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THE ABSTRACT CHANNEL MODEL

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ABSTRACT

In this paper we generalize the channel model to abstract case. The results are obtained by compared with Shannon model. The theory of communication has developed extensively and gained enormous importance. We should not imagine the act of coding in too obvious a manner, thus we simplity the problem. When we doesn't care how to code, we speak of a abstract codes. To a certain extent, this theory is, of course, the basis of any special theory that the code is given. Our study may not have seemed to be too complete, but it had as purpose, at least partly, to establish those properties that we want to use in any special theory.

1. SHANNON PROBLEM

We will use the following notations:

(X, Bx) : message source
(Y, By) : message sink
U : signal source
V : signal sink

Let (X, Bx) be a measurable space, Bx is a B-field over X. Similarly, assume that (Y, By) is the measurable space, By is a B-field over Y. Let U and V be two sets of real numbers.

ACS = (E, F) is said a abstract communication systems with measure structure with regard to X · U · V · Y, if

E = (X, Bx, Px(.)) is considered information source;

F = (U, P(v/u), V) is considered communication channel, where P(v/u) is the conditional distribution of V. Next,

f: X ⟶ U is called encoding

and

g: V ⟶ Y is called decoding.

If (f, g) is given, we define the measure m(x, y) on (X · Y, Bx · By) by

$$m(x \cdot y) = Px(x)P(g^{-1}(y)/f(x)) \qquad (1)$$

here

$$g^{-1}(y) = (v/g(v) = y)$$

The communication systems ACS is said to be determinate codes and is denoted ACS (f,g), (f,g) is given.

Let d(x y) be a measurable function. We should call d(x y) metric of distortion (MD). When we consider that ACS is a model of source sequance, we adopt the following convention. Let

$$d(x \cdot B) = \min_x(d(x \cdot y)/y \in B,\ B \subset Y)$$

such that

$$\mathrm{Lim}_c P_x(x/x \in H(B, C)) = 0$$

where

$$H(B,c) = (x/x \in Bx,\ d(x \cdot B) \le c)$$

Finality we define the rule of reliability of ACS (f,g) by

$$\int x \cdot y \ \ d(x \cdot y)\ dm \le c \qquad (2)$$

Now let us consider Shannon problem: Under what conditions there are proper coding (f,g) for communication systems ACS such that (1), (2) hold.

In this paper, we shall give conditions of existence of dx and dy, but not straightway find f and g. dx and dy is called MD on X and Y, if for d(x · y) have

$$d(x \cdot y) = dx\ (x) - dy\ (y). \qquad (3)$$

Hence f and g is functions of (x, dx(x)) and (y, dy(y)) respectively. That is

$$f(x) = f(x,\ dx(x)) \qquad (4)$$
$$g(x) = g(y,\ dy(y)) \qquad (5)$$

2. MEASURE

We begin the investigation of the measure m(x · y) from (2). If for d(x · y) there exists a measure $m^*(x \cdot y)$ such that

$$c(m^*) = \min \int x \cdot y \ \ d(x \cdot y)\ dm \qquad (6)$$

then (2) hold, so that $c(m^*) \le c$

In order to obtain dx and dy, we must have a restriction for m(x · y). If Py(.) be given, we put

$$m(x \cdot y) \le P\ \ x(x), \qquad x \in Bx \qquad (7)$$
$$m(x \cdot y) \ge P\ \ y(y), \qquad y \in By \qquad (8)$$

For example, if Px(x) Py(y) = 0, we have

$$m(x \cdot y) = cPx(x)Py(y) \qquad (9)$$

here, $c \in (1/x(x),\ 1/y(y))$. From this example we immediately conclude that if Px(.) Py(.) = 0 then (9) satisfies (7) (8).

Generally we have some measures m^n which satisfies (7) and (8).

On the other hand, if

$$m^n(X \cdot Y) \le \text{constant}$$

for all n and

$$m^n \longrightarrow m^*$$

By extended theory of Vitali-Hahn-Saks ([5], P.43), then m^* is measure on (X · Y, Bx · By). We wish to show that m^* satisfies (7), (8) and (6).

Let d be bounded continuous on X Y. Let Px(.) and Py(.) be finite measures. Then for (m^n) have

$$(\int x \cdot y \ \ d \ \ dm^n). \qquad (10)$$

If there are a subsequence of (10) which converges to least upper bound (or greatest lower bound) of (10), written as

$$\int X \cdot Y \, d\, dm^* \quad (11)$$

then m^* is called a optimization measure. It is known if the sequence (10) of real number is bounded. then (10) must contains a subsequence which converges to least upper bound (or greatest lower bound) of (10). (m^n) is called weakly converges and written as

$$m^n \xrightarrow{W} m^* \quad (12)$$

As regards this matter we will use the following result.

