AN OPTIMAL CONTROL OF AN INTEGRATED CIRCUIT-AND PACKET-SWITCHING SYSTEM

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ABSTRACT

We consider an optimal control of the integrated circuit-and packet-switched multiplexing system. The system is dynamically controlled by rejecting or accepting a new arrival voice call according to its current state. The control problem is formulated as the Markov decision process with the cost structure consisting of the holding and rejection costs. The properties of an optimal control policy are analyzed.

1. INTRODUCTION

Integrated switching, which includes circuit-switching for voice and packet-switching for data, is motivated by the desire to share transmission and switching facilities efficiently and to provide service to a variety of subscribers. The slotted enveloped network (SENET) [1] is an important early development. In SENET, a fixed number of time slots in a synchronous time division multiplexer (STDM) frame is reserved for voice while the rest of the frame is allocated to data. The traffic performance of the SENET-concept integrated voice and data system has been analyzed by many researchers [2]-[4].

Janakiraman, Pagurek and Neilson [5] have considered a variable-frame strategy and compared its performance with a SENET-like fixed-frame scheme.

Gitman, Hsieh and Occhiogrosso [6], Avellaneda, Hayes and Nassehi [7] and Woodside [8] have solved a capacity-allocation problem in which TDM slots in many links of a network are optimally allocated between voice and data to minimize a blocking probability for voice subject to a specified maximum mean delay to data. Konheim and Pickholtz [9] have proposed and analyzed a model of a moving-boundary, fixed frame length, integrated multiplexer. In their model, the assignment of slots within a frame to the voice and data sources is established anew at the beginning of each frame using an allocation function depending jointly on the number of voice and data packets stored at the start of the frame.

Weinstein, Malpass and Fisher [10] have investigated a scheme in which a voice flow rate is controlled depending on voice channel utilization to reduce data packet delays. However, they have not discussed an optimization of their scheme.

In this paper, we consider an optimal control of the integrated circuit-and packet-switched multiplexing system. The system is dynamically controlled by rejecting or accepting a new arrival voice call according to its current state. This control problem is formulated as a Markov decision process, and the properties of an optimal control policy are analyzed.

2. MODEL

The multiplexing technique under investigation is a time division scheme in which each fixed duration frame is partitioned into M time slots. For notational simplicity, the frame length is taken as a unit time. A certain number of time slots in the frame are allocated to the transmission of digitized voice. The voice is circuit-switched by assigning synchronous slots and is under the Lost-Calls-Cleared assumption. The voice traffic is modeled by a Poisson call arrival process with average arrival rate \( \lambda \) (calls/frame) and exponentially distributed call holding times with average service rate \( \mu \) (calls/frame). It is assumed that \( \lambda < \mu \) and \( M \mu < 1 \). This assumption is satisfied in practical systems [6].

The remaining time slots in the frame are dedicated to the transmission of data packets. Each data packet has a fixed size of one time slot and arrives in a Poisson process with average arrival rate \( \lambda_p \) (packets/frame). If no time slots are available for data packets, they wait for transmission in a buffer of size \( L \).

The system is observed at the beginning of each frame. According to the observed state, the system is controlled by rejecting or accepting a new arrival voice call so as to minimize an expected cost. The following costs are taken into consideration. A rejection cost \( c_0 \) is incurred by one rejected voice call and a loss cost \( c_L \) by one packet lost due to the fully occupied buffer. A holding cost of one data packet during one frame is taken as one unit cost. Then, an expected immediate cost in one frame is determined by three costs mentioned above. An optimal control to be analyzed in this paper is one that minimizes the total expected discounted cost over an infinite horizon.

3. FORMULATION

The optimal control problem stated in the preceding section is formulated as a Markov decision process. The state of the system is described by \( i = (i_1, i_2) \), where \( i_1 \) represents the number of voice calls in service and \( i_2 \) the number of data packets in service or in queue. The state space \( S \) is defined as a set \( \{ i = (i_1, i_2) ; i_1 = 0, 1, ..., N, i_2 = 0, 1, ..., L-M-i_1 \} \). When the system is observed in state \( i \) at the beginning of each frame, a control action \( a(i) \) is chosen from "0" or "1". Here, the action "0" or "1" means to
reject or accept a new arrival voice call if it occurs during the frame. A policy is defined as any rule \( \{a(i);i \in S\} \) depending on observed states. Let \( P(i,j;a(i)) \) denote the conditional probability that the system is in state \( j=(j_1,j_2) \) at the beginning of the next frame given that an action \( a(i) \) is taken in state \( i \) at the beginning of the current frame. For notational simplicity, define \( p_k \) by

\[
p_k = \frac{1}{p} \exp(-\lambda_k/p)/k!.
\]

for \( k=0,1, \ldots, L+M-j_1-(i_2-M+i_1)^+ \)

and

\[
P_{L+M-j_1-(i_2-M+i_1)^+} = \sum_{k=0}^\infty \frac{\lambda_k}{p} \exp(-\lambda_k/p)/k!.
\]

