THE ABSTRACT CHANNEL MODEL

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ABSTRACT

In this paper we generalize the channel model to abstract case. The results are obtained by compared with Shannon model. The theory of communication has developed extensively and gained enormous importance. We should not imagine the act of coding in too obvious a manner, thus we simplify the problem. When we don't care how to code, we speak of a abstract codes. To a certain extent, this theory is, of course, the basis of any special theory that the code is given. Our study may not have seemed to be too complete, but it had as purpose, at least partly, to establish those properties that we want to use in any special theory.

1. SHANNON PROBLEM

We will use the following notations:
(X, E) : message source
(Y, B) : message sink
U : signal source
V : signal sink

Let (X, Bx) be a measurable space. Bx is a B-field over X. Similarly, assume that (Y, By) is the measurable space. By is a B-field over Y. Let U and V be two sets of real numbers.

ACS = (X, P) is said a abstract communication systems with measure structure with regard to \( X \cup U \cup V \cup Y \), if

\[
E = (X, Bx, P_x(.)) \text{ is considered information source;}
\]

\[
F = (U, P_v(u), V) \text{ is considered communication channel, where } P_v(u) \text{ is the conditional distribution of } V. \text{ Next,}
\]

\[
f : X \rightarrow U \text{ is called encoding and}
\]

\[
g : V \rightarrow Y \text{ is called decoding.}
\]

If \( (f, g) \) is given, we define the measure \( m(x, y) \) on \( X \times Y, \text{ } Bx \times By \) by

\[
m(x, y) = P_x(x)P_v(g(y)|f(x))
\]

(1)

Here

\[
g^{-1}(y) = \{w \in V | g(w) = y\}
\]

The communication systems ACS is said to be determinate codes and is denoted ACS \((f, g), (f, g)\) is given.

Let \( d(x, y) \) be a measurable function. We should call \( d(x, y) \) metric of distortion (MD). When we consider that ACS is a model of source sequence, we adopt the following convention. Let

\[
d(x, B) = \min_{y} d(x, y) / y \in B, B \subseteq Y
\]

such that

\[
\lim_{P_x(x) \rightarrow 0} P_x(x)E(H(B, C)) = 0
\]

where

\[
H(B, C) = \{x / x \in E, d(x, B) \leq \epsilon\}
\]

Finally we define the rule of reliability of ACS \((f, g)\) by

\[
\int x, y d(x, y) \ dm \leq \epsilon
\]

(2)

Now let us consider Shannon problem: Under what conditions there are proper coding \((f, g)\) for communication systems ACS such that (1), (2) hold.

In this paper, we shall give conditions of existence of \( dx \) and \( dy \), but not straightforward find \( f \) and \( g \). \( dx \) and \( dy \) is called MD on \( X \) and \( Y \). If for \( d(x, y) \) have

\[
d(x, y) = dx(x) - dy(y)
\]

(3)

Hence \( f \) and \( g \) are functions of \( (x, dx(x)) \) and \( (y, dy(y)) \) respectively. That is

\[
f(x) = f(x, dx(x))
\]

(4)

\[
g(y) = g(y, dy(y))
\]

(5)

2. MEASURE

We begin the investigation of the measure \( m(x, y) \) from (2). If for \( d(x, y) \) there exists a measure \( m^0(x, y) \) such that

\[
oc(m) = \min \int x, y d(x, y) \ dm
\]

then (2) hold, so that \( c(m) \leq 0 \)

In order to obtain \( dx \) and \( dy \), we must have a restriction for \( m(x, y) \). If \( Py(.) \) be given, we put

\[
m(x, y) \leq P(x, y)
\]

(6)

Then (2) hold, so that \( c(m) \leq 0 \)

For example, if \( P_x(x) = 0 \), we have

\[
m(x, y) = 0
\]

(7)

Generally we have some measures \( m^0 \) which satisfies (7) and (8).

On the other hand, if

\[
m^0(x, y) \geq \text{constant for all } m
\]

\[
m \geq m^0
\]

By extended theory of Vitali-Hahn-Saks (15, P.43), then \( m^0 \) is measure on \( X \times Y, Bx \times By \). We wish to show that \( m^0 \) satisfies (7), (8) and (6).

Let \( d \) be bounded continuous on \( X \). Let \( P_x(.) \) and \( P_y(.) \) be finite measures. Then for \( m^0 \) have

\[
\int x, y d(m^0)
\]

(9)

If there are a subsequence of (10) which converges to least upper bound (or greatest lower

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We have
\[ m(x) \leq m(x,y) \leq m(x,y) + m(Y) \]
Then we have
\[ m(x) \leq m(x,y) \]
As regards this matter we will use the following result.

Theorem 1: ([5], P.196) Let \( m_1, m_2, \ldots \) and \( m^* \) be finite measures on the Borel sets \( B(X \times Y) \) of a metric space \( X \times Y \). For every \( x \in B(X \times Y) \) have
\[ m^*(x,y) = m^*(x) + m^*(y) \]
We now proceed to show that \( m^* \) satisfies (7) and (8).

Theorem 2: Let \( X \) and \( Y \) be metric space. Let \( d \) is continuous bounded functions. Let \( P_x(.) \) and \( P_y(.) \) are complete and satisfies (7) and (8). If
\[ m^* \rightarrow m^* \]
then \( m^* \) satisfies (7) and (8).

Proof: Since \( m^* \rightarrow m^* \) by (P.196), we have
\[ m^*(x,y) \rightarrow m^*(x,y) \]
and for every open subset \( 0 \subset X \times Y \), have
\[ \lim \inf m^*(x,y) \geq m^*(0) \]
for every closed subset \( c \subset X \times Y \), have
\[ \lim \sup m^*(o) \leq m^*(c) \]
We have \( P_x(0) \geq \inf m^*(0,y) \)
Since \( P_x(.) \) is complete, by (15), for any \( x \in B(Y) \), we have
\[ P_x(x) = \inf (P_x(y)/x \in B(x)) \]
Similarly let closed set \( C \subset B(Y) \), by (8) and (14), we have
\[ \lim m^*(x,c) \leq m^*(x,c) \]
Since \( P_y(.) \) is complete, by (16), for any \( y \in B(Y) \), we have
\[ Y(y) = \inf m^*(x,c)/y \in B(Y) \]
Thus \( m^*(x,y) \) satisfies (7) and (8).

3. METRIC OF DISTORTION

In this section, we first give the definition of the support concept. The support of the measure \( m \) on \( X \) is called measurable open neighborhood which has positive \( m \)-measure of \( X \). The set of all support on \( X \) is called the support of \( m \) on \( X \), denoted \( Sm(X) \).

Let \( m \) satisfy (7), (8) and let
\[ d(x) - dy \geq d(x,y) \]
hold. where \( (dx, dy) \geq 0 \). If
\[ \int m(x,y) (d(x,dy)) \, dm = 0 \]
We say that \( (dx, dy) \) is support of \( m \).