ABSTRACT

An idealised Automatic Telephone Exchange (ATE) is considered. A comparison is made between a model without repeated calls (with and without absence with uniform and ununiform activity of the subscribers) and a model with repeated calls. For some cases analytical dependences are given. Mainly a Poisson flow is considered but also Engset flow and a negative binomial case are touched upon. Considered are only losses due to mistakes of the A-subscriber, the absence and occupation of the subscribers, depending on the activity and duration of the occupation of the installations, i.e. only the natural factors limiting the quality of functioning of a switching telephone network.

The results obtained are significant for a better estimation of the real systems.

2. MAIN INPUT PARAMETERS

There are considerable differences in the experimental data received under different conditions by different authors because of which the chosen by us values and distributions of the input parameters shown on Figure 1 are illustrative. We have used for example [1, 2, 3] as well as our observations. When the value is given by an interval (for example 12-3) this means that a uniform distribution in the interval ([12-3, 12+3]) is used. If the value is denoted by "exp" (for example 180 exp) one must understand that the variable has the shown mean value (180) and has an exponential distribution. All the times are given in seconds.

The notations are:

N = number of all the subscribers in the system
A = mean intensity of the whole primary offered traffic in calls per second
A' = mean intensity of the primary offered traffic of a free source

Mean holding times:
Td = dialling performed by the calling A-subscriber (6-7 digits)
Tmd = dialling when the subscriber thinks he has misdialled the number
Tans = ringing tone when the called B-subscriber answers
Tbus = busy tone
Tc = conversation
Tm = conversation that finds misdialling

Times from the hanging up the receiver up to its picking up before the next new attempt, after:
Wno = conversation finding misdialling
Wbus = busy B-subscriber
Wans = no answer
Wind = misdialling
Wfree = waiting for the A-subscriber to be free, if he was busy in the interval between two attempts.

Probabilities for directioning of the calls to:
Fno = absent subscriber
Find = misdialling
Fm = conversation finding misdialling
3. MAIN OUTPUT PARAMETERS

- $P_{bus}$ = probability of finding the B-subscriber busy
- $P_{0}$ = probability of finding a busy not absent subscriber among all not absent
- $P_{2}$ = probability of finding a busy absent subscriber among all absent
- $\lambda R$ = average intensity of the whole flow of repeated calls
- $\beta = \frac{\lambda}{\lambda R}$
- $A$ = traffic of the A-subscribers
- $B$ = traffic of the B-subscribers
- $E$ = efficiency of calls
- $F$ = number of calls ended in a conversation
- $T_{A}$ = average occupation time of the A-subscriber (sec)
- $T_{B}$ = average occupation time of the B-subscriber (sec)
- $T_{AB}$ = average occupation time of a subscriber
- $E_{t}$ = efficiency of traffic
- $E_{c} = \frac{\sum T_{A}}{\sum T_{A}}$ for all occupations of the A-subscriber
- $P'_{ads}$ = number of all calls directed to an absent subscriber
- $P'_{ads} = P_{ads}$ if the system includes repeated calls

4. SIMULATION MODEL

is a development of the model [4]. A group of 2000 subscribers is modelled. Its essential particularity is that the subscribers are discernable, i.e. every subscriber has a unique number and other individual characteristics, for example whether he is absent or not. Each call is represented as a transaction in the GPSS-language. The flow of calls corresponds to the shown in Fig. 1. The original flow of calls is directed with equal probabilities only to free subscribers who are not noted as absent. The number of the B-subscriber is chosen with equal probability among all subscribers. The numbers of the A-subscribers and the B-subscribers are written in the parameters of the transaction and without changes are used with the repeated calls.

Since in the interval between two attempts the original A-subscriber can be seized as a B-subscriber or can generate another call as A-subscriber, it is necessary to check before the beginning of each repeated attempt whether the A-subscriber is free. The experiments show that $p = 1.854$, for one seizure of an A-subscriber the check cycle including (see Fig. 1) is fulfilled 1.245 times, i.e. it can be assumed that the interval between two calls is influenced directly by the loading of the telephone system (especially at great $p$).

