APPLICATION OF BAYESIAN TELETRAFFIC MEASUREMENT TO SYSTEMS WITH QUEUEING OR REPEATED ATTEMPTS

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ABSTRACT

In this paper Bayesian Statistical Inference is applied to the estimation of traffic parameters for systems with queueing and systems with repeated attempts. Algorithms for computing estimators of the mean and variance of the offered traffic and the mean service time are developed. A method for incorporating constraints on the variance to mean ratio of the offered traffic is also introduced.

1. INTRODUCTION

In References [1] through [3], the authors have described an application of Bayesian Statistical Inference to the problem of teletraffic measurement. The particular cases dealt with are loss systems with or without internal congestion. The theory on which this work is based can be found in References [4] through [7], and related work is described in References [8] through [10]. This method appears to offer some potential advantages over existing methods, as discussed in the concluding Section of this paper. However, it has not yet been applied in practice.

The work reported in the present paper extends the application of Bayesian Statistical Inference to two additional areas of practical significance: systems with queueing, and systems with repeated attempts. For the models of the two new applications, the algorithms for computing estimators are derived. The resulting estimation algorithms are somewhat similar to those previously derived for loss systems without repeated attempts.

The method of estimating parameters is extended to allow the variance to mean ratio of the offered traffic to be constrained either to a fixed value or to a range of values. Introducing a constraint on the variance to mean ratio eliminates the possibility of deriving unrealistic or physically impossible estimates of the mean of the offered traffic.

In Section 2 of this paper previous results are briefly summarised. At the end of Section 2, problems that can arise in the application of the basic results are discussed. In Section 3 systems with queueing are considered: unconstrained estimation is introduced in 3.1, and constrained estimation is introduced in 3.2. Section 4 deals with systems having repeated attempts. A simple model of repeated attempts is proposed in 4.1.

Unconstrained and constrained estimation of the parameters of the model are dealt with in 4.2 and 4.3 respectively. Numerical results for a simple example are given in Section 5, and conclusions are given in Section 6.

2. SUMMARY OF PREVIOUS RESULTS

For convenience, a brief summary is given of the relevant results reported in References [1] to [3]. The traffic process being observed is modelled as a Markov process specified by parameters \( \mathbf{a}, \mathbf{b}, \mathbf{c} \), where

- \( \mathbf{a} \) is an \( R+1 \) dimensional vector of arrival coefficients,
- \( \mathbf{b} \) is an \( R+1 \) dimensional vector of blocking probabilities (loss factors),
- \( \mathbf{c} \) is an \( R+1 \) dimensional vector of departure coefficients,
- \( \mathbf{r} \) is the state of the system (number of calls in the system),
- \( R \) is the maximum possible value of the state - for a queueing system this is the number of servers plus the number of waiting places; for a loss system it is just the number of servers.

Individual components of all vectors are denoted by subscripts, for example components of \( \mathbf{a} \) are:

\[
a_r \quad (r = 0, 1, \ldots, R).
\]

It is assumed that the group occupancy is observed as a function of time, and that the time of occurrence of every unsuccessful bid is also observed. Therefore, the observations are taken as: the time of occurrence of each event (bid or departure), denoted

\[
t(1) \quad (i = 0, 1, \ldots, k);
\]

and the state of the system prior to each event,

\[
r(1) \quad (i = 0, 1, \ldots, k).
\]

It can be shown that the following statistic constitutes an "information state" for the system - that is, a recursively computable, sufficient statistic of fixed, finite dimension:

\[5.3B-1-1]
where $s(k)$, $u(k)$, and $v(k)$ are each $R+1$ dimensional vectors with $r^\text{th}$ components defined by:

- $s_r(k)$ is the total time spent in state $r$,
- $u_r(k)$ is the total number of unsuccessful bids which occurred while the system was in state $r$,
- $v_r(k)$ is the total number of successful bids which occurred while the system was in state $r$,

with all statistics being collected over the period from the $0^\text{th}$ event to the $k^\text{th}$ event.

Let the $R+1$ dimensional vector $w(k)$ be defined by

- $w_r(k)$ is the total number of departures which occurred while the system was in state $r$.

