ABSTRACT

In a small zone system, the algorithm used to make the channel assignment for a call has a great effect on system performance. The efficiency of the channel usage in the dynamic channel assignment method at low blocking probability is superior to a fixed channel assignment method. At high blocking probability, the inequality of the efficiency is reversed. In this paper, the cause of the above results is theoretically studied. And it is shown that an adaptive channel assignment algorithm is necessary to maintain the high efficiency of the channel usage at any blocking probability. And also, it is shown that rearrangement of channels produce a more increased channel occupancy at high blocking probability than a usual dynamic channel assignment algorithm. At last, a simple adaptive channel assignment algorithm is proposed.

1. INTRODUCTION

The frequency bands in mobile radio communication systems are limited. Thus the efficient use of the frequency spectrum is one of the most important problems in the system. In a small zone (cell) system, it is possible to reuse the same channel (frequency or time slots) at the same time in the other zones, and the algorithm used to make the frequency assignment for a call has a great effect on system performance (traffic characteristics). That is, in the system, the telephone traffic-carrying capability varies with the method of channel assignment. The channel assignment methods are roughly classified into two types. One is a fixed channel assignment method and the other is a dynamic channel assignment method. It is known by computer simulations that the efficiency of the channel usage in the dynamic channel assignment method at low blocking probability is superior to a fixed channel assignment method. This means that at high blocking probability, a fixed channel assignment method is superior to a dynamic channel assignment method. We have not seen clearly why the inequality of channel usage of a dynamic channel assignment algorithm and a fixed channel assignment algorithm reverses. In this paper, we study theoretically the cause of the reversion of the inequality. If the blocking probability in the reversion, namely, the reversion point, rises, the efficiency of the channel usage is improved. We show that rearrangement of channels has a great effect on rising the reversion point. And also, we show that an adaptive channel assignment algorithm is necessary to maintain the high efficiency of the channel usage at any blocking probability. Furthermore, a simple adaptive channel assignment algorithm is proposed and a computer simulation results of a mobile radio communication system using this algorithm is presented.

2. THE CHANNEL ASSIGNMENTS

2.1 The Channel Assignment Methods

In a small zone mobile radio communication system, a service area is divided into a number of small zones. The method of the division depends on the property of the service area. If a channel frequency is issued in a zone, it may not be used in some other zones because of interference of the frequency. If the buffer zones of a given zone consist of all zones closer than n zones away from it, the buffering system is called the n-belt buffering. This means that a channel reuse interval is n+1.

There are some methods of channel assignment in a mobile radio communication system. They are roughly classified two types. One is a fixed channel assignment and the other is a dynamic channel assignment. In a fixed channel assignment method, a subset of channels available to the system is permanently reserved for use within each coverage zone. In the most general form of dynamic channel assignment, any channel can be used in any coverage zone. In the others (this may be contained in the class of the dynamic channel assignment methods), there is a hybrid channel assignment method containing both fixed channels and dynamic channels. And various algorithm of dynamic channel assignments have been proposed.

2.2 The Results of Computer Simulations

We begin with an example of computer simulations. The system adopted here has 61 zones as illustrated in Fig.1. Each zone is a regular hexagon and each edge of the hexagons is 1 km in length. The traffic carried per channel is much concerned in the buffering system. In this computer simulations, 2-belt buffering system is examined. The following assumptions are made.

(1) Telephone traffic distribution is uniform over the service area.

(2) The arrival of initiations (requests for service) forms a Poisson process and the holding time is an exponential random variable with the mean value of 1.5 minutes. And the system is a
loss system.

In general, it is not so easy to make a model of the movement of the mobile subscribers in the service area. Let us define the model by the following assumptions.

(3) At the time of call attempts, they are moving to uniform direction.

(4) They move at a constant speed in a straight line in a period which has a truncated Gaussian distribution with the mean value of 1 minute and standard deviation of 1 minute. Then, they change the speed and the direction. The speed distribution of the mobile is as follows. 0km/h is 30%, 15km/h, 30km/h, 45km/h are 20% respectively, and 60km/h is 10%.

The direction has the truncated Gaussian distribution with the mean value of zero and standard derivation of 90 degrees.

In this example, the following two channel assignment algorithms are examined.

(a) Fixed channel assignment.

(b) Dynamic channel assignment.

