DIMENSIONING ALTERNATE ROUTING NETWORKS
WITH OVERLOAD PROTECTION

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ABSTRACT

In hierarchical alternate routing networks traffic overload protection methods are to be used for protecting the traffic offered directly to the final routes. The paper presents a concept for direct dimensioning of the optional groups so that the cost and overload performances are in optimum relation. Taking into account the modularity aspects and full-grouping considerations a unified theorem and optimum design criteria are derived, as an extension of the theorem and criteria developed for cost-optimal dimensioning. The presented criteria are available to compile securized THF (tandem, high-usage, full-group) design diagrams to choose the type and size of the optional group concerned under a given nominal load and overload ratio. The cost-optimal and securized solutions and THF diagrams are extensively compared.

INTRODUCTION

In hierarchical alternate routing networks the transversal, optional circuit groups are established generally to provide cost reductions. Normally they are high-usage groups with overflow onto another optional or a final circuit group. The rapidly expanding use of digital switching and transmission facilities necessitates a reconsideration of dimensioning methods taking into account the digital peculiarities and involving the service protection requirements. The impact of the digital peculiarities, as 24 or 30 channel modularity of the circuit groups, preference of the both-way trunking over one-way trunking, etc. are handled in many papers (1 - 7).

As the minimum permitted size of the digital optional groups is greater than the minimum permitted group sizes in analogue networks, the number of economical optional groups will decrease and the established - final and optional - groups will be larger. This tendency is supported by the change of cost parameters, i.e. by the decrease of the line cost and tandem switch cost due to the digitalization (2, 3, 8). Thus, in general the economically optimum digital network will have a more vulnerable network structure than the economically optimum analogue network has. Therefore the network dimensioning should involve service protection methods, against traffic overloads and component failures. Regarding the overloads in hierarchical networks, preventive methods are to be used for protecting the traffic offered directly to a final route, as

a) limitation of the marginal load on the final groups (2, 9, 10),

b) splitting of the final groups into an overflow subgroup and a protective subgroup applied to the directly offered traffic portion only (10, 11, 12),

c) forced full-grouping of the optional groups to prevent very peaked traffic from overflowing (2, 4, 6).

Generally, a higher overload protection can be obtained by using high-usage groups of larger size than the size relating to the least-cost solution at the nominal load.

Usually we try to find a least-cost solution to a predetermined combination of the protection methods and use an iterative procedure including sizing and protection analysis phases [9, 13, 14]. In the following we present an efficiency concept and a procedure for direct sizing of the optional groups where the extra cost due to the scaling up of the optional groups and overload performances at a given overload ratio are in optimum relation. Taking account of the modularity aspects and full-grouping considerations a unified theorem and optimum criteria are derived. The criteria are available to construct decision diagrams to facilitate the design of optional groups with an efficient overload protection. These "securized" THF diagrams as an efficiently securized version of the cost-optimal THF diagrams provide the classification of the optional group tandem routing (T), high-usage group with overflow (H) and full-group without overflow (F), and the number of circuit modules optimum to a given nominal offered load, overload ratio and desired traffic performance level calculated from the alternate routing pattern. We point out some features of the securized modular THF diagrams to clarify some alternate routing and overload protection problems.

1. THE PRINCIPLE OF THE SIZING METHOD

Usually, the high-usage groups are dimensioned to least-cost; an optimum number of circuits and overflow probability are given at the nominal load. Overloading such a high-usage group, the overflow probability and the overflow traffic onto the final group are increased. The overload protection from the aspect of the sizing of the optional groups may be defined with the aid of excess overflow traffic, which causes an undesired increase of the grade-of-service over the final group.

Considering a high-usage group with N circuits nominal offered load M and overload ratio \( r \), and denoting the congestion function by \( B(N, M) \), the overflow traffic is \( U = M \cdot B(N, M) \) under nominal case, \( U = r \cdot M \cdot B(N, M) \) under overload condition.