This statistic can be computed from:

$$w_0(k) = 0,$$

and, for $r = 1, \ldots, R$,

$$w_r(k) = v_{r-1}(k) + e(r(0), r(k), r),$$

where the function $e(\cdot, \cdot, \cdot)$ is defined by

$$e(r(0), r(k), r) = \begin{cases} -1 & \text{if } r(0) < r \leq r(k) \\ +1 & \text{if } r(k) < r \leq r(0) \\ 0 & \text{otherwise}. \end{cases}$$

It is desired to estimate the unknown parameters $(a, b, c)$ from the sufficient statistic given above. For this purpose, the following function is introduced:

$p_k(a, b, c)$ is the likelihood kernel, at stage $k$, of the unknown parameters.

The likelihood kernel may be written directly by taking the probability density function of the observations as a function of the unknown parameters $(a, b, c)$, and eliminating any factor which does not depend on the unknown parameters. In this case, it is given by the expression below, in which the dependence on $k$ of $s_r, u_r, v_r, w_r$, and $p(a, b, c)$ is not shown for brevity:

$$p(a, b, c) = \prod_{r=0}^{R} \left\{ (1-b_r)^{v_r} b_r^{u_r} \right\} \left( u_r + v_r \right)^{w_r} a_r^{s_r} c_r^{w_r} \exp(-a_r (v_r + c_r))$$

The probability density function of $(a, b, c)$, conditional on the observations, is equal to the function $p(a, b, c)$ multiplied by a normalising constant and by the prior density function of the unknowns. The normalising constant is computed by numerical integration of the function $p(a, b, c)$. The prior density function must be assigned, and may incorporate prior information regarding $(a, b, c)$.

The results summarized above are based on a general model. Several special cases have also been dealt with in [1] to [3]. In this paper we consider models which involve negligible internal congestion. This is appropriate for modern digital switching systems with reasonable levels of traffic loading. Hence, as explained in [3], the term $v_r^{(1-b_r)} u_r^{b_r}$ is set equal to unity.

The offered traffic is modelled as being either Binomial, Poisson, or Negative Binomial. Hence, the arrival coefficients are

$$a_r = (S-r)^\alpha$$

where $S$ is the effective number of sources,

$\alpha$ is the arrival rate per free source.

Note that $S$ and $\alpha$ may be both positive (Binomial), or both negative (Negative Binomial). Poisson traffic corresponds to the limit as $S$ approaches infinity and $\alpha$ approaches zero, with the product $SA$ remaining constant.

Assuming Negative Exponential service times, the departure coefficients are

$$c_r = r \mu$$

where $\mu$ is the mean service rate,

$h$ is the mean service time,

and $\mu$ is related to $h$ by

$$\mu = 1 / h$$

Estimators of the unknown parameters can be found by computing the values of the parameters which maximise the logarithm of $p(a, b, c)$. The resulting estimators may be interpreted as Maximum Likelihood Estimators, or, equivalently, as Maximum Aposteriori Probability Density Estimators for the case of uniform prior density. The logarithm of the function $p$ can be written as:

$$\ln(p) = \sum_{r=0}^{R} \left\{ n_r \ln((S-r)) + w_r \ln(h \mu) \right\}$$

where

$$n_r = a_r (v_r + c_r)$$

Necessary conditions to be satisfied by the estimators are found by equating the partial

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derivatives of $\ln(p)$ to zero. Firstly, differentiating with respect to $\mu$ and equating to zero yields

$$\sum_{r=0}^R (-r \alpha + \beta) = 0. \quad (10)$$

Differentiating with respect to $\alpha$ yields

$$\sum_{r=0}^R (-s_r \alpha + n_r) = 0, \quad (11)$$

and with respect to $S$ yields

$$\sum_{r=0}^R (-\alpha s_r + n_r) = 0. \quad (12)$$

Defining

$$N = \sum_{r=0}^R n_r, \quad (13)$$

$$W = \sum_{r=0}^R w_r, \quad (14)$$

$$T = \sum_{r=0}^R s_r, \quad (15)$$

and average occupancy

$$\bar{\mu} = W / \{T \bar{\tau}\}, \quad (16)$$

it follows that

$$\hat{\mu} = W / \{T \hat{\tau}\}, \quad (17)$$

$$\hat{\alpha} = N / \{T (\hat{S} - \hat{\tau})\}, \quad (18)$$

and $S$ satisfies

$$\sum_{r=0}^R \left( \frac{n_r}{\hat{S} - \hat{\tau}} \right) = T \hat{\tau}, \quad (19)$$

$$= N / \{\hat{S} - \hat{\tau}\}. \quad (20)$$

The last equation does not have a general closed form solution for $S$. However, it can be solved iteratively as described in [3].