After terminating of an unsuccessful attempt, in this case with repeated attempts, the call remains in the system with probability $R_{md}$, $P_{obs}$, $R_{bus}$ or $R_{me}$, depending on the cause for
failure. These probabilities are equal for every attempt with two exceptions: 1) Means for registration of up to 10 attempts are provided and for this reason the transactions for the 11-th attempt are rejected by the system. The experiments show that up to \( \lambda = 2.522 \) calls/sec, \( P_{\text{busy}} = 39.059 \% \), 0.07% of all calls have made the 10-th attempt. These data show that the registration of more than 10 attempts is indefensible.

2) The other exception is the cases with repeated attempts due to absent subscriber. The known data and our observations show that when the B-subscriber does not answer the A-subscriber makes after a short interval one more (as an exception) attempt, mainly to be sure he has not misdialled. We assume that the attempts made after more than 60 minutes are new attempts and not repeated, that is why we do not admit more than 2 attempts ending with "no answer". Of course, as before, so between the two attempts there can be losses due to any of the other described reasons, but it is not possible for the B-subscriber to answer, because he is noted as absent. Naturally the telephone of an absent B-subscriber can be occupied when it rings (average \( T_{\text{obs}} \) sec). A considerable part of the telephones of the absent subscribers can be occupied due to this reason (see §6.2). The data received from the simulation show that the second call ended in "no answer" under the described conditions can occur even at the 8-th attempt.

The model includes about 1000 punch cards in the GPSS-language, uses about 480 K bytes storage of a computer ES 1040 and needs about 0.07 sec for processing of a call, including input/output operations. For a time unit is chosen a nanosecond (0.015). The shown results are received on the basis of 10000 transactions that left the system through the blocks "end" in Fig.1 after the processes have reached statistical equilibrium. Since statistics are gathered after the end of every attempt, for a system with repeated calls this means that at least 10000 \( \beta \) measurements have been made for every value of \( \lambda \).

5. MODEL WITHOUT REPEATED CALLS

5.1. Without Absent Subscribers

Let in Fig.1 \( P_{\text{obs}} = R_{\text{md}} = R_{\text{af}} = \frac{R_{\text{busy}}}{R_{\text{md}}} = R_{\text{af}} = 0 \). In this case obviously \( P_{\text{af}} = P_{\text{busy}} \). We shall follow the approach offered in [5]. We consider every phase of service like a multichannel device with unlimited number of channels in which at every moment can be served independently from each other unlimited number of calls. We consider as logically different holding times even such which are received in the same way, if this increases the clearness and convenience at examination and does not change the logic of functioning of the system. For example
To consider the conversation and so on.

where and the saving laws we derive at a stationarity: 

$$A + B = \lambda \left[ C_1 - P_{busy} C_2 \right],$$  \hspace{1cm} (2)

where $C_1$ and $C_2$ are random variables

$$C_1 = \frac{P_{md} T_{md}}{1-P_{md}} + \frac{1-P_{md}}{2T_d + T_d};$$

$$C_2 = \frac{(1-P_{md})}{2T_d - T_{busy}};$$

$$T_d = T_{ans} + T_{mo} T_{mo} + \frac{(1-P_{mo}) T_o}. \hspace{1cm} (3)$$

The coefficient 2 before $T_d$ is required, since while a B-subscriber is busy, an A-subscriber is busy too. In this case all subscribers are equal and the calls are directed uniformly to all the subscribers, therefore we can consider:

$$P_{busy} = \frac{A + B}{N},$$  \hspace{1cm} (4)

From (2) and (4) we receive:

$$P_{busy} = \frac{\lambda}{N} \left[ C_1 - P_{busy} C_2 \right]$$  \hspace{1cm} (5)

In the case of a Poisson input $\lambda = \lambda N$, from where:

$$P_{busy} = \frac{\lambda C_1}{1 + \lambda C_2} \hspace{1cm} (6)$$

By analogy for Engset input and input of negative binomial type we receive correspondingly (7) and (8):

$$P_{busy} = \lambda' \left(1 - P_{busy}\right) C_1 \hspace{1cm} (7)$$

$$P_{busy} = \lambda' \left(1 + P_{busy}\right) C_1 \hspace{1cm} (8)$$

The expression (5) makes it possible to consider other types of input flows. The simulation results confirm fully the received results [7, 8, 9, 10]. In [11] is considered the case of an Engset input flow, as in case (7), so when the intensity of the input flow decreases proportionally to the absent subscribers. The results show that in Engset the absence of the A-subscribers must not be taken into consideration, since a subscriber who has lost his telephone can remain a source of calls, creating in this way additional loading of other telephones. The model with Engset input and decrease of flow proportionally to the absence of subscribers gives lower losses in comparison with our observations.

