ABSTRACT

The paper deals with statistical properties of service quality characteristics and capacity of full-available group with repeated calls and partly or completely faulty lines. A new method is applied to study of Markovian teletraffic systems. Values of means and variances of characteristics for lost calls' coefficients, blocking probability and average number of busy lines are obtained. Problems of engineer algorithm constructing to detect non-blocking faults in telephone and telegraph circuits are discussed.

1. INTRODUCTION

At last years due to telephone and telegraph circuits' development it becomes necessary to elaborate and use in practice methods of automatic fault detection in lines and equipment. If faulty lines are not detected, the calls continue coming to them not being satisfactory serviced. Such calls cause repeated ones that additionally load the circuit, decrease service quality and leads to untime wear of equipment. Fault detection method based on measurement analysis of service quality characteristics (lost calls coefficient and blocking probability) and capacity (average number of busy lines) is considered to be economic at last years. To provide high reliability of detection it is necessary to obtain enough accurate characteristics' measurements. To make an optimal plan for the measurement accuracy and duration required means and variances of characteristics should be studied. The paper presents results of such study for full-available group model with partly or completely faulty lines. Some numerical data are also presented.

2. MODEL OF SYSTEM

Consider a full-available group of V lines and a buffer with n < \infty expectation places (see Fig.1). The group of lines can be conditionally presented as consisting of 3 groups with V_k, k=1,3, \(V=V_1, V_2, V_3\) of good, partly and completely faulty lines each correspondingly.

Assume that the faulty lines are not detected and receive calls as good ones. Line occupation in the third group never ends in conversation. Lines of the second group are faulty with probability \(\mathcal{F}\). A Poissonian flow of primary calls of intensity \(\lambda\) comes to the group of lines. The incoming call with probability \(\mathcal{D}_k, k=1,3\), (see later) occupies any line of the k-th groups for preservice time, exponentially distributed with a parameter \(\mathcal{A}\). Preservice time is the mean time from the moment of line occupation up to its liberation due to subscriber non-answer, subscriber line being busy, unsatisfactory service quality. After preservice in the k-th group with probability \(Q_k, k=1,3\), a call leaves the line and with probability \(H_3\) forms one source of repeated calls (SRC) and with probability \(1-H_3\) leaves the system finally. Values \(Q_k\) are determined from equations

\[
Q_r = Q, \quad Q_2 = Q + (1-Q) \cdot \mathcal{F}, \quad Q_3 = 1,
\]

where \(Q\) is mean probability of subscriber non-answer, subscriber line being busy, or call blocking in network sections following considered group. After preservice in k-th group with probability \(1-Q_k, k=1,3\), the line does not become free and the main service - conversation starts. Such service duration is exponentially distributed with parameter equal to one.

Calls, coming at times, when V lines are busy, occupy one place each in the buffer. If there are no free places then the primary call with probability \(H_2\) forms one SRC and with probability \(1-H_2\) leaves the system finally. The call's waiting time in the buffer is bounded by exponentially distributed random value with parameter \(\mathcal{Y}\). Having not received service during this period the call leaves the buffer and with probability \(H_2\) forms one SRC and with probability \(1-H_2\) leaves the system finally. Each SRC sends repeated calls in independent exponentially distributed intervals with parameter \(\mathcal{Y}\). Service algorithm of repeated calls differs from one of primary calls by the fact that blocking repeated call with probability \(\mathcal{H}\) should be repeated and with probabil-
lity 1-H2 leaves the system finally that decreases the number of SRC by a unit. Occupation of line or of expectation place by a repeated call is accompanied by immediate liquidation of one SRC.

3. SYSTEM CHARACTERISTICS AND OTHER RANDOM VALUES

Consider a stepped continuous in the left part process J(\(z\), \(z\geq 0\), with states \(i=(i_1,k_1; k_2,7)\in G\), where

\[ i_1=0,1; i_2=0,1; i_3=0,1; i_4=0,1; i_5=0,V_0 \]

is the number of lines, occupied by preservice in the first, second and third groups correspondingly; \(i_4=0,V_1\), \(i_4=0,V_2\) is the number of lines occupied in the first and second groups correspondingly; \(i_2=0,n\) - is the number of busy places in the buffer; \(i_5=0,n\) - is the number of SRC. It is not difficult to see that the process \(J(\cdot)\) is homogeneous transitive conservative Markovian process.