The offered traffic is defined to be the traffic that would be carried on an infinite group of servers with the arrival and departure coefficients as above. For cases where the inequality

$$\alpha h > -1 \quad (21)$$

is satisfied, the mean of the offered traffic is given by:

$$m = S \alpha h / (1 + \alpha h) \quad (22)$$

and the variance is given by

$$v = S h / (1 + \alpha h)^2 \quad (23)$$

Hence, estimators of $m$, $v$, and $h$ can be written as:

$$\hat{m} = S \alpha \hat{h} / (1 + \alpha \hat{h}) \quad (24)$$

$$\hat{v} = S \alpha \hat{h} / (1 + \alpha \hat{h})^2 \quad (25)$$

$$\hat{h} = 1 / \mu \quad (26)$$

For cases where

$$\alpha h < -1 \quad (27)$$

then the mean of the corresponding offered traffic is undefined. The physical interpretation of this is that the arrival rate, as a function of the state, increases faster than the departure rate. Hence as more servers are made available to handle the traffic, the carried traffic increases without limit. This problem arises from the physically untenable assumption that the arrival rate as a function of the state can be extrapolated linearly to any value of $r$. Clearly, from physical arguments, the arrival rate cannot continue to exceed the departure rate as the number of servers increases without limit.

Even as the term $\hat{\alpha} \hat{h}$ approaches $-1$, $\hat{m}$ and $\hat{v}$ become very sensitive to small errors. A practical means of overcoming these problems of sensitivity to errors and undefined offered traffic is to introduce a realistic constraint on the variance to mean ratio of the offered traffic. This is dealt with in following Sections.

3. SYSTEMS WITH QUEUEING

The systems considered are those with a single queue which has a finite number of waiting places, and which provides access to a finite group of servers. Internal congestion is taken to be negligible. The offered traffic is modelled as being either Binomial, Poisson, or Negative Binomial. Hence, the arrival coefficients are as given in the previous Section.

Assuming Negative Exponential service times, the departure coefficients are

$$c_r = \mu \quad (28)$$

where

$$x \quad \text{is the number of busy servers, given by}$$

$$x = \begin{cases} \quad r & \text{for } 0 \leq r \leq R-Q \\ R-Q & \text{for } R-Q \leq r \leq R \end{cases} \quad (29)$$

where

$Q$ is the number of waiting places,

$R-Q$ is the number of servers.

5.3B-1-3
3.1 Unconstrained Estimation of Parameters

The logarithm of \( p(\alpha, \beta, \gamma) \) can be written as:

\[
\ln(p) = \sum_{r=0}^{R} \{ n_r \ln(\alpha(S-r)) + \gamma_r \ln(x \mu) - s_r(\alpha(S-r) + x \mu) \} \tag{30}
\]

Necessary conditions to be satisfied by the estimators are found by equating the partial derivatives of \( \ln(p) \) to zero. Differentiating with respect to \( \mu \) and equating to zero yields

\[
\sum_{r=0}^{R} (-s_r x \alpha + \gamma_r \mu) = 0. \tag{31}
\]

Hence, the estimator of \( \mu \) is

\[
\hat{\mu} = W / (T \overline{x}) \tag{32}
\]

where

\[
\overline{x} \triangleq \frac{1}{(1/T) \sum_{r=0}^{R} x s_r}. \tag{33}
\]

The estimators for \( \alpha \) and \( S \) are exactly as given in Section 2, since the derivatives of the logarithm of \( p \) with respect to these two variables are unchanged.

3.2 Constrained Estimation of Parameters

As discussed in Section 2, it is possible to produce unrealistic estimates of the mean offered traffic in some cases. To overcome this problem, the constraint

\[
\frac{v}{m} = z \tag{34}
\]

is introduced, where

\( z \) is the variance to mean ratio (or peakedness) of the offered traffic, and is assumed to be known.