In the mentioned works, in contrast to this work, is considered the case of the subscriber's absence with equal probability. In [11] is given a comparison between the cases with and without absence of the subscribers at Engset and Poisson flows.

As it is seen in Fig.1 the knowledge of $P_{busy}$ gives us the opportunity to calculate all important characteristics of the telephone system in stationarity, since the values of the other input parameters are relatively independent of $\lambda$. For example, the conversational traffic $(A_0)$ of the A-subscribers (simultaneously occupied A-subscribers, whose occupation will end with conversation) will be (see Fig.1 with assumptions made):

$$A_0 = \lambda \left(1 - P_{md}\right) (1 - P_{busy}) \left[ T_d + T_{ans} + T_{me} T_{me} + (1 - P_{me}) T_o \right]$$

and the specific conversational traffic will be $A_0/N$. After determining $\lambda N$ from (5), we receive:

$$A_0 = \frac{P_{busy} \left(1 - P_{md}\right) \left(1 - P_{busy}\right) \left(1 + T_{ans} + T_{me} \left(1 - P_{me}\right) T_o \right)}{N} \left[ C_1 - P_{busy} C_2 \right]$$

$A_0/N$ is 0 for $P_{busy} = 0$ and 1 and has only one maximum, for example when $P_{busy} = 0.8$. It is important to note that (9) does not depend on the type of input flow (Poisson, Engset or negative binomial case) if it is stationary. Other similar results are shown in [12]. All given results are confirmed with a relative error at 6000 measurements not greater than 2% for all simulated values of $A$.  

5.1.1. Model when the activity of the subscribers is not uniform

Let us consider the subscriber's traffic with the assumptions made, but when all $N$ subscribers are divided into $k$ categories. The uniform activity is expressed with regard to the intensity of the offered flow in calls per sec, its directioning and the duration of occupation of the installations. For each category remain in force the already des-
scribed input parameters and one or two indexes are added to the notations, the first showing the category of the A-subscriber, the second - the category of the B-subscriber. For example, \( T_d(I,J) \) is the duration of dialling the number of the B-subscriber from category \( J \), by the A-subscriber from category \( I \). If there is only one index, it refers to the category of the A-subscriber, for example \( P_{busy}(I) \) depends only on the behaviour of the A-subscriber.

The required additional parameters are:

\[ P(I,J) = \text{matrix of the traffic interest. It shows the probability of directioning of a call originating from category } I \text{ to category } J \]

\[ L(I) = \frac{A(I)}{\lambda}, \text{ where } A'(I) \text{ is the offered traffic (calls/sec) by one subscriber of category } I \text{ and } \lambda \text{ is the whole offered traffic by the } N \text{ subscribers.} \]

\[ Q(m) = N(m)/N, \text{ where } N(m) \text{ is the number of the subscribers from category } m. \]

In [5] is shown that at a Poisson flow the \( \lambda < \) unknowns \( P_{busy}(I) \) can be determined as a solution of the following system of \( k \) linear equations:

\[
\sum_{m=1}^{k} A(I) P_{busy}(J) + P_{busy}(I) [T_B(I,m) - T_{busy}(I)] = B(I),
\]

where

\[
A(I) = L(I) Q(I) \sum_{m=1}^{k} P(I,m) [T_B(I,m) - T_{busy}(I)],
\]

\[
D(I) = \sum_{m=1}^{k} Q(m) P(m,I) T_B(m,I),
\]

\[
B(I) = L(I) Q(I) \sum_{m=1}^{k} P(I,m) [T_d(I,m) + T_B(m,I)] + \sum_{m=1}^{k} P(m,I) L(m) Q(m) T_B(m,I).
\]

After knowing all \( P_{busy}(I) \), we can determine all important characteristics describing the modelled system and for each of the categories. In [13] is used this result for 3 categories of subscribers and is shown that not taking into account the ununiform activity of the subscribers when dimensioning PABX, can lead to considerable errors. The data received as a result of the simulation confirm (10) with a relative difference under 2% in the whole work interval of \( \lambda \).

Another important conclusion for the PABX is that the losses due to "busy" must be accounted not according to the occupation of the connective lines, but according to the occupation of the subscribers from the PABX.