Theorem 1: ([5]. P.196) let m^1, m^2 ... and m^* be finite measures on the Borel sets B (X · Y) of a metric space X · Y. For every x · y ∈ B (X · Y) have

(i) $m^n(x \cdot y) \longrightarrow m^*(x, y)$

(ii) $m^*(d\,(x \cdot y)) = 0$

then

$$m^n \xrightarrow{W} m^*$$

We now proceed to show that m^* satisfies (7) and (8).

Theorem 2: Let X and Y be metric space. Let d is continuous bounded functions. Let Px(.) and Py(.) are complete and satisfies (7) and (8). If

$$m^n \xrightarrow{W} m^*$$

then m^* satisfies (7) and (8)

Proof: Since $m^n \xrightarrow{W} m^*$ by 5 (P.196), we have

$$m^n(X \cdot Y) \longrightarrow m^*(X \cdot Y)$$

and for every open subset $O \subset X \cdot Y$, have

$$\lim_n \inf m^n(O) \geq m^*(O) \quad (13)$$

or for every closed subset $c \subset X \cdot Y$, have

$$\lim_n \sup m^n(c) \leq m^*(c) \quad (14)$$

Let open set $O \in B(X)$, by (7) and (13), we have

$$PX(O) \geq \lim_n \inf m^n(O \cdot Y)$$
$$\geq m^*(O \cdot Y) \quad (15)$$

Since P(.) is complete, by (15), for any $x \in B(Y)$, we have

$$Px(x) = \inf\,(Px(O)/x \subset O \in B(X))$$
$$\geq \inf\,(m^*(O \cdot Y)/x \subset O \in B(X)) \geq m^*(x \cdot y)$$

Similarly let closet set $C \in B(Y)$, by (8) and (14), we have

$$Py(c) \leq \lim_n \sup m^n(X \cdot C) \leq m^*(X \cdot C) \quad (16)$$

Since Py (y) is complete, by (16), for any $y \in B(y)$, we have

$$Y(y) = \sup\,(Py(c)/c \subset y \in B(Y))$$
$$\leq \sup\,(m^*(X \cdot c)/c \subset y \in B(Y)) \leq m^*(X \cdot Y)$$

Thus $m^*(x \cdot y)$ satisfies (7) and (8).

3. METRIC OF DISTORTION

In this section, we first give the definition of the support concept. The support of the measure m on $x \subset X$ is call measurable open neighborhood which has positive m-measure of X. The set of all support on X is called the support of m on X, denoted Sm(X).

Let m satisfy (7), (8) and let

$$d(x) - dy(y) \geq d(x\ \ y) \quad x \in Bx,\ y \in By \quad (17)$$

hold. where (dx, dy) ≥ 0. If

$$\int_{Sm(X \cdot Y)} (d-(dx-dy))\, dm = 0 \quad (18)$$

We say that (dx, dy) is support of MD.

Theorem 3: Let Tx Bx be topology over X, and let $Ty \subset By$ be topology over Y. Let be continuous with respect to $Tx \cdot Ty \subset Bx \cdot By$. Then there exist functions Bx and By such that (dx, dy) is support of MD.

Proof: Let $(Xik \cdot Yik) \in Sm(X \cdot Y)$, k=1,2,..., ni(ni be finite). For any $x \in Bx$, we define

$$dx(x) = \sup_i \left(-d(x \cdot yi1) + \sum_{k=1}^{ni} d(X_{ik} \cdot Y_{ik}) - \sum_{k=1}^{ni-1} d(X_{ik} \cdot Y_{ik+1})\right)$$

By (17), for any y BY, we define

$$d(y) = \inf_{x \in Bx} (dx(x) + d(x\ \ y))$$

We imitate [6], then obtain that if d is bounded then dx is bounded. It follows that dy is bounded too. Next since

$$dx(x) \geq -d(x \cdot yi1) + d(xi1 \cdot yi1) + dx(xi1)$$

that is

$$dx(x) + d(x \cdot yi1) \geq d(xi1 \cdot yi1) + dx(xi1)$$

$$\inf_{x \in Bx} (dx(x) + d(x \cdot yi1)) = d(xi1 \cdot yi1) + dx(xi1)$$

Hence

$$dy(yi1) = d(xi1 \cdot yi1) + dx(xi1)$$

and the proof is complete.

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