The case of \( z=1 \) must be dealt with separately later. For other cases, it is equivalent to use the constraint

\[
\mu = \frac{z}{1-z} \alpha \tag{35}
\]

The logarithm of \( p \) now becomes

\[
\ln(p) = \sum_{r=0}^{R} \{ n_r \ln(\alpha(S-r)) + \gamma_r \ln(x \alpha \frac{z}{1-z}) - s_r(\alpha(S-r) + x \alpha \frac{z}{1-z}) \} \tag{36}
\]

Equating the appropriate partial derivatives to zero yields the following necessary conditions for the estimators:

\[
\hat{\alpha} = (N\gamma W) / (T (s - \overline{\alpha} + \frac{z}{1-z} \overline{x})). \tag{37}
\]

and

\[
\sum_{r=0}^{R} \frac{n_r}{S-r} = T \alpha \tag{38}
\]

\[
= (N\gamma W) / (s - \overline{\alpha} + \frac{z}{1-z} \overline{x}). \tag{39}
\]

This last equation can be solved iteratively by adjusting the value of \( S \) until

\[
S = (N\gamma W) / \sum_{r=0}^{R} \frac{n_r}{S-r} = \overline{\alpha} + \frac{z}{1-z} \overline{x}. \tag{40}
\]

For the case of \( z=1 \), \( S \) must approach infinity and \( \alpha \) must approach zero in such a way that

\[
S \alpha = a_0. \tag{41}
\]

Any non-negative value of \( \mu \) will satisfy \( z=1 \). Hence, the arrival rate is estimated by

\[
\hat{\mu} = N / T \tag{42}
\]

and the service rate by

\[
\hat{\mu} = W / (T \overline{x}). \tag{43}
\]

4. SYSTEMS WITH REPEATED ATTEMPTS

4.1 Repeated Attempt Model

When congestion is high, it is possible that repeated attempts will be a significant proportion of all bids. The objective adopted here is to eliminate the effect of repeat attempts on estimates of the parameters of the offered traffic. The assumption being that if sufficient servers are provided then repeated attempts will become insignificant.

For the limited objective described above, the following model of repeated attempt behaviour is proposed. While there is no congestion the arrival rate is modelled as before. Once an unsuccessful bid occurs the arrival rate jumps to some constant value, presumably higher than normal. The arrival rate remains at this higher value until the next successful bid, at which time it reverts immediately to the normal level depending on the state of the system.

It is emphasized that this approach is proposed as a means of eliminating the effect of repeated attempts on the estimation of offered traffic. It is not intended to provide a good model for predicting repeated attempt behaviour.

To analyse the model of repeated attempt behaviour, a second state variable is introduced, namely:

\( q \) is a state variable which takes the value 0 for normal arrival rate, and 1 for higher than normal arrival rate.

\( \rho \) is the value of arrival rate for \( q=1 \).
Figure 1 shows the state transition diagram for this model.

\[ \text{Fig. 1 State Transition Diagram for Repeated Attempt Model.} \]

It is desired to solve for \( P(r,q) \) for \( r=0,\ldots,R \) and \( q=0,1 \), where

\[
P(r,q) \quad \text{is the steady state probability of the state (r,q).}
\]

First consider the subset of the state-space defined by

\[
\{ (i,1) \mid i < r, \text{ for some } r < R \}.
\]

This subset can be described as the set of all states \( (i,1) \) such that \( i \) is less than some fixed value \( r \), which in turn is less than \( R \). This subset may be entered only if a departure occurs while the system is in state \( (r,1) \). Hence the rate of entry into this subset is just \( \rho \). An arrival occurring while in any state within the subset will cause an exit from the subset, hence the rate of exit is given by

\[
r^{-1} \rho \sum_{i=0}^{r-1} P(1,1)
\]

Equating the rate of entry to the rate of exit (for steady state) gives

\[
r^{-1} \rho \sum_{i=0}^{r-1} P(1,1) = \rho \sum_{i=0}^{r-1} P(1,1)
\]

Thus, starting with an arbitrary constant for \( P(0,1) \), the values of \( P(r,1) \) for \( r=1,\ldots,R-1 \) can be computed.

Consider the state \( (R,1) \). This state will be entered if and only if an unsuccessful bid occurs while the system is in state \( (R,0) \). Hence the rate of entry into this state is \( (S-R) \alpha \). The state will be exited if and only if a departure occurs. Thus the rate of exit is \( \rho \). Equating the rates of entry and exit, we get

\[
P(R,0) = \frac{\rho}{(S-R) \alpha} P(R,1)
\]

which gives the value of \( P(R,0) \) (still in terms of an arbitrary constant for \( P(0,1) \)).

