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# APPROXIMATION TECHNIQUES FOR QUEUEING SYSTEMS WITH FINITE WAITING ROOM

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The paper presents a short overview of two approximation techniques that may be appropriate to estimate performance measures of queueing systems with non-Poisson offered traffic and finite queue. The techniques evolved from similar techniques originally designed for pure loss systems [1,3], recently generalized to systems with infinite queue [4].
Consider the queueing system GI/M/N/N+Q, in which renewal traffic is offered to N exponential servers and a finite number Q of waiting places. Customers that arrive when the total system capacity N+Q is fully occupied are lost from the system and do not return. We report a study on approximation techniques to estimate the performance measures of the GI/M/N/N+Q-queueing system, viz., the blocking probability B, the waiting probability W and the mean waiting time T, based on non-lost arrivals. In the approximation models offered traffic is supposed to be sufficiently described by its mean M and its peakedness Z (Z=V/M, where V represents the variance). The approximations are such that the performance measures of the original GI/M/N/N+Q-system are expressed in terms of the corresponding measures B', W' and T' of an adjoint system with Poisson input, viz., the Markovian queue M/M/N'/N'+Q' (quantities in the adjoint system are indicated with ' in the sequel). The adjoint system provides us with exact formulas [2] that can be advantageously expressed in terms of Erlang loss functions. Fast algorithms are available to compute the Erlang formula for integral as well as nonintegral values of N'. In order to evaluate the approximations, exact analysis of GI/M/N/N+Q [5], in general complicated, has been performed for hyperexponentially and gamma distributed interarrival times in case of peaked (Z>1) and smooth (Z<1) offered traffic, respectively.
The perfomance measures B', W' and T' of the adjoint system, with offered Poisson traffic M', are:
(1) $B'=b(M',N',Q')$, $W'=w(M',N',Q')$, $T'=t(M',N',Q')$,
where the functions b, w and t are known [2].
First, we introduce an approximation technique that is a new application of the decomposition method as explained in [3]. The approximation formulas are ('decom' in the example):
(2) $B \simeq Z.B' = Z.b(M',N',Q')$,
(3) $W \simeq \alpha.Z.W' = \alpha.Z.w(M',N',Q')$,
(4) $T \simeq Z.T' = Z.t(M',N',Q')$, where
(5) $M'=V$, $N'=N-M+V$, $Q'=Q$, $\alpha=N/N'$.
Formulas (2) and (4) follow from the rather simple decomposition argument [3,4]. The introduction of $\alpha$ in (3), however, needs a more complicated argument. Approximation (3) with $Q'=\infty$ and $\alpha=1$, for the class of pure waiting systems $GI/M/N/\infty$, has been envisaged in [4]. Also in this case $\alpha=N/N'$ gives a better estimate to the waiting probability.
Second, in the Fredericks-Hayward approximation to pure loss system [1], the notion 'equivalent congestion' can be generalized to 'equivalent delay'. The transformation formulas are ('EDM' in the example):
(6) $B \simeq B' = b(M',N',Q')$,
(7) $W \simeq W' = w(M',N',Q')$,
(8) $T \simeq T' = t(M',N',Q')$, where
(9) $M'= M/Z$, $N' = N/Z$, $Q' = Q/Z$.
For the class of pure waiting systems $GI/M/N/\infty$, approximation (7) with $Q'=\infty$ has also been introduced in [4].
Third, we take into consideration a Hayward-variation of (6), recently published by Whitt [6], where:
(10) $M' = M/Z^2$, $N' = N/Z^2$, $Q' = Q/Z$.
We present several numerical results of these approximations and exact analysis for Z<1. It can be seen that the approximations perform well (and often extremely well) in a large range of the server occupancy $\rho=M/N$ (low and heavy traffic).

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Example: N=25 Q=5 Z= .8

| ρ | blocking probability B | | | | waiting probability W | | | mean waiting time T | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | exact | decom | EDM | Whitt | exact | decom | EDM | exact | decom | EDM |
| 0.6 | .0001 | .0001 | .0001 | .0000 | .0047 | .0057 | .0053 | .0003 | .0004 | .0004 |
| 0.7 | .0012 | .0014 | .0013 | .0007 | .0334 | .0359 | .0359 | .0026 | .0032 | .0030 |
| 0.8 | .0081 | .0086 | .0085 | .0064 | .1236 | .1252 | .1284 | .0110 | .0125 | .0122 |
| 0.9 | .0301 | .0304 | .0306 | .0268 | .2788 | .2781 | .2848 | .0289 | .0317 | .0310 |
| 1.0 | .0715 | .0708 | .0717 | .0677 | .4418 | .4427 | .4483 | .0530 | .0583 | .0560 |
| 1.1 | .1258 | .1245 | .1259 | .1228 | .5581 | .5641 | .5649 | .0775 | .0866 | .0812 |
| 1.2 | .1838 | .1824 | .1837 | .1818 | .6181 | .6284 | .6247 | .0984 | .1126 | .1026 |