephone calls. This is especially true in France with the wide scale introduction of small terminals at the subscriber’s home (the “Minitel” terminals used for the Electronic Directory). Local communications due to the utilization of the “Minitel” for the electronic directory may be quite short (1 or 2 minutes) compared with the holding time of a local call. Conversely, a terminal to computer connection in the toll network may be very long (half an hour or more) compared with the inter-city calls. It is thus natural to study the grade of service of a network when these kinds of communications are introduced. A related problem is the dimensioning of such networks.

The study will be conducted in the following way:

1- the study of the overflowing process when Poisson traffics are offered. The steady state system of equations which can be derived leads to a linear system which can only be solved numerically by matrix inversion. An approximate solution is then presented, to evaluate the mean and variance of the overflowing traffic for each class of traffic. The results are compared with the exact results mentioned above and show a maximum relative error of about 10% for the second moment.

2- when a trunk group receives several overflow streams, it is necessary to calculate its blocking probability. The first part of the study has given the mean and variance of each flow, for each customer class. We can therefore establish the global characteristics of the flow offered to the last choice trunk group. From this information, we calculate, as in the ERT method, an “equivalent trunk group” which allows us to calculate the blocking probability. The result is obtained by inversion of the system of equations of the approximate method developed in part 1. This result is in very simple calculations and numerical values are compared with simulations.

3- The last part of the paper validates the method and gives some considerations on the dimensioning problem, when the proportion of each traffic varies.

The assumptions of the study are the following: two classes of customer are considered, each one being characterized by the mean call holding time, whose values are 1/\mu_1 and 1/\mu_2. For each class, the duration of the calls are supposed to be exponentially distributed. The values of \mu_1 and \mu_2 must be considered as different.

To model the network, the method will be copied on classical techniques used for telephone networks: it is supposed that trunk groups can be ordered, so that trunk group 1 can be calculated as soon as all trunk groups \text{j}, \text{j} < 1 have been evaluated. The problem is then to calculate the mean and variance of the overflowing traffic for each trunk group when the offered traffic is Poisson. When the traffic is peaky (in the case of an overflow), it is necessary to evaluate the blocking probability of the trunk group. These are classical assumptions used to calculate telephone networks, at least for hierarchical networks when end to end blocking probability is not the criterion. This classical theory will be extended here for the case of a mixture of traffics.
2- CALCULATION OF OVERFLOWING TRAFFICS

We suppose now that the two flows are offered to a trunk group of size $N$. We will use the following notations:

- $\mu_1$ and $\mu_2$ are the rates of service for the two classes of customers,
- $\lambda_1$ and $\lambda_2$ are the arrival rates, which are supposed Poisson,
- $m_{11}$, $m_{12}$, $v_i$ are mean, second moment and variance of overflow traffic $i$,
- $A_i$ is the offered traffic for class $i$, equal to $\lambda_i/\mu_i$,
- $A$ is the total traffic, equal to $A_1 + A_2$.

The following results can be derived from the classical theory:

$$P(N) = E(A,N) = \frac{A^N}{N!} \sum_{i=N}^{\infty} \frac{A^i}{i!}$$ (2-1)

$$m_{11} = A_1 P(N), \quad i=1,2$$ (2-2)

$$P(i,j) = P(O)(A_1/i!)(A_2/j!)$$ (2-3)

$P(i,j)$ is the steady state probability of having $i$ customers of class 1 and $j$ customers of class 2 on the first choice trunk group. $P(i)$ is the steady state probability of having a total of $i$ customers.

We also have:

$$P(i) = P(O) A_i^i/i!$$

The only real problem is thus to calculate the variance or the second moment of each overflowing traffic.

2-1 EXACT SOLUTION

Define $P(i,j,k)$ to be the steady state probability of having:

- $i$ customers of class 1 on the first choice trunk group,
- $j$ customers of class 2 on the first choice trunk group,
- $k$ customers of class 1 overflowing.