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The protection level at overload ratio $\tau$ can be defined as

$$S(N) = \frac{U-U}{\tau M} = \frac{Y+U}{\tau M} = 1-B(N,\tau M) + \frac{B(N,M)}{\tau},$$  

where $U$ is the excess overflow traffic, $Y = (N-M)U$, is the carried traffic of the optional group under overload condition. Obviously $S(N)=1$ if $\tau=1$, for higher $\tau$ values $Y>Y(\tau=M-U$, $S(0)=1/\tau$, $S(1)=1$ and $S(N)/\tau=n=0$. Thus if the size of the concerned optional group is increased, the overload protection is improved, i.e. the excess overflow traffic is decreased (the carried overflow traffic over the high-usage group is increased). This overload protection consideration stimulates to have an optional group with a larger size than that relating to the least-cost solution at nominal load. The overall cost of handling $M$ Erlangs by an optional high-usage circuit group of size $N$ and a tandem route of marginal capacity $m$ can be estimated as

$$C(N) = N \cdot c_N + \frac{M B(N,M)}{\alpha} \cdot c_c$$

where $c_N$ is the cost per circuit of the concerned optional high-usage group, $c_c$ is the average cost per circuit in the overflowing network of the high-usage group. The least-cost continuous solution is given at size $N_c$, where:

$$\sum_{n=1}^{N_c} M \cdot B(N_c, M) = c_N \cdot c_c$$

with simple notations:

$$d(N_c, M) = L_1,$$

where $d$ is called marginal occupancy, $L=8c_N/c_c$ is called the desired traffic performance level. To find the most efficient solution we define a qualification factor $Q$, expressing the ratio of the protection level and the relative cost of handling traffic load $M$, as:

$$Q(N, M) = \frac{S(N)}{C(N)/C(0)} = 1-B(N,\tau M)+B(N,M)/\tau$$

Maximizing $Q$ according to $N$ under $\tau<1$, $N_c N_c$ is yielded, so that

$$S(N) C(N)/C(0) = 0 \quad \text{if} \quad N_c N_c,$$

Thus the scaling up of the cost-optimal optional group to $N_c$ results in an efficient improvement in protection level, the additional circuits provide greater improvement in protection level than the relative increment in the overall cost. If $\tau=1$, then $N_c = N_c$ (Fig. 1).

In the following taking into account the modularity (that the size of the circuit group is to be in multiple of a fixed module size $m$), the permitted loss probability of the optional groups $B_0$, as well as the feasible minimum group size

* The marginal capacity is the additional traffic that can be offered to the route by adding a circuit under condition of constant blocking. The marginal capacity of a circuit group is a theoretical maximum value, depending on the mean and the peakedness of the offered traffic, which is usually not fully utilized in the course of the dimensioning, but a practical constant value ($0.7 \ldots 0.9$) is taken.
Because $a^*(n, M)$ is decreasing versus $n$, and $d^*(n, M) = d^*(n, M) \Delta d^*(n, M)$ if $\forall 2 \times 1 > 1$, then we have that $n_{o} \leq n_{r} < n_{x}$.

3. FULL GROUPING

If the permitted loss probability of an optional group is $B_0$, then we have an upper limit for the number of circuits and modules, $n_k$ and $n_{o} - \text{ent}(n_{o}/m)$ respectively. To exploit the permitted increase of the point-to-point congestion we may establish a fully provided group without overflow. These so-called full-groups provide an overload protection for final routes, the peaked traffic being prevented from overflowing, $WU=0$. Thus we can define $S(n_o) = S(n_o) = 1$, and $Q(n_o) = M$.

(1) Taking account of the possibility of full-grouping a high-usage group, we obtain the efficient solution, where $n = \max_{n=0, 1, \ldots, n_0} Q(n) = \max(Q(n_o), Q(n_o))$.

Generalizing the partial average capacity [4]

\[ \gamma(n, n_o, n, M) = \frac{\text{MB}(n)}{n \cdot (n_o n)} \]

in the form of

\[ \gamma(n, n_o, n, M) = \frac{\text{MB}(n)}{n \cdot (n_o n)} = \gamma(n, n_o, n, M) \]

we obtain that $Q(n_o) \geq Q(n)$ for all $n < n_0$, i.e. it is efficient to establish full group, if:

\[ \gamma(n, n_o, n, M) \rangle L \text{ for all } n < n_0. \]

Introducing

\[ \gamma_{\max}(n_o, M) = \min \{\gamma(n, n_o, n, M) | n \} \]

where obviously $n_v(n_o, n) < n_0$, we can write (7) as

\[ \gamma_{\max}(n_o, M) \rangle L. \]

With the parameters $n_o$ and $M$ omitted, if $\forall 2 \times 1 \rangle 1$, then $\gamma(n) \rangle \gamma_{\max}(n_o, M)$. Thus $\text{MB}(n) \rangle \gamma_{\max}(n_o, M) \text{ and } \text{MB}(n) \rangle \gamma_{\max}(n_o, M) \text{ and } \text{MB}(n) \rangle \gamma_{\max}(n_o, M)$.