To work down from state \( (R,0) \) to state \( (1,0) \), consider subsets of the form

\[
\{ (i,q) \mid q=0,1,\ldots,R \\
q=1,2,\ldots,R-1 \}
\]

The only way of entering this subset is for an arrival to occur while in state \( (r-1,0) \). The rate of entry is thus

\[
(S-r+1) \alpha P(r-1,0)
\]

Exit from the subset may be effected by a departure from state \( (r-1,1) \) or a departure from state \( (r,0) \). Thus the rate of exit is

\[
r \mu P(r,0) + (r-1) \mu P(r-1,1)
\]

Equating the rates of entry and exit gives

\[
P(r-1,0) = \frac{\mu}{(S-r+1) \alpha} P(r-1,1)
\]

Finally, by similar argument,

\[
P(0,0) = \frac{\mu}{S \alpha} P(1,0)
\]

All the values of \( P(r,q) \) are thus computed in terms of an arbitrary constant assigned to \( P(0,1) \). The values obtained are then normalised to give the steady state probability distribution. From which time congestion is given by

\[
E = P(R,0) + P(R,1)
\]

and call congestion is given by

\[
B = \rho \sum_{r=0}^{R} P(r,1) + \sum_{r=0}^{R} (S-r) \alpha P(r,0)
\]

In computing the offered traffic, the effect of repeated attempts is ignored, since congestion will never occur with unlimited servers. Hence the expressions for mean and variance of the offered traffic remain unchanged. Similar problems still arise for \( a \) and \( h \).

4.2 Unconstrained Estimation of Parameters

The notation used previously must be extended to cope with the two-dimensional state-space for the model introduced in the previous Section. The statistics to be collected and processed are now denoted by:

\[
S_{r,q}(k) \quad \text{is the total time spent in state (r,q)},
\]

\[
u_{r,q}(k) \quad \text{is the total number of unsuccessful bids which occurred while the system was in state (r,q)},
\]

\[
v_{r,q}(k) \quad \text{is the total number of successful bids which occurred while the system was in state (r,q)},
\]

\[
u_{r,q}(k) \quad \text{is the sum of } u_{r,q}(k) \text{ and } v_{r,q}(k),
\]

To work down from state \( (R,0) \) to state \( (1,0) \), consider subsets of the form
\( w_{r,q}(k) \) is the total number of departures which occurred while the system was in state \((r,q)\).

with all statistics being collected over the period from the 0'th event to the \(k\)'th event.

The likelihood kernel is now given by (with the dependence on \( k \) omitted for brevity):

\[
p = \prod_{r=0}^{R} \left\{ \left( (S-r) \alpha \right) w^r \right\}
\]

\[
\exp(-s_{r,0}((S-r)\alpha + r \mu))
\]

\[
p_{n,r,1} \left( r \mu \right) w^r \exp(-s_{r,1}(p + r \mu) ) \}
\]

As a notational convention, the sum of a variable over all possible values of one subscript is denoted using a period to replace the appropriate subscript. For example,

\[ s_{r} = s_{r,0} + s_{r,1} \]

and

\[ s_{0} = \sum_{r=0}^{R} s_{r,0} \]

Using this notation for \( s, u, v, w, \) and \( n, \) the logarithm of \( p \) can be written as

\[
\ln(p) = \sum_{r=0}^{R} \left\{ w_{r} \ln(r \mu) - s_{r} \mu \right. \\
+ n_{r,0} \ln((S-r)\alpha) - s_{r,0} (S-r)\alpha \\
+ n_{r,1} \ln(p) - p s_{r,1} \}
\]

Taking the partial derivative with respect to \( p \) and equating to zero gives

\[
\hat{\alpha} = \frac{n_{1}}{s_{1}} \]

Following a similar procedure for \( \mu, \) it is also found that

\[
\hat{\mu} = \left( \frac{w_{0} + w_{1}}{T + r} \right) \]

where, in this context,

\[
T = \sum_{r=0}^{R} s_{r} \]

and

\[
T = s_{0} + s_{1} \]

Similarly, it can be shown that

\[
\hat{\alpha} = \frac{n_{0}}{s_{0} (S - \overline{r})} \]

where

\[
\overline{r} = \frac{1}{T} \sum_{r=0}^{R} r s_{r} \]

By a similar procedure, \( S \) must satisfy

\[
S = \frac{n_{r,0}}{(S-r)} \]

As before, this may be solved iteratively.