$i$ and $j$ can vary from 0 to $N$, and $k$ varies from 0 to infinity. The following system (equations 2-4) can be derived from the Chapman-Kolmogorov theory:

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2)P(0,0,0)
= \lambda_1 P(1,0,0) + \mu_1 P(0,1,0) + \mu_2 P(0,0,1)$$

$$P(i,j,k) = \lambda_1 P(i-1,j,k) + \lambda_2 P(i,j-1,k) + (i+1) \mu_1 P(i+1,j,k) + (j+1) \mu_2 P(i,j+1,k) + (k+1) \mu_1 P(i,j,k+1)$$ (2-4)

$i+j+k < N$

$$i+j=N$$

Define $f(i,j) = \Sigma_k P(i,j,k)$

$$f(i,j) = \Sigma_k P(i,j,k)$$ (2-5)

We multiply each equation of the previous system by $k$, and sum over $k$ to find the following system (2-5):

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2) f(0,0,0) = \mu_1 f(1,0,0) + \mu_2 f(0,1)$$

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2) f(i,j) = \lambda_1 f(i-1,j) + \lambda_2 f(i,j-1) + (i+1) \mu_1 f(i+1,j) + (j+1) \mu_2 f(i,j+1) + (k+1) \mu_1 f(i,j,k+1)$$ (2-4)

$i+j+k < N$

$$i+j=N$$

By this method, we obtain a system of $(N+1)(N+2)/2$ equations with the same number of unknown variables $f(i,j)$. A numerical resolution by inversion of the linear system is then possible. The summation of all equations of system 2-5 leads to:

$$m_{11} = \Sigma_{i,j} f(i,j) = \frac{\lambda_1}{\mu_1} P(N)$$

This relation may be used to verify the accuracy of the variables $f(i,j)$. To calculate the second moment of overflow traffic 1, it is necessary to multiply each equation of system 2-4 by $k^2$ and to sum all these equations over $i$ and $j$. The terms of the form $k^2 P(i,j,k)$ will disappear after summation, and the following result is finally obtained after some simplifications:

$$m_{12} = \Sigma_{i,j,k} k^2 P(i,j,k) = \frac{\lambda_1}{\mu_1} (P(N) + \Sigma_{i,j} f(i,j))$$ (2-5)

The previously calculated coefficients $f(i,j)$ are thus sufficient to calculate the first two moments.

$$m_{11} = \Sigma_{i,j} f(i,j)$$ (2-6)

$$m_{12} = m_{11} + \Sigma_{i,j} f(i,j)$$ (2-7)
2-2 APPROXIMATE CALCULATION.

The exact method presented in the previous section is certainly not satisfactory from a practical point of view. It is thus necessary to imagine an approximate method, which should be fast enough and whose accuracy will be evaluated by comparison with the exact analysis.

The basic idea which will be developed now is to model the first choice trunk group by a switch, which will remain closed during a negative exponentially distributed time, with a rate $\alpha$, and will remain closed during an exponential time with rate $\beta$. This method is derived from the classical simplified IPP (Interrupted Poisson Process) of Kuczura [1]. Figure 1 explains the model.

The full IPP calculates three parameters $(\lambda, \alpha, \beta)$ to match the three moments of the overflowing traffic. Here, as in the simplified IPP, $\lambda$ is kept to its real value, $\alpha$ and $\beta$ are calculated to match the two first moments. However in our problem, $\lambda$ is not defined since there are two traffics. We have therefore to evaluate the parameters of the IPP using an "equivalent flow", whose arrival rate is $\lambda = \lambda_1 + \lambda_2$ and whose mean service time $\mu$ is such that $A = \frac{\lambda}{\mu}$; this is the exact flow when $\mu_1 = \mu_2$. The switch is in fact defined as if both traffics have the same holding time. This is justified by the fact that the steady state probabilities of the trunk group are independent of the service rates for each class.