We can observe that the function $\gamma(n, n_o, M)$ starts from $\gamma(n, n_o, M)$, the average load of the circuits of the full group under overload condition, i.e.

\[ \gamma(n, n_o, M) = \gamma_{\min}(n, n_o, M) \leq \gamma(n, n_o, M) \]

and the relationship between $\gamma(n, n_o, M)$ and $\gamma(n, n_o, M)$ functions, as follows:

\[ \gamma(n) \geq \gamma(n-1) \geq \gamma(n) \]

Consequently if $\gamma(n) = \gamma_{\max}(n)$, then the function $\gamma(n)$ will monotonously increase and $n \leq 0$. In these cases the function $\gamma(n)$ dominates over $d^*(n)$ for all $n$, a full group provides better qualification than a high-usage one. The condition $\gamma(n) \rangle d^*(n)$ is obviously found for $n \leq 0$ or at extremely large offered load $M$. For example, if $\forall 1 \rangle 1$ and the offered load is of random type this is the case when $n_0 \leq \gamma_{\max}$. In other cases, if $\gamma(n) \rangle d^*(n)$, then $\gamma(n)$ is decreasing to $\gamma(n)$. Here from (9) we find that

\[ \gamma(n) \rangle d^*(n) \geq \gamma(n) \geq d^*(n-1) \]

Since after the intersection $\gamma(n)$ is monotonously increasing, dominates over $d^*(n)$, thus it is efficient to scale up a high usage group to satisfy $B_0$.

In sum, the full grouping involves the high-usage groups of size $n > n_r(r)$, and the ones of size $n_r(r)$ for which $\gamma(n) > d^*(n)$ is decreasing versus $r$.

For practical way to full grouping, we express from (7) $n_o = \text{ent}(n_{o}/m)$, the size of the full group, which would be desirable to the efficient full-grouping of a high usage group of size $n$ modules. We obtain:

\[ n_o = \frac{1}{S(n_o)} \left( \frac{\text{MB}(n)}{L} \right) \]

Thus, after determining $n_o$, the optimum number of high-usage modules, we try to find a number of modules not higher than $n_o$, which satisfies the congestion objective $B_0$. If the congestion of the circuit group of size $n_o$ modules is greater than $B_0$, then the efficient full-grouping is not possible.

The cost penalty in case of efficient full-grouping of a cost-effective high-usage group of size $n$ modules, i.e. when $Q(n_o) \geq Q(n_o)$ is given as:

\[ C_v(n_o) \leq \frac{1}{C_v(n_o)} \leq 1 \]

Consequently, choosing the value of $r$, the overload ratio we limit the extra cost.

4. TANDEM ROUTING

It is usual to set a lower limit $n_{min}$ below which an optional group will not be established. This means that we try to find the maximum of $Q(n)$ with respect to $n_o = n_{min}, n_{min} + 1, \ldots, n_0$ and makes necessary the conversion of circuit groups having 1, 2, ..., $n_{min}-1$ modules according to (5) and (6) into a high-usage group of $n_{min}$ modules or a full group of $n_o = \max(n_o, n_{min})$ modules or the offered traffic will be routed on tandem route ($n_0$).

To evaluate the tandem routing, the qualifications $Q(n) = 1/r, Q(n_{min})$ and $Q(n_{max})$ can be compared.

The tandem route is more efficient than the high-usage group if $Q(n) > Q(n_{min}), i.e.$

\[ L \cdot \gamma_{M}(n) = Q(n_{min}, M) = \gamma_{min}(n_{min}, M) \]

where $\gamma_{min}(n_{min}, M)$ means the average occupancy of the circuit group of size $n_{min}$ under overload condition, as an extension of $\gamma_{max}(n_{min}, M)$, the average occupancy.

When $n_{min}$, then $\gamma_{min}(n_{min}, M) = d^*(n_{min})$, otherwise $\gamma_{min}(n_{min}, M) > d^*(n_{min})$, i.e. there is a range in $L$, where the rounding up to $n_{max}$ modules is advantageous. Seeing that $\gamma_{min}(n_{min}, M) > \gamma_{max}(n_{min}, M)$, the overload protection stimulates to establish a high-usage group against tandem routing.