Consider the most typical system characteristics corresponding to measurement interval \([0,t]\). The characteristics have the form

\[ \mathcal{J}_i(t)=S_{z_i}(t)/S_{f_i}(t), \quad t=1,3 \quad (2) \]

and mean the following. Denote by \(G_r=\{i;i_1=i_2=\in\}\) a subset of states all places in the buffer being busy, in which the incoming calls are blocked. Characteristic \(\mathcal{J}_i(t)\) means lost calls' coefficient in states \(i\in G\). Values \(S_{z_i}(t)\), \(S_{f_i}(t)\) in \((2)\) mean corresponding number of blocking (primary and repeated) calls in states \(i\in G\), and of all the calls incoming during time \(t\). Characteristic \(\mathcal{J}_i(t)\) is loss probability (probability of blocking) of the system, and \(S_{z_i}(t)\), \(S_{f_i}(t)\) in \((2)\) is the duration of stay of process \(J(\cdot)\) in states \(i\in G\), during time \(t\).

Consider a vector of random values \(S(t)=[S_{0}(t),S_{1}(t),...,S_{4}(t)]\) (3) where

\[ S_{1}(t)=S_{z_1}(t), \quad S_{2}(t)=S_{z_2}(t), \quad S_{3}(t)=S_{z_3}(t), \quad S_{4}(t)=S_{z_4}(t) \]

\[ S_{0}(t)=S_{f_0}(t), \quad S_{5}(t)=S_{f_5}(t)=1 \quad (4) \]

Denote by

\[ h_{ij}=[h_{ij}(0),h_{ij}(1),...,h_{ij}(4)] \]

the vector of random values \(h_{ij}(r)\), \(r=0,4\), corresponding to one-step stay of process \(J(\cdot)\) in state \(i\in G\) with condition that the process will transfer into state \(j\in G\). Value \(h_{ij}(0)=h_{0}(0)\) is the duration of one-step stay of process \(J(\cdot)\) in state \(i\), exponentially distributed with a parameter

\[ \lambda_i=\lambda_i/\mu_i + \alpha(1, +i_4 +i_5) +1_{4} -i_4 +i_5 \quad (6) \]

Value \(S_{0}(t), \ r=0,4, \) in \((3)\) is equal to the sum of values \(h_{ij}(r), \ r=0,4, \) at time \([0,t]\).

In \((5)\) values \(h_{ij}(r), \ r=1,2\) are equal

\[ h_{ij}(1)=X_i h_{ij}(2), \ h_{ij}(2)=X_i(\lambda, \mu) \quad (7) \]

where \(X_i\) - is the indicator of subset \(G_i\) blocking states and \(X_i(\lambda, \mu)\) is the process \(J(\cdot)\) transition from state \(i\) to state \(j\) for one step (later we shall denote it by \(i\rightarrow j\)) takes place due to primary or repeated call incoming, otherwise \(X_i(\lambda, \mu)=0\). Values \(h_{ij}(r), \ r=3,4, \) are determined from equation

\[ h_{ij}(3)=h_{ij}(4)=X_i h_{ij}(0), \quad i,j\in G, \]

\[ h_{ij}(h_{ij}(4)=X_i h_{ij}(0), \quad i,j\in G. \]

From \((7)\), \((8)\) it is seen that values \(h_{ij}(r), \ r=0,4, \) corresponding to two-step transition \(i\rightarrow j\) of process \(J(\cdot)\) are independent.

Consider a vector of random values

\[ h_i=[h_i(0), h_i(1),...,h_i(4)], \quad i\in G. \]

Vectors \((5)\) and \((9)\) are connected by stochastic equation

\[ h_i=\sum_{j\in G} X_{ij} h_j, \quad i\in G, \]

where \(X_{ij}\) is the indicator of \(i\rightarrow j\) transition of process \(J(\cdot)\). The indicator has distribution

\[ P_{ij}=Pr(X_{ij}=1)=\lambda_i h_{ij}/\lambda_i \]

\[ Pr(X_{ij}=0)=1-P_{ij}, \quad i,j\in G, \]

where \(\lambda_{ij}\) is intensity of \(i\rightarrow j\) transition of the process \(J(\cdot)\). Values \(h_{i}(0), \ r=0,4, \) in \((9)\) are determined by equations \((6), (8)\). Values \(h_{i}(1), h_{i}(2) \) are determined from equations

\[ h_{i}(1)=X_i h_{i}(2), \ h_{i}(2)=X_i(\lambda, \mu), \]

\[ i\in G, \]

where \(X_i(\lambda, \mu)\) is the indicator of primary or repeated call incoming to the state \(i\in G\) of process \(J(\cdot)\).