4.3 Constrained Estimation of Parameters.

As before, the constraint

\[
\mu = \frac{z}{1-z} \alpha
\]

is introduced. The logarithm of \( p \) is now

\[
\ln(p) = \sum_{r=0}^{R} \left\{ w_{r} \ln(r \mu) - s_{r} \mu \right. \\
+ n_{r,0} \ln((S-r)\alpha) - s_{r,0} (S-r)\alpha \\
+ n_{r,1} \ln(p) - p s_{r,1} \}
\]

By the procedure of equating partial derivatives to zero, it is found that the estimator for \( \mu \) is the same as in the case of unconstrained estimation, and that, for \( z \) not equal to one,

\[
\hat{\alpha} = \frac{w_{0} + w_{1} + n_{0}}{s_{0} (S - \overline{r} + r - T z) \frac{z}{1-z}}
\]

and finally that \( S \) must satisfy

\[
S = \frac{n_{r,0}}{S - \overline{r} + r - T z \frac{z}{1-z}}
\]

which may be solved iteratively.

As before, the case of \( z = 1 \) must be treated separately. The estimators of \( \rho \) and \( \mu \) are the same as for unconstrained estimation, and the estimator of \( \alpha \) is

\[
\hat{\alpha} = \frac{n_{0}}{s_{0} (S - \overline{r})}
\]

5. Computation

A computer program has been written to compute estimates as described above, and has been tested using simulation. The results of a small example problem are presented here by way of illustration. The problem considered is a system with repeated attempts and having the following parameters:

Number of Servers, \( R = 3 \)

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Effective Sources, \( S = 6 \)
Arrival rate per free source, \( \alpha = 1.0 \)
Mean service rate, \( \mu = 1.0 \)
Arrival rate after unsuccessful bid, \( \rho = 10.0 \)

From the values given above, the offered traffic has the following parameters:

Mean, \( m = 3.0 \)
Variance, \( v = 1.5 \)

For this system, twenty batches were run, each representing about twenty mean service times. For conversation traffic with a mean holding time of three minutes this represents an hour of measurement. The results for unconstrained and constrained estimation of the mean and variance of the offered traffic are shown in Table 1, below. The numbers in that table represent the sample mean of twenty estimates, and their sample standard deviation.

<table>
<thead>
<tr>
<th>Constraint</th>
<th>None</th>
<th>( z = 0.5 )</th>
<th>( z = 1.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>3.122</td>
<td>3.025</td>
<td>3.828</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.581</td>
<td>0.312</td>
<td>0.604</td>
</tr>
<tr>
<td>( v )</td>
<td>1.826</td>
<td>1.513</td>
<td>3.828</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.178</td>
<td>0.156</td>
<td>0.604</td>
</tr>
</tbody>
</table>

Constrained estimates may be used in practice when the variance to mean ratio is known (or assumed) to be equal to a specified value, or to lie in a specified range. In the latter case, a search must be performed over the specified range of \( z \) values to find the constrained estimator that yields the greatest value of the likelihood function.

These results demonstrate the working of the estimation algorithms. The accuracy of the estimators has not yet been studied quantitatively.

6. CONCLUSIONS

The method described in this paper allows Bayesian Statistical Inference to be applied to the two applications of systems with queueing and systems with repeated attempts. The additional refinement of constraining the estimated variance to mean ratio of the offered traffic eliminates the possibility of producing an unrealistic estimate of offered traffic due to a poor estimate of arrival and departure rates.

The method has been tested on small problems by simulation. Further studies using real data are needed before recommending adoption of the method as a practical alternative to classical methods.

There are several potential advantages offered by the method such as the following. It processes all available information, and therefore may prove more accurate than present methods. The conditional probability density function of the parameters of the offered traffic can be computed. This could be used for dimensioning based on Decision Theory (as described in [3]). Also, it is possible that the method could be extended to allow for time-varying traffic. This could be of use in applications such as Network Management.

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REFERENCES