Comparing to full group, the tandem routing is more efficient if $Q(n) > Q(n_{max})$, i.e.

\[ \gamma_{M} = \frac{1}{n_{max}} = Q(n_{max}, M) = \gamma_{max}(n_{max}, M) \]

where $\gamma_{max}(n_{max}, M)$ is the average offered traffic under overload condition, as an extension of $\gamma_{max}(n_{max}, M)$, the average offered traffic.

\[ \gamma_{max}(n_{max}, M) = \gamma_{max}(n_{max}, M) \]

where $\gamma_{max}(n_{max}, M)$ is the average offered traffic under overload condition, as an extension of $\gamma_{max}(n_{max}, M)$, the average offered traffic.
significantly emphasizes the full-grouping against tandem routing.

The minimal size \( \min_1 \) and the permitted loss probability \( B_0 \) determine a traffic limit \( \min_1 \) so that \( B(\min_1, \min_1) = B_0 \).

So, if \( M < \min_1 \), then \( \min_1' = \min_1 \), otherwise \( \min_1' = \min_1 \).

Summarizing the relations (10) and (11) we can write that the tandem routing is efficient, if

\[ L > \max(\min_1'(\min_1, \min_1, MH); \min_1(\min_1, \min_1, TM)) \]  

(12)

Accordingly, the criterion of tandem routing under enhanced overload protection is equivalent to the criterion calculated at \( \gamma \) times traffic load. The right side of the relation (12) gives a value of \( \gamma M/\min_1 \) in the domain \( M < \min_1 \) (otherwise it is less). Hence if \( M < \min_1/L/\gamma \), then the traffic load \( M \) should be tandem routed.

Introducing the limit \( \min_1 \), the criterion to establish full group has been directly effected according to (11). Generally we must find the minimum of \( \min_1'(n) = \min_1'(n, \min_1, MH) \) with respect to \( n=0, \min_1, \min_1+1, \ldots \). Cancelling the not permitted group sizes instead of (8)

\[ \gamma \min_1'(n, \min_1, MH) \]  

is valid, i.e. the criterion to establish full-group against a high-usage group is also influenced. Taking account of the relationship

\[ \gamma \min_1'(\min_1) \geq \gamma \min_1'(\min_1) \geq \gamma \min_1'(\min_1) \]  

(13)

which can be easily proven, and that

\[ \gamma \min_1'(n, \min_1, MH) = \gamma \min_1'(n, \min_1, MH) \] when \( n = \min_1 \) we have, with notation \( P(\gamma \min_1'(\min_1)) \):

\[ \gamma \min_1'(\min_1) \geq \gamma \min_1'(\min_1) \geq \gamma \min_1'(\min_1) \]  

(14)

where the conditions are corresponding to \( n=0, \min_1, \min_1 \) and \( n=\min_1, \min_1 \) respectively. Additionally, the criterion of tandem routing (12) can be written as

\[ L > \left\{ \begin{array}{ll} \min_1'(\min_1, \min_1, TM) & \text{if } n=0 \\ \min_1'(\min_1, \min_1, TM) & \text{if } n=\min_1, \min_1 \end{array} \right. \]  

(15)

If \( \min_1 = \min_1 \), i.e. \( M = MH \) then we have the case \( n=0 \) and \( \gamma \min_1'(\min_1, \min_1) \). If \( \min_1 < \min_1, \min_1 \), i.e. \( M > MH \) then \( P(\gamma \min_1'(\min_1)) \) and \( \gamma \min_1'(\min_1, \min_1) \).
5. SECURIZED THF DIAGRAMS

As a result of dimensioning taking the protection considerations also into account, the optional groups are not established (tandem routing T), or it is realized as a high-usage group with overflow (H) or as a non-overflowing full-group (F).