The equivalent flow generates an overflowing traffic whose mean and variance are known from the classical formulas:

$$ m = AP(N) $$
$$ v = m(1 - m + \frac{A}{N + 1 + m + A}) $$

The IPP for this equivalent flow gives a renewal process whose Laplace transform is given by:

$$ g(s) = \frac{\lambda (\beta + s)}{s^2 + s (\lambda + \alpha + \beta)} $$

From this last formula, it is known that:

$$ P(N) = \frac{\alpha + \beta}{\lambda} $$
$$ m_2 - m = v + m^2 - m = m - \frac{g(\mu)}{1 - g(\mu)} $$

This gives:

$$ g(\mu) = \frac{m_2 - m_1}{m_2} $$

So $g(\mu)$ can be calculated directly from (2-8) and (2-10). Then, in (2-9), $\alpha \beta$ can be replaced by $\beta P(N)$ or by $\beta A/m$ to obtain a linear equation in $\beta$.

Finally, the solution for $\alpha$ and $\beta$ is:

$$ \beta = \frac{\lambda \mu - g(\mu)\mu^2 - \lambda \mu g(\mu)}{\lambda g(\mu) + \lambda g(\mu)/m - \lambda} $$
$$ \alpha = \frac{(A \beta - \beta m)}{m} $$

We apply this simplified model to each individual flow. The IPP defined by $\alpha$ and $\beta$ is applied to each individual traffic 1 and 2 to obtain:

$$ m_{i2} = m_{i1} + m_{i1} \frac{g_i(\mu_i)}{1 - g_i(\mu_i)} $$

with:

$$ g_i(\mu_i) = \lambda_i (\beta + \mu_i) $$
$$ \mu_{i1}^2 + \mu_i (\lambda_i + \alpha + \beta) + \lambda_i \beta $$

Thus, we obtain an approximation for the second moment and variance of each individual overflowing traffic. Of course, this method does not consider the covariance between overflowing traffics, but it will be shown in the following sections that this approximation is reasonable.

2-3 EVALUATION OF THE APPROXIMATE SOLUTION

The first remark is that the proposed model gives exact results in the special case where $\mu_1 = \mu_2$. This property can easily be verified, but the proof will not be given here. It consists in verifying that the known formulae are found, that is (cf [2]):

$$ E(\eta^2) = p_1 E(\eta_1^2) - p_1 (1 - p_1) E(\eta_1) + p_2 E(\eta_2) $$

with $p_1 = A_1/A$.

For the general case where $\mu_1$ and $\mu_2$ are not equal, we give a comparison of results in Table 1. Several cases are studied, with $A = 10$ and $N = 10$. First, we examine the case where $\mu_1 = \mu_2$, then the case where the traffics are not equal. For each case, the ratio $\mu_1/\mu_2$ varies from 1 to 20. We have only shown the results concerning the second moment, since the first is given exactly.

The results are certainly not perfect, but give only a 10% maximum relative error. The error is less than that in most cases, however it is increasing with the ratio of the holding times. We will show below that the results are sufficient to solve the problem of cluster networks, when several trunk groups overflow on a common trunk group.
Table 1: Comparison of second moments: exact and approximate solutions

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3- MIXTURE OF OVERFLOW TRAFFICS.

3-1 THE MODEL.

