ITC 11, Minoru Akiyama (Editor)
Elsevier Science Publishers B. V. (North-Holland)

ON BOUNDS OF RANDOM NUMBER GENERATORS WITH FINITE CYCLE LENGTH FOR TRAFFIC SIMULATION

J. SÄGEBARTH

INSTITUTE FOR ELECTRONIC SYSTEMS AND SWITCHING, UNIVERSITY OF DORTMUND

GENERAL CONSIDERATIONS

This paper deals with properties of random-number-generators (RNGs) used in traffic simulations, and with new dimensioning methods for such generators with regard to the mutual independence of the generated random numbers (RNs) and the random events (REs) in the actual simulations, respectively. The methods presented, which can be only summarized here, will be treated in more detail in separate papers /1,2,3/. From the properties of RNGs with finite cycle length (CL) it can be concluded that the REs which are determined from the RNs produced by such a generator can only be mutually independent, if the number of events in the actual simulations does not exceed a certain limiting value.

In a first method, an upper limit is determined for the maximum number of independent random events (ire) for arbitrary generators with finite CL. This value ire depends only on the CL of the used generator but not on the chosen generating algorithm. In simulations with the usual assignments of RNs to REs, however, the actual admissible maximum number of REs can in most cases be considerably lower than this upper limit.

GENERATING ALGORITHMS

Usually a new RN y_k is a function of the preceeding r RNs, i.e.:

$$y_k = f(y_{k-1}, \dots, y_{k-r}) \mod m .$$

If a fast generating algorithm is desired, an algorithm of the following type is often used:

$$y_k = a\, y_{k-1} \mod 2^e .$$

In this algorithm n succeeding RNs always form a lattice after some simple transformations /1/.

Based on the use of RNGs with a lattice structure, a generalized simulation model (GSM) which is commonly used can be defined as follows:

- o only neigbouring values of RNs can be comprised in a RE.
- o the sequence of each j-th selected RN is desired to be statistically independent.

For such GSMs, a second upper limit mnoc (maximum number of classes) can be derived /1/ which is considerably sharper than the limit ire. This limiting value mnoc is very suitable for dimensioning RNGs such that the independence of the REs in a simulation, and, in consequence, the accuracy of the simulation results are guaranteed.

SIGNIFICANCE OF STATISTICAL TESTS AND VALUES

Up to now statistical tests like POKER-test, RUN-tests etc. are the most used tools to qualify RNGs. Any statistical test represents a special application of the RNG which can be considered as a special simulation. As, however, the application of the RNs in such a test can differ considerably from the application in user simulations, it can easily be shown /1/ that, in general, such statistical tests are of little significance for qualifying RNGs.

Other methods often used for qualifying RNGs are based on statistical values like serial correlation of pairs. For mixed congruential RNGs with not interlocked hyperplanes it can be shown /1/ that the correlation factor can be adjusted to arbitrary values within a wide range, even to zero by means of an additive value. It is wellknown that this additive value has low significance to the maximum number of classes. This shows that statistical values like serial correlation of pairs are also not suited for qualifying RNGs.

CONCLUSION

To get simulations more reliable it is suitable to use a monitor watching the critical access distances. If each distance is known the value mnoc can be calculated. In user-simulations the minimal interval of RNs forming a RE must be greater than 1/mnoc. If this cannot be accomplished with the generator used the simulation must be carried out with a different generator which is dimensioned optimally for this application.

Any method which destroys the lattice, like scrambling RNs, coupled generators, etc., leads only to structures with uncalculable properties but does not lead to better results than an optimized generator with equivalent CL. Consequently, the user has lost a valuable tool for checking the accuracy of simulation results.

The presented methods enable an efficient implementation of RNGs in simulation programs and in complex universal simulation systems. They offer new facilities to get simulations more reliable.

REFERENCES

/1/ J. Sägebarth: Über Eigenschaften und Dimensionierung von rekursiven Zufallszahlengeneratoren und ihre Anwendung in der Verkehrssimulation. Ph. D. Thesis, University of Dortmund, 1985.

/2/ H. D. Ide, J. Sägebarth: On Properties of Random Number Generators and their Influence on Traffic Simulation. 10th ITC Montreal, 1983, and AEÜ 38, 3, 1984.

/3/ J. Sägebarth: Über Methoden zur Steigerung der Zuverlässigkeit bei der Simulation stochastischer Prozesse. 3rd GI/NTG Technical Conference, Dortmund, Oct. 1-3, 1985 (to be published).