The first available method is used in these algorithms. In the case of fixed channel assignment, the number of channels is 15 per zone and in the case of dynamic channel assignment, the number of channels is 100 in whole system. The results of computer simulations are shown in Fig.2. The axis of ordinates denote the blocking probability and the axis of abscissa denote traffic carried per channel. The traffic carried per channel is calculated by the following equation.

\[ a_c = na(1-B)/n_c \]  \hspace{1cm} (2 - 1)

where \( a_c \) : traffic carried,

\( n \) : the number of zones,

\( a \) : traffic intensity offered in each zone,

\( B \) : blocking probability and

\( n_c \) : the number of channels in whole system.

From the result of computer simulations in Fig.2, at low blocking probability, the efficiency of channel usage using dynamic channel assignment method is superior to fixed channel assignment method. However, when the blocking probability becomes high, the traffic-carrying capability using fixed channel assignment method is superior to dynamic channel assignment method. So, two curved lines are intersected in Fig.2. Let us call the point of intersection 'reversion point'. In Fig.2, the reversion point is at about 4% of blocking probability. In general, a point of the reversion varies with the number of channels and the structure of system. If the number of channels increase, the reversion point shifts to the left (decrease) generally. This reversion of the inequality of channel usage of a dynamic channel assignment and a fixed channel assignment has been seen in many computer simulations[1][5]. However, we have not seen clearly why the inequality reverses. We will theoretically study the cause at Sec.4.

3. REARRANGEMENTS

If the blocking probability of the reversion of the inequality namely, the reversion point, rises, then the efficiency of the channel usage is improved. In this section, we show that rearrangement of channels has a great effect on rising the reversion point. We begin with introduction of rearrangements of a mobile radio communication system.

3.1 A Formulation of Rearrangement in a Mobile Radio Communication System[5][6]

Rearrangement is a method of breaking through a blocked call by changing routes of calls already in progress[8]. This method is applicable to various systems that adopt a common control switching system. A mobile radio communication system using dynamic channel assignment is a common control system. So, the rearrangement can be applied to the system and by which the performance of the system may be considerably improved.

It is convenient to consider a channel assignment in a small zone mobile radio communication system by a following nonoriented graph \( G \).

\[ G = (V,E), |V| = n \]  \hspace{1cm} (3 - 1)

where \( V \) and \( E \) are the set of vertices and edges respectively.

(i) Calls attempts occur on each vertex.

(ii) \( n_c \) channels are assigned one by one to each call on each vertex, where \( n_c \) is the number of
channel of the system. Only the calls on the vertices which are adjacent can use the same channel. If the \( n_{c}+1 \)st channel is required for a new call, the new call is blocked.

In a mobile radio communication system, the vertices \( v_1, \ldots, v_n \) of \( G \) correspond to the zone \( z_1, \ldots, z_n \) of a service area, and an edge \( e_{kl}(v_i, v_j) \) of \( G \) represents that a zone \( z_k \) does not belong to the buffer zones of \( z_l \). For example, consider a service area of Fig.3(a). If the buffer zones of a zone consist of all zones closer than one zones away from it, the graph \( G \) is Fig.3(b).

In this definition of a graph \( G \), \( G \) has the following properties.

(a) The vertices which have calls using the same channel form a clique (a complete subgraph).

(b) The calls on vertices in an independent set of vertex can not use the same channel.

From these properties, the rearrangement problem can be considered as the channel assignment problem to cliques in \( G \).

For example, we consider a channel assignment with rearrangement in the case of Fig.4. The system in Fig.4 is 1-belt buffering system, and it has three channels in whole system and let the labels of these channels be A, B and C respectively. The letters in a zone show that a channel of the letter is used in the zone. Suppose that a new call attempt occurs in a zone \( z_4 \). The buffer zones of a zone \( z_4 \) consist of zones \( z_1, z_3, z_4, z_5, z_7 \) and \( z_8 \). Since channels A, B and C are used in the buffer zones of \( z_4 \), there is no channels for the new call of \( z_4 \). However, if some channels are permitted to reassign, the new call may not be blocked. This channel assignment with rearrangement can be shown on graph \( G \). A state before rearrangement is shown in Fig.5(a). In this graph, there are a number of cliques, for example, a clique \( A \) is used on a clique \( C = \{v_1, v_2, v_5, v_7\} \) which is drawn with thick lines. A channel assignment is performed as follows:

(1) A clique \( C_A \) that is a set of vertices using the channel B can be enlarged to a set of vertices including \( v_1 \). That is to say, the channel B is assigned instead of the channel A since the channel B can be used in \( v_1 \). Thus, a clique \( C_A \) becomes \( \{v_2, v_6, v_7\} \) and a clique \( C_B \) becomes \( \{v_1, v_2, v_5\} \).