For network dimensioning, it is necessary to evaluate classical clusters. We now examine the case of several trunk groups overflowing onto a final choice trunk group. The following figure 2 also gives the notation used:

The problem is to evaluate the lost traffic on the last choice trunk group of size N. In a first step, we will calculate the parameters of a switch (α and β) and equivalent input rates λ1 and λ2, by a method which is the exact converse of that presented in section 2-2. The four unknown variables α, β, λ1, λ2 must be derived from the equations:

\[
M_1 = \lambda_1 \frac{1}{\beta} \quad M_2 = \lambda_2 \frac{1}{\beta} \\
V_1 = \frac{1}{\lambda_1} \quad V_2 = \frac{1}{\lambda_2}
\]

It is in fact quite easy to solve this system. First, it can be noted that \(g_1(\mu_1)\) and \(g_2(\mu_2)\) are known from (3-4) and (3-5). Define \(X_1\) by the following equation which can be obtained from (3-6):

\[
X_1 = M_1 \left(\frac{1}{g_1(\mu_1)} - 1\right) = M_1 \frac{\mu_1(\lambda_1 + \alpha + \beta)}{\lambda_1(\beta + \mu_1)} = \frac{\beta}{\alpha + \beta} \frac{\mu_1 + \alpha + \beta}{\mu_1 + \beta}
\]

We find:

\[
1 - X_1 = \frac{\alpha}{\alpha + \beta} \frac{1}{\mu_1 + \beta}
\]

and in a similar way:

\[
1 - X_2 = \frac{\alpha}{\alpha + \beta} \frac{1}{\mu_2 + \beta}
\]
As $X_1$ and $X_2$ are known, $\beta$ can be obtained from:

$$\frac{1 - X_1}{1 - X_2} = \frac{\mu_1}{\mu_2} + \beta$$

(3-10)

All other variables are then derived directly from (3-8) (to get $a$) and (3-2) and (3-3) to obtain $\lambda_1$ and $\lambda_2$.

The problem is now to calculate the blocking probability of the final trunk group. The first idea, which seems natural, is to calculate it directly from the IPP. Define $\lambda = \lambda_1 + \lambda_2$ and $\mu = \lambda / A$. These data will be the input of the model described in Figure 3:

$$\lambda, \mu \rightarrow N$$

Figure 3: The model to calculate the blocking probability.

We define $P_1(i)$ to be the probability that the trunk group is in state $i$ when the switch is closed, and $P_2(i)$ the corresponding probability when it is open. The following system of equations (3-11) can be established:

$$(\lambda + \mu)P_1(0) = \beta P_2(0) + \mu P_1(1)$$

$$\beta P_2(0) = \beta P_1(0) + \mu P_2(1)$$

......

$$(\lambda + i + \mu)P_1(i) = \beta P_2(i) + \lambda P_1(i-1) + (i+1)\mu P_1(i+1)$$

$$(i + \mu)P_2(i) = a P_1(i) + (i+1)\mu P_2(i+1)$$

......

$$(N_i + a)P_1(N) = \beta P_2(N) + \lambda P_1(N-1)$$

$$(N_i + \beta)P_2(N) = a P_1(N)$$

The matrix of this system is triangular, so that values of $P_1(i)$ and $P_2(i)$ can be obtained with a single loop, with a complexity no greater than the calculation of the Erlang formula. The blocking probability is then equal to $P_1(N)/(a+\beta)/\beta$.

This solution will be referred as solution I in the following section which analyses the results. It will be shown that in some cases this solution, although it is simple, gives erroneous values.

It is also possible to imagine a solution consisting in calculating an "equivalent trunk group", following the idea of the ERT method, which corresponds to our IPP. Two methods are then possible:

a) The total traffic offered to the equivalent trunk group is known: $A = \lambda_1/\mu_1 + \lambda_2/\mu_2$. It is also known that the blocking probability of the equivalent trunk group $P(N^*)$ is equal to $\beta/(a+\beta)$. $N^*$ can thus be obtained by inversion of the Erlang formula $P(N) = E(A,N)$.

b) We define $\lambda = \lambda_1 + \lambda_2$ and $\mu = \lambda / A$. It is then possible to calculate the variance of the "equivalent overflowing traffic" using:

$$g(\mu) = \mu^2 + \mu(\lambda + \alpha + \beta) + \lambda$$

The usual formula for the variance,

$$V = M (1 - M) \frac{N^* + 1 + M - A}{A}$$

allows us to calculate $N^*$ in a linear way without the use of the Erlang formula.