Consider the vector of random values

\[ S_{ij}=[S_{ij}(0), S_{ij}(1),...,S_{ij}(4)], \]

\[ i,j\in G, \]

that is obtained by summing of the vectors \((5)\) during the time \(S_{ij}(0)\) of one transition of process \(J(\cdot)\) from state \(i\) to state \(j\).

4. MEANS AND VARIANCES OF CHARACTERISTICS

Introduce the notation

\[ b_{ij}(u)=h_{ij}(x_u)-\theta_u h_{ij}(x_u), \quad i,j\in G, \]

\[ u=1,3, \]

5.3B-4-2
Theorem. If for homogeneous part of Markovian process \( J(\tau) \) the conditions hold:

a) values \( h_j(r), i,j \in G, r=r_{ia}, u=1,q, \) are non-negative;

b) values \( h_j(r), i,j \in G, r=r_{ia}, u=1,q, \) and for each fixed value \( r_{ia}, u \), such value of \( i \) exists that \( Mh_i(r) > 0 \);

c) means \( Mh_i(r) \) are such that \( 0 \leq Mh_i(r) < \infty, i \in G, r=r_{ia}, u=1,q, \) and for each fixed value \( r_{ia}, u \), \( Mh_i(r) \) is asymptotically normal, i.e. \( \{ \lambda \in [\mathbb{L}(t) - \beta_j], u=1,q \} \to \mathbb{N}(0, 1/\nu u, q), (15) \)

where \( \beta_j = L(r_{ia}) / L(f_{ia}), u=1,q, \)

d) \( Mh(r)h_i(f) < \infty, i \in G, r=r_{ia}, u=1,q, \) for then \( t \to \infty \) with probability equal to unit, \( \mathbb{L}(t) = \beta_j, u=1,q, \) and the vector of characteristics \( (\mathbb{L}(t), u=1,q) \) is asymptotically normal, i.e. \( \{ \mathbb{L}(t) - \beta_j \} \to \mathbb{N}(0, 1/\nu u, q), (16) \)

having for fixed value of \( u \) the only solution.

For the model of system \( p \) it is not difficult to make sure that the conditions a, b) of the theorem hold. Find moments for values \( h_j(r) \) and control the realization of conditions c, d). From (5)-(8) we can find

\[ Mh_i(r) = \frac{1}{\nu u} \mathbb{L} \]

\[ Mh_i(r) = \frac{1}{\nu u} \mathbb{L} \]

Formulae for moments \( M[h_i(r)h_i(f)] \), \( r \neq f \), we omit. From (17) and the formulæ

omitted follows that the conditions c), d) of the theorem hold.

From (14), (15), (17) we can find means of characteristics for \( t \to \infty \)

\[ \beta_j = \sum_{i \in E} \mathbb{L}(i) / \sum_{i \in F} \mathbb{L}(i), \]

\[ \beta_j = \sum_{i \in E} \mathbb{L}(i) / \sum_{i \in F} \mathbb{L}(i), \]

Values \( d_{ua} \) in (15) for \( u=\tau_{1,3} \) are the main members in the decomposition of variations of characteristics

\[ D \mathbb{L}(t) = d_{ua} / t + o(1/t), t \to \infty. (19) \]

Means \( Mh_{i1}(r), u=\tau_{1,3}, \) in (15) we can find from (6)-(8), (14)

\[ Mh_{i1}(r) = \mathbb{L}(r) / \mathbb{L}(f), (15), (16) \]

we can find from (14), (17)

\[ Mh_{i1}(r) = \mathbb{L}(r) / \mathbb{L}(f), (15), (16) \]