(2) Similarly, the channel B is assigned instead of the channel A to a call on \( v_7 \). Therefore, a clique \( C_A \) becomes \( \{v_2, v_6\} \) and a clique \( C_B \) becomes \( \{v_1, v_2, v_5, v_7\} \).

(3) After all, a clique \( C_A \) can be enlarged to set of vertices including \( v_4 \) and the channel A can be assigned to a new call on \( v_4 \).

The thick lines in Fig.5(b) show cliques \( C_A \) and \( C_B \). The rearrangement problem is formulated as follows.

Let a set of maximal cliques in \( G \) be \( Q \).
T is a (nxm) matrix as

\[ T = \{t_{ij}\}, \quad t_{ij} = 1 \text{ for } v_i \in q_j \]

\[ t_{ij} = 0 \text{ for } v_i \notin q_j \]

For two vectors

\[ X = [x_1, x_2, \ldots, x_m]^T \]

\[ W = [w_1, w_2, \ldots, w_m]^T \]

the problem becomes:

\[ \text{Minimize } Z = \sum_{i=1}^{m} x_i \]

subject to a constraint

\[ TX \leq W \]

\[ \sum_{i=1}^{m} x_i \leq nc \]

where \( x_i \) = 0, 1, 2, ..., and \( w_i \) is the number of calls on a vertex \( v_i \).

The solutions \( X \) and \( Z \) represent the assignment of channels to maximal cliques of \( G \) and the minimum number of channels which is necessary for calls of \( G \), respectively. Although this formulation is expressed in a simple form, it is difficult to solve it. Because this problem is equivalent to the well known problem of colorability of a graph. The colorability problem is NP-complete[9]. The NP-complete problem is known as one of the intractable problem. So the rearrangement problem is very difficult to obtain the solution for large system, and we need an approximate algorithm.

### 3.2 The First Level Rearrangement Algorithm

(A) Generally, it is possible to assume that the origin of calls is at random and the probability of the origin of two or more calls at a time is negligibly smaller than the probability of the origin of no or only one call. Then, the rearrangement problem in a mobile radio communication system is finding an algorithm to assign a channel to one new call under the condition that no call in progress is forced to terminate.

Let \( G \) be a subset of cliques of \( G \).

\[ C = \{c_1, c_2, \ldots, c_{nc}\} \]

where \( c_i \in C \) is a clique consisting of vertices which have calls using a channel \( i \). When a channel \( j \) is not used in \( G \), \( c_i \in C \) is \( \phi \). Let a new call attempt occurs in a vertex \( v_i \). If a clique \( c_k \in C \) such that \( c_k \cap v_i \neq \phi \) and

\[ (v_i) \cup c_k \cap v_i \cap q_j = d_j \]

exists in \( G \), then \( c_k \cup \{v_i\} \) is a clique (this operation is called the enlargement of a clique). So, a channel \( k \) can be assigned to the new call and in this case the rearrangement is unnecessary. (B) If \( c_k \) does not exist in \( G \), the rearrangement is necessary. In this case, if the rearrangement of all calls in progress is permitted, the algorithm of rearrangement is complicated as mentioned above. The algorithm in which candidates of rearrangement are restricted to \( r \) channels in calls \( r \)th level algorithm of rearrangement. In this \( r \)th level algorithm, if \( r = 1 \), the algorithm is very simple. Let us consider the 1st level algorithm.

Let \( W \) be a vector which represents the number of calls (include the new call) on each vertex.

Step 1. Select a channel \( k \) as a candidate, where \( q_j \) containing the maximum vertices which use the channel \( k \) and are adjacent with \( v_j \).

Step 3. Let \( X_r \) be a vector in which \( j \)th component is 1 and the other are zero.

\[ X_r = [0, 0, \ldots, 1, 0, \ldots, 0]^T \]

Step 4. Set \( W_r = W - TX_r \).

Step 5. The components of the \( W_r \) are 1 or 0. Reassign the call on vertices of the \( "m" \) component of \( W_r \) using the (A) technique (the operation of the enlargement of a clique) if all of the above calls can not reassigned, return Step 1.

Step 6. Assign the channel \( k \) to the new call (If all channels are examined, the new call is blocked).