In fact, in the general case, the two values obtained for $N$ may be slightly different, leading to different values for the blocking probability. Numerical calculations, compared with simulations, show that Erlang inversion gives better results for the blocking probability. But the conclusion is that the equivalent trunk group and the IPP are not in exact correspondence. To match the two models, it may be necessary to modify the value of the ratio $\mu_1/\mu_2$.

It seems in fact unnatural to modify the values of $\mu_1$ and $\mu_2$. The following argument shows that it may be necessary to match the model of section 2: Consider a cluster with two overflowing trunk groups where $\lambda_2 = 0$ and $\lambda_1 = 0$. In this case, $\mu_2$ has no significance in trunk group 1 and similarly $\mu_1$ for trunk group 2. Only the ratios $\lambda_1/\mu_1$ and $\lambda_2/\mu_2$ affect the characteristics of the overall overflowing traffic. It is then necessary to calculate the ratio $\mu_1/\mu_2$ to obtain a realistic model. The problem is to calculate this ratio, knowing that the system of equations of the system is not linear. The equation $\beta/(a+\beta) = E(A,N)$ must be added to the system of equations (3-2) to (3-7), $\mu_2$ being an unknown variable. The way to solve the system is the following:

1- choose $\mu_1 = 1$, $\mu_2$ large enough,

2- increase $\mu_1$ by a step (1 for example),

3- At each step, calculate $a$, $\beta$, $\lambda_1$, $\lambda_2$ as in the beginning of this section, and $N$ as in solution b,

4- when the sign of the difference between $\beta/(a+\beta)$ and $E(A,N)$ changes, modify the step of evolution of $\mu_1$ (divide by 10 for example) and modify the sense of variation of $\mu_1$.

5- iterate steps 2 to 4 until the difference between $\beta/(a+\beta)$ and $E(A,N)$ is as small as desired.

The resulting model is then such that it corresponds exactly to the case of section 2. The blocking probability of the last choice trunk group is equal to $E(A,N^*)/(a+\beta)/\beta$.

5.2B-1-5
3-2 EVALUATION OF THE METHOD

Some results and comparisons with simulation runs are summarized in table 2. The relative proportion of the holding times for each type of traffic seems to be the critical parameter of the problem. In the first part of table 2 (examples 1 to 5), \( x \) is the same on each trunk group and the results (theoretical and simulation) show that the blocking probabilities are constant and therefore independent of \( x \). This is probably a quite important result and the following assertion can be formulated:

"If the proportion \( x \) of the traffic of each class is the same on all trunk groups in a cluster, then the blocking probability is independent of \( x \), and therefore the performance can be evaluated using the equivalent random theory."

It is certainly not possible to prove this assertion with the model presented here since the values for the blocking probabilities may vary with \( x \), but with a very small relative variation. I have tried to prove this result directly but without success.

The second part of table 2 (examples 6 to 11) shows the results when the coefficients \( x \) are not equal for each trunk group. It can be observed that now the blocking probabilities vary quite widely: from 0.082 to 0.123 with the model, from 0.083 to 0.097 with the simulation. The conclusion is that the mixture of traffics may have a significant effect on blocking probabilities and therefore on network dimensioning.