Denote by \( X_k, k=2,4, \) the indicators of stay of process \( J(\tau) \) in states \( i \in G_k, k=2,4, \) where

\[ G_2 = \{ i: i \neq 3 \}, G_3 = \{ i: i \neq 3 \} \]

and by \( \lambda_k \) vector

\[ \lambda_k = \{ \lambda_k = 1, i_0 = 1, l_1, \theta \} \]

Then equation system (16) for \( Mh_{i1}(r) = \mathbb{L}(r), u=1,3, \) has the form

\[ \mathbb{L}(r) = \mathbb{L}(f) / \mathbb{L}(f), (15), (16) \]

\[ \mathbb{L}(r) = \mathbb{L}(f) / \mathbb{L}(f), (15), (16) \]

5.3B-4-3
\begin{align}
+&q_n x_n (i+1, -l) + \alpha (1, q_n + q_{n+1}) (1 - x_n H_n) x_n (i+1, -l) + \alpha (1, q_n +
+&q_n q_{n+1} H_n) x_n (i+1, -l) + \alpha (1 - x_n H_n) x_n (i+1, -l) + \\
+&q_n x_n (i+1, -l) + \alpha (1, q_n + q_{n+1}) (1 - x_n H_n) x_n (i+1, -l) + \alpha (1 - x_n H_n) x_n (i+1, -l) + \alpha (1 - x_n H_n) x_n (i+1, -l)
\end{align}

Members without sense in (23) should be omitted. Stationary probabilities \( P(i) \) are determined from the equation system

\[ P(i) - \sum_{j \in G} A_{ij} P(j) = 0, \quad \sum_{i \geq 0} P(i) = 1. \] (24)

Intensities \( \lambda_j \); in the evident form are not difficult to write down, if evident form of intensities \( \lambda_{ij} \) from equation system (23) is used. We omit the full writing of system (24). Equation systems (23), (24) were computed by the method of successive approximate Gauss-Seidell (it is proved that this method is convergent to systems (16), (24) with approximately the same rate).

5. NUMERICAL RESULTS

Computed numerical results are presented on Fig. 2-7. On Fig. 2, 4, 6 curves of values of means \( \Delta_1 \), \( u_1 \), \( u_3 \), and on Fig. 3, 5, 7 of variances \( d_{u_1} \), \( u_1 \), \( u_3 \), of characteristics \( \theta_1 (t) \), \( u_1 \), \( u_3 \). The first curve on Fig. 2-7 corresponds to the following set of parameters of the model:

\( V=V_1=4, n=0, \mu=5, \alpha=8, H_1=H_2=H_3=0.83, \) \( Q=0.5 \).

Let us point for curves 2-4 the values of parameters that are not equal to the ones of the curve 1: the second curve - \( n=0, H=0.83 \);
the third curve - \( V=V_3=4 \);
the fourth curve - \( V=V_3=4, n=0, H=0.83 \).

On Fig. 2-7 it is seen the great difference between the means' and variances' values in the systems without faulty lines (curves 1, 2) and in the systems with completely faulty lines (curves 3, 4). Such difference can be used for the systems automatic non-blocking fault detection in telephone circuit. For this purpose any of characteristics \( \theta_1 (t) \), \( u_1 \), \( u_3 \), may be used and faith interval

\[ \theta_1 \pm \frac{u_{\alpha}}{\theta_1} E_\alpha = \theta_1 \pm \frac{u_{\alpha}}{\theta_1} E_\alpha \]

may be built, where \( u_{\alpha} \) is twosided \( \alpha \) - quantil of normal distribution \( N(0,1) \) (for example, for \( \alpha=0.95 \) \( u_{\alpha}=1.96 \)). For the given absolute \( E_\alpha \) (or relative \( \Delta_\alpha \) \( E_\alpha / \theta_1 \)) width of the faith interval measurement duration \( T_\alpha \), \( u=1,3 \), can be found by formula

\[ T_1 = \frac{u_{\alpha}^\Delta}{\theta_1} \]
6. CONCLUSION

The authors suppose that the full analysis of numerical results, partly presented on Fig.2-7, makes possible to build a convenient engineering algorithm for non-blocking fault detection in the telephone and telegraph circuits. Using the method of statistical study of teletraffic Markovian systems, presented above, admits the investigation of the other interesting models of the systems.

REFERENCES