The assumptions in the preceding section imply that for \( i \in S \) and \( j_1, j_2 \) as above, the following hold:

- **Suppose** that for \( i=(i_1,i_2); i_1=1,2, \ldots, M, i_2=0,1, \ldots, L+M-i_1 \) and \( j_1=(i_2-M+i_1)^+, \ldots, L+M-(i_1+L) \),

\[
P(i,(i_1-1,i_2);a(i)) = \lambda(i)(1-\alpha(i)) + \lambda(i-a(i))(1-\alpha(i)),
\]

for \( j_1=(i_2-M+i_1)^+, \ldots, L+M-i_1 \)

and

\[
P(i,(i_1,i_2+1);a(i)) = \lambda(i)(1-\alpha(i)) + \lambda(i-a(i))(1-\alpha(i)),
\]

for \( j_2=(i_2-M+i_1)^+, \ldots, L+M-(i_1+1) \),

where \( (\alpha(i)) \) is the set of all subsets \( \alpha \) of \( S \). When the system is in state \( i \) and an action \( a(i) \) is taken, a holding cost \( (i_2-M+i_1)^+ \) is incurred by data packets during the frame and an expected rejection cost \( \alpha(i) \) by lost data packets. Consequently, an expected cost over the frame, denoted by \( C(i;a(i)) \), is given by

\[
C(i;a(i)) = (i_2-M+i_1)^+ + \alpha(i-a(i)) + \lambda(i-a(i)) + \delta(i),
\]

for \( i=(i_1,i_2), a(i) \) and \( \delta(i) = 0 \). Let \( v_i(n) \) be the minimal expected discounted cost over \( n \) frames given that the initial state is \( i \). For simplicity, \( v_i(0) \) is defined as 0 for \( i \in S \). Then, \( v_i(n) \) satisfies the following recursive relation \([11]\): for \( n=0,1, \ldots, i \in S \),

\[
v_i(n+1) = \min\{v_i(n+1;0), v_i(1;1)\},
\]

where for \( a=0 \) or 1 and discounted factor \( \beta \),

\[
v_i(1;1+a) = C(i;a(i)) + \beta v_i(1;1).
\]

Moreover, it holds that for the minimal total expected discounted cost \( v_i^* \),

\[
v_i^* = \lim_{n \to \infty} v_i(n).
\]

Thus, the problem is to find a policy minimizing the right hand side of the above optimality equation. An optimal control policy can be determined by a modified policy iteration algorithm with a suboptimality test \([12]\).

4. CONTROL LIMIT POLICY

Let a partial ordering \( \preceq \) on the state space \( S \) be defined by

\[
i = (i_1,i_2) \preceq (j_1,j_2) \text{ if and only if } i_1 \leq j_1 \text{ and } i_2 \leq j_2.
\]

A policy \( \{a(i);i \in S\} \) satisfying for \( i \in S \)

\[
a(i) = 1 \text{ for } i \in S_1
\]

is called a control limit policy, and the state \( \hat{i} \) is called its control limit \([13]\). Moreover, define \( F \) as the set all non-negative increasing functions with respect to the partial order \( \preceq \) on \( S \).

Lemma \([14]\)

(a) Let \( K \) be the set of all subsets \( \alpha \) of \( S \) such that if \( i \in K \) and \( i \preceq \hat{i} \), then \( \hat{i} \in K \). Then, for each \( f(i) \in F \) there exists a non-negative sequence \( \{\alpha_k\} \) such that \( f(i) = \alpha_k(i) \), where \( K(i) \)

\[
\alpha_k(i) = 1 \text{ for } i \in K, \text{ } 0 \text{ otherwise.}
\]

(b) Suppose that \( \sum P(i,j) K(j) f(j) \in F \) for \( i \in K \) and \( j \in S \). Then, \( f(i)(j) f(j) \in F \).