### Table 2: Validation of the method

<table>
<thead>
<tr>
<th>example nb.</th>
<th>1</th>
<th>2</th>
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<th>10</th>
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<tbody>
<tr>
<td>A ( A_{11} )</td>
<td>0.8</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>7.2</td>
<td>2</td>
<td>6</td>
<td>4</td>
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<td>7.2</td>
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<td>7.2</td>
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<td>4</td>
<td>2</td>
<td>0.8</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>7.2</td>
<td>0.8</td>
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<tr>
<td>T ( x_1 )</td>
<td>10%</td>
<td>25%</td>
<td>50%</td>
<td>75%</td>
<td>90%</td>
<td>25%</td>
<td>75%</td>
<td>50%</td>
<td>10%</td>
<td>90%</td>
<td>50%</td>
</tr>
<tr>
<td>T ( A_{21} )</td>
<td>0.6</td>
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<td>T ( A_{22} )</td>
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<tr>
<td>( x_2 )</td>
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<td>25%</td>
<td>50%</td>
<td>75%</td>
<td>90%</td>
<td>75%</td>
<td>25%</td>
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<td>1%</td>
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<td>Method I ( P_1 )</td>
<td>0.011</td>
<td>0.011</td>
<td>0.011</td>
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<td>0.011</td>
<td>0.012</td>
<td>0.010</td>
<td>0.015</td>
<td>0.014</td>
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<td>0.017</td>
<td>0.017</td>
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<td>0.022</td>
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<td>0.016</td>
<td>0.016</td>
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<td>0.088</td>
<td>0.088</td>
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<td>0.087</td>
<td>0.103</td>
<td>0.093</td>
<td>0.113</td>
<td>0.130</td>
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<td>Simulation ( P_1 )</td>
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<td>0.012</td>
<td>0.012</td>
<td>0.012</td>
<td>0.012</td>
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<tr>
<td>Simulation ( P_2 )</td>
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<td>0.016</td>
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<td>0.014</td>
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<tr>
<td>Simulation ( P )</td>
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<td>0.090</td>
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</tbody>
</table>

**Legend:**
- \( P_1 \): proportion of lost traffic for traffics offered to the first trunk group
- \( P_2 \): " " " " second trunk group
- \( P \): Blocking probability of the last choice trunk group

Table 2: Validation of the method

5.2B-1-6
The method I (using the initial values for $\mu_1$ and $\mu_2$) seems to be more stable when the parameter $x$ varies. However the validity of this model can be questioned when the coefficient $x$ for one stream takes a very small value compared to the other stream: this is the case of example 11 with $x_1 = 50\%$ and $x_2 = 1\%$.

Model II gives perfect results when all the $x$'s are equal, but is unstable in the converse case. We have found no rational explanation for this phenomenon. The reason is probably that the covariance between overflow traffics has not been considered. Further research in that direction is necessary.

4 CONSIDERATIONS ON DIMENSIONING AND CONCLUSIONS.

Still with the same examples as in section 3-2, we have tried to dimension the network such that the blocking probability of the last choice trunk group is equal to 1%. The same set of 11 examples has been considered, in two cases: firstly with exactly the same traffics as those presented in table 2 (data part); secondly with a Poisson additional traffic of 3 Erlangs offered directly to the last choice trunk group such that $x=50\%$.

The results for the dimensioning of the last choice trunk group are summarized in table 3. The first line corresponds to the first case without additional traffic, and the second line to the case where the three Erlang traffic is added. We have used model I to solve this problem. It can be considered that example 11 is suspicious since the results of table 2 were not in agreement with the simulation.

These two sets of examples show that the dimensioning of the network may vary (in these examples by about 20%) with the variation of the proportion of each class of traffic.

The whole study can easily be extended to the case where more than two classes of customers are considered. To get the mean and variance of overflow traffics, the same model as in section 2-2 can be used without difficulties. The converse problem (for the blocking probabilities and for dimensioning) imply the recalculation of the ratios of the holding times.

This study can be considered as a first approach to the problem of mixture of traffics. It may probably be fruitful to investigate the effect of covariance of flows, which was not considered here. It seems very difficult to obtain a better model than the one presented without adding a lot of complexity. It can be also concluded that mixture of traffics has no influence on grade of service as soon as the proportions of each class of traffic remain in a narrow space for each stream.

BIBLIOGRAPHY


<table>
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<th>example nb.</th>
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<td>12</td>
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</table>

Table 3: Size of the last choice trunk group for 1% blocking probability.