Evaluating \(\max\{e_{m,n}(n_0,\tau M); n_{m,n}(n_0,\tau M)\}\) versus \(M\), a curve \(T^*(M)\) is derived, which represents maximum overload capability of optional groups with \(n > n_{m,n}\), taking the value of \(B_0\) into account. \(T(M)\) is the limit curve of tandem routing. Calculating \(e_{m,n}(n_0,\tau M)\) versus \(M\), a curve \(F^*(M)\) can be given, which provides capability of the optional groups of size \(n > n_F\) scaled up to full-group of size \(n_0^*\), taking account of the change in overload protection. \(F^*(M)\) indicates the limit of the full-grouping. Drawing the curves \(T^*(M)\) and \(F^*(M)\) on \(L-M\) plane, three domains are designated: above \(T^*(M)\) the domain of \(T\)-class, between \(T^*(M)\) and \(F^*(M)\) the domain of \(H\)-class, below \(F^*(M)\) the domain of \(F\)-class. Also drawing \(e_{m,n}(n_0,\tau)\) for \(n = n_{m+1}, \ldots, n_F\) in the \(H\)-domain a so-called securized THF diagram is compiled. The diagram gives the efficient routing of the offered traffic \(M\), and the number of modules at a given module size and overload ratio \(\tau\) and at any desired performance level \(L\). Figs. 2-6 present THF diagrams for some practical cases. All diagrams suppose a blocking objective \(B_0=0.01\) and a pure-chance offered traffic \(M\). Figs. 2-3 are related to the unit-size module with \(N_{m,n}=6\). Figs. 4-6 present diagrams for module size \(m=30\) with \(n_{m,n}=1\), which are typical to the design of two-way digital optional groups.

For comparison we show diagrams for \(\tau=1\) (cost-optimal solution [4]), and \(\tau=1.2\), as well as the \(F^*(M)\) and \(T^*(M)\) curves for various \(\tau\) values under \(m=30\) in Fig.6.

We can see that the curves \(F^*(M)\) have saw-toothed form; a lead is at load \(M\) if \(N_0 = N(M,B_0) = n_0^*\) independently of \(\tau\). The lead is increasing with \(m\) and \(\tau\), and if \(e_{m,n}(n_0^*) \leq 1\), we have got \(F\)-domain at any \(L\) in a certain domain of the load \(M\).

The curves \(T^*(M)\) and \(F^*(M)\) coincide in certain traffic ranges. From (13) and (14) we obtain that coincidence is at a traffic load \(M\), if and only if

\[ a_{m,n}(n_0^*) \geq a_{m,n}(n_{m,n}) \]  

(15)
and then $T'(M) = F'(M) = \alpha(n_1)$. 

(Otherwise $T'(M) = \eta_1(n_1) > \alpha(n_1) > F'(M)$.)

The condition (15) can be written as 

$$n_1 > \eta_1(1-B(n_1,n_1,M)),$$

(16)

Analysing this condition we can define an upper and a lower critical load $M_u$ and $M_*$ respectively, so that $T'(M) = F'(M)$ for $M < M_u$ and $M > M_*$, hence $H$-domain is found only if $M_u < M < M_*$. Condition (16) is obviously satisfied in the range $0 < M_*$, because $n_1 = n_1(n_1,M)$. In general, $M_u > G_u$ only if $n_1(n_1,M) > n_1$. If $M$ tends to infinity, $\eta_1(n_1)$ tends to 1, $\alpha(n_1)$ approaches $1/(1-B_0)$, consequently $M_u$ exists necessarily. With the aid of the asymptotical features of the congestion function we obtain from (15) approximately $T'(M) = n_1(n_1,M) + 1$. If $m=1$ then $M_u = 285$ erlangs, if $n_1 = 30$, $M_2 = 285$ erlangs are given, under $T=1.2$, $B=0.01$. Studying Fig. 6 from practical point of view, above 170 erlangs F-domain is found. If $T=1.5$ we can consider pure F-domain above 60 erlangs (Fig.6). Fig.6 also shows that the $T'(M)$ curves can be well approximated as:

$$T'(M) \approx \min(n_1(n_1,M), 1).$$

6. CONCLUSION

The 24 or 30 channel modularity of the circuit groups is an essential feature of digital network planning, and has significant impact on network planning, mainly due to the security requirements. The paper presented unified criteria for optimum modular engineering an optional circuit group in an alternate routing network, involving overload protection considerations. The developed criteria are the extension of the criteria of the cost-optimal engineering. They are based on an efficiency concept and give an optimum combination of overload protection provided and extra cost required from increasing the size of the optional group concerned. The criteria are valid for random and non-random traffic load and available to construct securized modular THF design diagrams for calculating the number of circuit modules optimum to a given nominal load and overload ratio under an alternate routing pattern. It is found that main features in the design criteria of the generalized, securized dimensioning.

REFERENCES