In order to apply the results to local exchanges we need much more and more suitable data in comparison with the available data characterizing the ununiform activity and in particular the mutual interest among the subscribers from different categories. Similar data are given in [14] and are approximately valid for Bulgaria, where the difference between the home, business and administrative categories of subscribers is too small.

5.2. Model with Absence of Subscribers

Let us consider the model in Fig.1 under the condition \( R_{md} = R_{abs} = R_{busy} = R_{mc} = 0 \). The results from § 5.1 are not applicable directly, since here the subscribers are not equal. But we can consider the subscribers consisting of two homogeneous categories: 1) present and 2) absent (with \( A = 0 \) ) and use the results from § 5.1.1. In Fig.1 we can easily see that:

\[
P_{busy} = (1 - P_{md}) [P_{abs} P_{busy} + (1 - P_{abs}) P_{01}] (11)
\]
The equation (11) is valid for the measurement of subscribers, which are homogeneous:

\[
\frac{A+B}{N} = P_{\text{abs}} \cdot \frac{g_2}{N} \cdot (1 - P_{\text{abs}}) \cdot P_{\text{busy}}
\]

In Fig. 2 and 3 are shown the results from the simulation. With a dotted line are plotted the values of \((A+B)/N\) in %. The equation (11) is valid for the measured values up to the first 5 digits and the equation (12) has a relative difference of 1.9% for \(A = 0.326\) that decreases with augmentation of \(A\) (for \(A = 0.606\) it is 1.16%) and reaches 0.69% for \(A = 5.133\) calls/sec. The losses from "no answer" decrease with augmentation of \(A\), since they increase as the common losses from "busy subscriber", so the probability to dial an occupied telephone of an absent subscriber (\(P_{\text{busy}}\)).

In Fig.2 it is seen that the probability of failing upon a busy subscriber among the present subscribers (\(P_{\text{busy}}\)) in the work area of \(A\) can exceed 1.26 times the average probability \(P_{\text{busy}} = 12.23\%\) while \(P_{\text{busy}} = 2.15\%\) is 5.68 times lower than \(P_{\text{busy}}\) and causes 0.28% from all losses. It is seen from (11) that the importance of \(P_{\text{busy}}\) increases with the increase of \(P_{\text{busy}}\). On the other hand, \(P_{\text{busy}}\) can exceed in some cases even 60%.

\(E_1\) is practically a straight line and decreases more slowly than \(E\). \(E_1\) depends strongly on the duration of the conversation \([8]\). In Fig.4 is seen the decreasing of the mean occupation times separately for \(A, B\) and all the subscribers. The decrease of \(T_A\) is the greatest. This decrease is due to the increase of \(P_{\text{busy}}\) and \(P_{\text{busy}}\) (Fig. 1, 2), at constant mean values of the holding times.

6. MODEL WITH REPEATED CALLS

Let us consider the full model in Fig.1. The results from the simulation are shown in Fig. 3, 4, 5, 6, 7. In Fig.3 is seen considerably more steep increase of \(P_{\text{busy}}\) and \(P_{\text{busy}}\) in comparison with the case without repeated calls (Fig.2). The values of \((B/N)\) (dotted line) approach \(P_{\text{busy}}\). This is due to the uneven distribution of the phenomena of the repeated attempts. The dependence (11) is fulfilled again with the same precision, if \(P_{\text{busy}}\) is replaced by \(P_{\text{busy}}\) (plotted on Fig.3). It is seen that only one repeated call due to "no answer" can considerably increase the percentage of calls directed towards absent subscribers. This increase is registered more strongly for the low (near the work area) values of \(A\).

The equation (12) shows, as it can be expected, is not valid in the case of repeated attempts - the measured relative error is 20%. The observed values of \(\beta\) are a little bit lower than the published experimental data \([3,15]\) but this can be explained by the lack of losses in our model, due to technical losses and shortage of installations.

\(E\) and \(E_1\) are lower and decrease more rapidly than in the case without repeated calls. The shown efficiency (\(E\)) is lower in comparison with some published data \([3,15]\) but well agrees with our observations upon a system with direct control, without noticeable technical losses.