Let \( S_0 \) denote the set \( \{i; (i_2-M+i_1)^+ = 0\} \), and \( S_+ \) the set \( \{i; (i_2-M+i_1)^+ > 0\} \). It follows from \( \{4\} \) and \( \{5\} \) that

\[
v_1(1) = C(i;a(i)) + \beta f(i(j)).
\]

Moreover, for each \( i=(i_2-M+i_1)^+, \ldots, L+M-j_1 \) and \( j_1=1,2, \ldots, i_1+i_2 \),

\[
\sum_{j_2} P(j_2-(i_2-M+i_1)^+) v_j = \sum_{j_2} \lambda_j \exp(-\lambda_j)/j! \cdot C(i_2-M+i_1)^+ + \lambda(i-a(i)) + \lambda(i-a(i)) + \delta(i).
\]

In the following it is assumed that

\[
\min(\exp(-\lambda_j), \exp(-\lambda_j)/(L+M)) > M/(1-M_j).
\]

Then, it follows that for each \( i=(i_2-M+i_1)^+, \ldots, L+M-j_1 \) and \( j_1 \in S_+ \),

\[
\{1-(i_2-M+i_1)^+\} \sum_{j_2} P(j_2-(i_2-M+i_1)^+) v_j \leq \{1-(i_2-M+i_1)^+\} \sum_{j_2} \lambda_j \exp(-\lambda_j)/j! \cdot \lambda(i-a(i)) + \lambda(i-a(i)) + \delta(i).
\]
\[ -(1-i_1) \mu_k \sum_{p=0}^{\infty} \frac{k^p \exp(-\lambda_p)}{k!} \leq 0. \]  

That is, for each \( k = (i_2 - M + i_1) \), \( \ldots, L + M - j \), and \( S \),
\[ \sum_{j_2=1}^{L-M-j} \sum_{j_1=1}^{L+M-j} P(i,j) \leq \sum_{j_2=1}^{L-M-j} \sum_{j_1=1}^{L+M-j} P(i,j). \]  

On the other hand, for \( i \in S_+ \) and \( j \in S_0 \),
\[ \begin{align*}  
& -(1-i_1) \mu_k \sum_{j_2=1}^{L-M-j} \sum_{j_1=1}^{L+M-j} P(i,j) \\
& = \lambda \sum_{j_2=1}^{L-M-j} \sum_{j_1=1}^{L+M-j} P(i,j). 
\end{align*} \]  

Equations (9), (12), (13) and (20)–(22) yield for \( \mathbb{E} \),
\[ \sum_{j \in S} P(i,j) I_k(j) \leq \sum_{j \in S} P(i,j) I_k(j). \]  

Since (6) leads to
\[ v_1(n+1;1) - v_1(n+1;0) = -\alpha \lambda + \sum_{j \in S} P(i,j) v_1(n), \]  

(15), (23) and lemma imply that \( v_1(n+1;1) - v_1(n+1;0) \) increases with respect to the partial order \( \leq \). Thus, there exists a control limit.

5. NUMERICAL RESULTS

The optimal control policy was computed by the modified policy iteration algorithm [11] on a FACOM M-382 computer at the Data Processing Center, Kyoto University. In the computation, the values of parameters \( \mu, L, M, \alpha \) and \( \rho \) were set as
\[ \mu = 10^{-3}, L = M = 10, \alpha = 100 \rho. \]

Figure 1(a) shows control limit lines of the optimal policies in cases where \( \beta = 0.9, \rho = 10, \lambda_p = 3 \) and \( \alpha = 1, 10, 100 \). In the figure, action 'reject' is optimal in states above and on the control limit lines. The figure shows that the control limit lines go up as the rejection cost \( \lambda_p \) increases. Figure 1(b) shows a change of the control limit lines as the value of \( \lambda_p \) changes from 1 to 3 in the case of \( \beta = 0.9, \rho = 10 \) and \( \alpha = 100 \). The control limit lines go down with increasing \( \lambda_p \). Figure 1(c) describes the dependence of the control limit line on the value of \( \beta \). The larger \( \beta \) leads to the lower control limit.

6. CONCLUSIONS

We considered an optimal control of the integrated circuit-and packet-switched multiplexing system. The control problem is formulated as the Markov decision process with the cost structure consisting of the holding and rejection costs. Under condition (10), it is proved that a control limit policy is optimal. The control limit is affected by a rejection cost, an offered load of data packet and a discounted factor.
Fig. 1 Control limit line. (a) Influence by $\alpha$ where $\beta=0.9, \rho=10$, and $\lambda_p=3$; (b) Influence by $\lambda_p$ where $\beta=0.9, \rho=10$, and $\alpha=1$; (c) Influence by $\beta$ where $\rho=10, \lambda_p=3$ and $\alpha=1$.

ACKNOWLEDGEMENT

We would like to thank Professor T. Hasegawa and Dr. Y. Takahashi in Kyoto University for their invaluable suggestions.

REFERENCES