The first level rearrangement does not contain the repetition except the Step 1, and the selection of a candidate channel at Step 1 is not more than \( n_c \) times. The worst-case complexity is \( O(N \cdot n_c) \), where \( N \) is the maximum number of buffer zones of a zone. So, this may be one of the simplest algorithm in this rearrangement problem.

An example of the computer simulations is shown in Fig.6. Channel assignment algorithms are fixed channel assignment, dynamic channel assignment with rearrangement and hybrid channel assignment. Assumptions in this simulations are similar to assumptions in Sec.2.2. 15 channels per zone is used in a fixed channel assignment method and 100 channels is used in a dynamic channel assignment method. In the case of hybrid channel assignment, each zone can use 10 fixed channels and 5 dynamic channels, and after the fixed channels can not use, a dynamic channel is assigned to a new call.

From this result, the dynamic channel assignment method with rearrangement is superior to a fixed and a hybrid channel assignment method. And the reversion point shifts to the right, that is, the traffic characteristics using dynamic channel assignment with rearrangement is superior to without rearrangement. Therefore, the traffic characteristics can be considerably improved by rearrangement. However, the channel assignment with rearrangement can't also avoid the reversion. That is, a dynamic channel assignment method with rearrangement is inferior to fixed channel assignment at higher blocking probability. Thus, it is necessary to propose a method not producing the reversion.

### 4. The Reversion of the Efficient of Channel Usage in Two Channel Assignment

In this section, we study the reversion of the inequality of channel usage of a dynamic channel assignment and a fixed channel assignment.

The traffic characteristics of the mobile radio communication system with fixed channel assignment can be calculated by Erlang's B formula with the number of channels and offered traffic intensity. On the other hand, the traffic
characteristics of the mobile radio communication system using dynamic channel assignment method have not been obtained analytically. However, the exact solution is known in a system which use only one channel[4]. So, using this solution, it can be known that a method of finding approximately properties of a system with multi-channels[4].

Consider a nonoriented graph $G$ as in Sec.3.1 and define $\theta_x$ and $\theta_y$.

$\theta_x$: A set of $X$-cliques of $G$.

$\theta_y$: A set of $X$-cliques including $v_j$ of $\theta_x$ in $G$.

$k_x$: The number of vertices of maximum cliques of $G$.

$\theta_s = \{v_{i1}, v_{i2}, \ldots\}$ \hspace{1cm} (4-1)

Then origination of calls in each zone forms a Poisson process with the coefficient of $\lambda_i$ and the holding time is an exponential random variable with the mean value of $1/\mu_i$. Therefore, the offered traffic intensity is $a_i = \lambda_i/\mu_i$ (erl), and the traffic carried in zone $z_j$ is

$$a_{c,j} = \frac{\sum_{x=1}^{k_x} \sum_{x \in \theta_x} a_{i1} a_{i2} \ldots a_{ix}}{1 + \sum_{x=1}^{k_x} \sum_{x \in \theta_x} a_{i1} a_{i2} \ldots a_{ix}}$$ \hspace{1cm} (4-2)

And the blocking probability in the zone $z_j$ is

$$B_j = (a_j - a_{c,j})/a_j$$ \hspace{1cm} (4-3)

The blocking probability in the whole system can be calculated with similar method. But in the case of a system with a large number of channels, the number of available states (that is, cliques) of channel usage becomes very large. So, the equation of states can't be found in the system with multi-channels. From this equation of states, we can see the cause of the reversion of the inequality of channel usage of a dynamic channel assignment and a fixed channel assignment. Let us consider by an example. Let us consider graphs in Fig.7. They correspond to the systems of 6 and 7 zones, respectively. The graph $G$ represents the relations between a zone and its buffer zones. In these systems, let the number of channels be two, $A$ and $B$, and let a dynamic channel assignment method use. For simplicity, assume that the mobile unit does not cross a zone boundary during a communication and a channel can use in any zones. There are many states of channel usage. In these systems, a state is changed when a call arrives or departs.
In 6 zones system and 7 zones system, the maximum number of zones that can use a same channel at the same time is 3. That shows the numbers of zones using the channel A and of this most right circle, the traffic of (1,4) and (2,4,6) can not flow (shift) to the of this circle according to the increase of the traffic characteristics using fixed channel assignment method. However, in the system of graph (A), the inequality is reserved. In the system of graph (B), dynamic channel assignment method has higher efficiency of channel usage than a fixed channel assignment method at any blocking probability, since a maximal clique is identical with a maximum clique in the graph of the system.

