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QUEUEING SYSTEMS WHERE ARRIVAL OR SERVICE TIMES ARE MARKOV-DEPENDENT

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1 INTRODUCTION

It is of interest to study the influence on queueing processes of dependencies between interarrival times and/or service times, i a in modelling the behaviour of telecommunications control systems.

In a contribution to ITC 10 the author formulated a class of queueing problems involving dependencies, which should allow fairly simple generalizations of classical analyses, where full independence assumptions are made. It was proposed to study one-stage systems under stationary conditions, where it is assumed that the sequence of interarrival times and that of service times are independent of each other, that one of them is a sequence of independent, identically distributed positive r v (random variables) (as in the classical theory) but that the other is a stationary sequence of Markov-dependent times. In the paper were given, i a, simple generalizations of the P-K formulae for the single server system having Poisson arrivals and general Markov-dependent service times with linear regression between successive service times.

Further research has since been done and in this short paper we will report on the analysis of the single server system where the interarrival times form a general stationary Markov sequence and the service times are independent of one another and of the interarrival times and exponentially distributed.

2 ASSUMPTIONS AND NOTATIONS

Let T_n = length of the interval between the nth and the (n+1)th arrival, X_n = nth service time and K_n = number of customers in the system immediately before the nth arrival (n=.., -1,0,1,..). The sequences (T_n) and (X_n) are independent of each other. (X_n) is a sequence of independent r v and $P(X_n \leq x) = 1-\exp(-\mu x)$, while (T_n) is a stationary sequence of Markov-dependent r v, completely described by $A(t) = P(T_n \leq t)$ and $A_1(t|y) = P(T_n \leq t | T_{n-1}=y)$. We will also use the alternative conditional distribution function $A_2(y|t) = P(T_{n-1} \leq y | T_n=t)$. We treat the absolutely continuous case with the probability density functions $a(t) = dA(t)/dt$, $a_1(t|y) = dA_1(t|y)/dt$ and $a_2(y|t) = dA_2(y|t)/dy$. Interpretations for cases where interarrival times are discrete or generally distributed should be obvious.

We denote: $E(T_n) = 1/\lambda$, $\rho = \lambda/\mu$=traffic offered.

3 SOLUTION

From the assumptions follows that (K_n, T_n) is a two-dimensional Markov process. We let $P(K_n=k, t<T_n \leq t+h) = r_k(t)h + o(h)$. Then we have $r_k = P(K_n=k) = \int_0^\infty r_k(t)dt$. Using the appropriate transition probabilities we get:

$$r_0(t) = \int_0^\infty \sum_{j=0}^{\infty} r_j(y)a(t|y) \sum_{l=j+1}^{\infty} q_l(y)dy$$

$$r_k(t) = \int_0^\infty \sum_{j=k-1}^{\infty} r_j(y)a(t|y)q_{j-k+1}(y)dy \quad (k=1,2,..)$$

where $q_s(y) = ((\mu y)^s/s!)\exp(-\mu y)$

The solution is (k=0,1,..):

$$r_k(t) = (C(t))^k(1-C(t))a(t); r_k = \int_0^\infty r_k(t)dt$$

$$C(t) = \int_0^\infty a_2(y|t)\exp(-\mu y(1-C(y)))dy$$

This generalizes the classical case where $a_2(y|t)=a(y)$ and $C(t)=C$ for all t.

4 EXAMPLE

We use a four-point distribution for (T_{n-1}, T_n) with $E(T_n)=1/\lambda$, $Var(T_n)=1/\lambda^2$ and varying $r=Corr(T_{n-1}, T_n)$, and such that $r_k = \rho^k(1-\rho)$ (classical M/M/1 state distribution) if T_{n-1} and T_n are independent ($\Longrightarrow r=0$). This example gives quite simple calculations by which we can study how, for example, the mean waiting time varies when ρ and r are varied.