In \([3,14]\) is indicated that the multiplication \(\beta E\) gives an impression about the traffic which is proportional to the charged traffic. We think that the charged traffic is represented \(\beta\) better through the multiplication \(\beta E\), where \(\beta = 1.24\) is the minimal measured by us value of \(\beta\), when \(P_{\text{busy}} \geq 3\%\).

The variables \(E_1, \beta E\) and \(\beta E\) are practically straight lines. Round the work point, for which is accepted the model \(A\), creating \(P_{\text{busy}} = 2\% = 15\%\) \(P_1\), \(P_{\text{busy}}\) and \(P_2\) are in ratio 1.31/1/0.23.

Fig. 4 shows considerably lower values of the average holding times in comparison with the case without repeated calls.

The ratio \(A/(A+B)\) increases slightly faster than in the case without repeated attempts. In both cases, in the work point depicted on Fig. 2 and 3 interval of \(A\), the ratio \(A/(A+B)\) varies from 51.92% to 53.89%. For \(A = 1\) calls/sec without repeated attempts \(A/(A+B)\) is 52.197% and with repeated attempts - 52.573%.

In Fig.5 is shown the distribution of the occupation time of the \(A\)-subscribers in the case of an unsuccessful attempt. The cases of 1 to 9 sec are due to dialling, considered wrong, from 9 to 22 sec are caused by busy B-subscriber and over 40 sec by not answering B-subscriber. The losses due to conversation, finding wrong dialling are not included, because they pertain to the charged traffic. It is evident that it is very difficult to approximate with only one analytical distribution the results in Fig.5.

In this respect the results shown on Fig. 6 and 7 look better. In Fig. 6 is depicted the distribution of the number of all simultaneously occupied subscribers and in Fig.7 of the number of simultaneously occupied \(A\)-subscribers. The distributions in Fig. 5, 6 and 7 change as their mean value, so their type, depending on \(A\). The ratio \(P_{\text{busy}}/\text{mean decreases from 1.45} \text{ at } P_{\text{busy}} = 6.67\% \text{ to } 1.11 \text{ at } P_{\text{busy}} = 39.06\% \text{ when } P_{\text{busy}} = 11.94\% \text{ it is } 1.32.

7. CONCLUSION

A simulation model of a telephone system is worked out, including idealised Automatic Telephone Exchange (ATE). Comparatively in detail described the behaviour of the subscribers in the case without repeated calls: considered are the subcases with and without absence of the subscribers as well as their ununiformity.Offered are analytical dependences describing the model without repeated attempts, confirmed with great precision by means of the data received from the simulation models. These dependences are in force for the mean values of the participating in them varia-
bles, at a statistical equilibrium of the processes in the modelled system and are independent of the distribution of the values of the input parameters in the same degree in which the formula of Little is independent.

It is shown that not taking into consideration the ununiform activity of the subscribers can lead to considerable mistakes when dimensioning PABX. The available data from measurement of the uniform activity in real local exchanges are not suitable and not sufficient for the full application of the model worked out. In order to consider the case of permanently absent (in the considered time interval) subscribers it is necessary to use the model with ununiform activity considering the absent subscribers as a separate category. In this case the hypothesis that \( P_{\text{busy}} = (A+B)/N \) [16] is not confirmed.

Given are results from the simulation in the case with repeated attempts and they are compared with data received for the same input parameters, but without repeated attempts. Shown is the considerable deterioration of the considered indices as losses due to busy and absent subscribers, efficiency and average occupation time. Makes an impression a considerable increase of the calls directed towards absent subscribers, for example 23% against 15% in the case without repeated attempts. This is important, having in mind that in today's dynamic world the absence of the subscribers is considerable and shows the necessity of decreasing the cases "the subscriber is busy" and "the subscriber is absent" [17, 18]. Shown are the distributions of the time of the lost occupations and the traffic of the A-subscribers, as well as the traffic of all the subscribers.

In the chosen by us approach, the received dependencies have the character of natural laws, since they do not depend on shortage of installations and technical losses. This gives an opportunity to dimension the real ATE reaching optimum. For example, there is a direct relation between the traffic of the traffic of the A-subscribers and the number of the necessary internal connective lines in ATE. Because of this the found mean values, maximum values and the distribution of the A-subscribers' traffic for each A have a great practical significance. The worked out model is foreseen to reflect the interval structure of a real ATE with different types of control and in this way it will become convenient instrument in aid of the telephone administration.

REFERENCES