5. AN ADAPTIVE CHANNEL ASSIGNMENT

In general, it is rare that in the graph of a system a maximal clique is identical with a maximum clique. So, most of the system have a reversion point (blocking probability of the reversion of the inequality of channel usage of a dynamic channel assignment and a fixed channel assignment). Thus, in order to maintain the high efficiency of the channel usage at any blocking probability, we assign a channel for a new call according to traffic offered. That is, an adaptive channel assignment method (to traffic offered) channel assignment algorithm is necessary. The reversion point varies with the zone structure and the number of channels. So, it is difficult to find the reversion point analytically.

In this section, we propose a simple adaptive algorithm. The algorithm consists of two parts, that is, a dynamic channel assignment part and a fixed channel assignment part. The choice between two parts depends on traffic offered in the system. In the system of graph (A), the inequality is not reserved in a system with a graph in which a maximal clique is identical with a maximum clique. Thus, we get the following results: "The inequality is not reserved in a system with a graph in which a maximal clique is identical with a maximum clique".

In the system of graph (A), the inequality is reserved. However, in the system of graph (B), the inequality is not reserved. And in the system of graph (B), dynamic channel assignment method has higher efficiency of channel usage than a fixed channel assignment method at any blocking probability, since a maximal clique is identical with a maximum clique in the graph of the system.

A part of the state flow diagram of 6 zones system is shown in Fig.8. In 6 zones system and 7 zones system, the maximum number of zones that can use a same channels at the same time is 3. That is, the number of vertices of maximum cliques is 3. The sets of the numbers in the circle in Fig.8 show the numbers of zone using the channel A and channel B. For example, the most right circle shows that the channel A is used at zones z1, z3 and z6, and the channel B is used at zones z2, z4 and z5. An arrow shows next state after $\Delta t$, and its subscript shows the flow (transition) probability.

The state of the most right circle shows that of the most efficiency of channel usage. In the fixed channel assignment, the state of the channel assignment is a subset of the state in the most right circle in Fig.8, and it flows to the state of this circle according to the increase of the traffic offered. In the case of dynamic channel assignment, if a state flows smoothly to the state of this most right circle, the traffic characteristics will not be inferior to the traffic characteristics using fixed channel assignment. However, in 6 zones system, the state of (1,4) and (2,4,6) can not flow (shift) to the most right circle directly. If such states do not exist in the system, the system does not produce the reversion point of the inequality of channel usage of a dynamic channel assignment and a fixed channel assignment. The existence of these state is a cause of the reversion. For example, the results of the computer simulations of 6 zones system and 7 zones system is shown in Fig.9 and Fig.10, respectively. The reversion point is produced in 6 zones system and it is not produced in 7 zones system.

Thus, we get the following results: "The inequality is not reserved in a system with a graph in which a maximal clique is identical with a maximum clique".

The results of the computer simulations using the adaptive channel assignment is shown in Fig.11 and Fig.12. The system has 61 zones. It is the same as Fig.1. The system is 1-belt buffering system. The number of channels in Fig.11 and Fig.12 are 18 and 36 in whole system respectively.

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The axis of abscissa denote the blocking probability and the axis of ordinate denote traffic carried per channel. A solid line is the result using dynamic channel assignment, the broken line is the result using fixed channel assignment, and the result using the adaptive channel assignment is shown as "o".

From the results of the computer simulations, the traffic characteristics using an adaptive channel assignment method is similar to that using dynamic channel assignment without rearrangement at low blocking probability, and it is similar to that using fixed channel assignment at high blocking probability. So, the algorithm using adaptive channel assignment is superior to the dynamic channel assignment without rearrangement because of it can prevent the "reverse" at any blocking probability. This adaptive channel assignment is not using rearrangement, but it can be used with rearrangement. The adaptive channel assignment with rearrangement may be more superior to a method without rearrangement and is also able to prevent the reversion. And, the method has the following disadvantage. It must change the channel assignment step by step, so it must spend any time until the channel assignment is completed and it must always calculate the blocking probability using fixed channel assignment method, so this algorithm is more complex than dynamic channel assignment.

6. CONCLUSION

The inequality of the efficiency of the channel usage of a dynamic channel assignment and a fixed channel assignment reverses at some blocking probability in most of mobile radio communication systems. The zone structures in which the reversion does not occur are revealed. And it is shown that rearrangement of channels has a great effect on rising the reversion point. And a simple adaptive channel assignment algorithm in which the reversion does not take place is proposed and the validity of the channel assignment algorithm is confirmed by computer simulations.

7. REFERENCES