ENGINEERING TRAVERSE TRUNKS IN A TRANSIT NETWORK

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ABSTRACT

A new network design method called the "Skewer Connection Method" is proposed for constructing future networks which must meet the rapidly growing demand for new service traffic. In this method, traffic from one center to another is carried by a tandem route switched through a third same-level center like a skewer. The method makes use of the traffic bundling effect which enables the number of direct trunks to be reduced. The economic advantage, i.e., cost saving, of this method is analyzed by derived formulas and numerical examples. It is shown that the skewer connection method is advantageous when: i) the node-to-node traffic is small, ii) the nodes are located far apart, and iii) skewer tandem node cost is small. This method improves existing hierarchical networks and enhances the possibilities of constructing even higher-quality networks in the future.

1. INTRODUCTION

Most existing public telephone networks have hierarchical structures with an alternate routing scheme, in which overflow traffic from a direct traverse trunk is routed to a second choice path, if such exists. This alternate routing scheme results in economical network engineering and operation. However, predicted cost reductions in network components through rapidly changing technology coupled with high growth in traffic are now provoking studies on new network structures and trunk engineering methods.

The above trends suggest that future network nodes will be densely connected to each other with direct trunks, i.e., quasi-complete graph networks ([1] - [3]). However, applying this type of structure to the ever-growing large-scale transit networks will require highly sophisticated network design and administration methods. To make both network facilities and administration as economical as possible, simpler regularized topologies for practical networks are desirable. As one such topology, a hierarchical mesh structure constructed according to the following rules is presented:

1) Transit centers closely related to each other in terms of both traffic flow and geographical location are connected in a mesh, and

2) Traffic to/from other areas flows via a specified transit center.

This paper proposes the "skewer connection method", a new engineering method for using direct traverse trunks suitable to the above type of hierarchical mesh structure. In this method, small traffic loads which cannot afford direct traverse trunks to their destined center are carried by a tandem route switched through a third center on the same level like a skewer. The method is compared with a conventional alternate routing method mainly from an economical viewpoint to clarify its area of application. For this purpose, we have formulated and closely examined the advantages of the skewer connection and conventional alternate routing methods over the method in which only direct final trunks are utilized.

2. DESIGN METHODS AND NETWORK MODEL

2.1 Network Design Methods

The following four basic network design methods using direct trunks in different ways are considered:

Method A: Traffic to/from other areas is carried only by basic trunks via higher level transit centers (no direct trunks).

Method B: Traffic to/from other areas is carried only by direct trunks used as final routes.

Method C: Traffic to/from other areas is first offered to direct trunks used as high-usage routes, then overflows to basic trunks if necessary.

Method D: Traffic to/from other areas is carried by direct trunks used as final routes with skewer connections.

In terms of network costs, Method C is generally advantageous over Methods A and B because of the increased trunk efficiency provided by alternate routing. Method D is more economical than Method B because of the increased efficiency obtained from the traffic bundling by the skewer connection. The merits and demerits of Methods C and D cannot be discussed by simple
qualitative observations, however; it is necessary to compare them quantitatively. In Section 3, the cost advantages of the alternate routing and skewer connection methods over Method Bare examined. A hybrid method which uses both alternate routing and skewer connection is not discussed in this paper.

2.2 Network Model

An essential model of a hierarchical mesh structure which can be analyzed easily with no loss in generality is shown in Fig.1. This structure is a part of a large transit network consisting of higher-level RC's (Regional transit Centers) and lower-level TC's (Transit Centers). The TC's in an RC area are fully interconnected by direct final trunks because these nodes are located in short distance and have large traffic demands between one another. It is assumed that both RC-RC and RC-TC basic trunks provide high trunk efficiency. The above three methods B, C and D differ only in how the T1-T2 or T1-T3 traffic in Fig.1 is carried. Therefore, in Section 3, the design methods are compared in terms of the relative network (switching and transmission) cost required for carrying these two traffic. The network cost realized by each design method was formulated in Table 1 using the following parameters:

- \( a_1(a_2) \): node-to-node traffic for T1-T2 (T1-T3),
- \( a_0 \): background traffic for T2-T3,
- \( C_s \): switching cost per erlang
- \( C_c \): transmission cost of one trunk,
- \( \alpha \): ATC (additional trunk capacity) of basic trunks,
- \( n_1(n_2) \): the numbers of T1-T2 (T1-T3) direct traverse trunks,
- \( a_1'(a_2') \): overflow traffic from T1-T2 (T1-T3) direct traverse trunks,
- N(a): the numbers of trunks required for offered load "a" under the constraints of link blocking probability b0.

Trunk dimensioning methods employed are:
1) The number of high usage (direct traverse) trunks \( n_1 \) and \( n_2 \) in the alternate routing method are engineered with the so-called ECCS method.
2) The number of final direct traverse trunks is calculated under the constraints of the link blocking \( b_b \) (set 0.01 in the numerical examples in this paper).
3) The grade of service for calculating the number of basic trunks accepting overflow traffic from high usage routes is the same as the above 2). This number is usually calculated by using the mean and the variance

\[
\begin{align*}
N(a_0) &= \text{the numbers of trunks required for offered load } a_0 \\
N(a_0 + a_2) &= \text{the numbers of trunks required for offered load } a_0 + a_2
\end{align*}
\]

Table 1 Comparison of network resources (except common resources)

<table>
<thead>
<tr>
<th>Resources</th>
<th>Method</th>
<th>(a) Basic trunk only</th>
<th>(b) Direct final trunk only</th>
<th>(c) Alternate routing</th>
<th>(d) Skewer Connection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switching</td>
<td>T1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( a_0 \times C_s )</td>
</tr>
<tr>
<td>cost</td>
<td>R1</td>
<td>((a_1 + a_2) \times C_s)</td>
<td>0</td>
<td>((a_1' + a_2') \times C_s)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>R2</td>
<td>((a_1 + a_2) \times C_s)</td>
<td>0</td>
<td>((a_1' + a_2') \times C_s)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>T1 - R2</td>
<td>((a_1 + a_2) \times C_s)</td>
<td>0</td>
<td>((a_1' + a_2') \times C_s)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>T1 - R1</td>
<td>((a_1 + a_2) \times C_s)</td>
<td>((a_1' + a_2') \times C_s)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>R2 - T2</td>
<td>( a_1' \times C_s )</td>
<td>0</td>
<td>( a_1' \times C_s )</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>R2 - T3</td>
<td>( a_2' \times C_s )</td>
<td>0</td>
<td>( a_2' \times C_s )</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>T2 - T3</td>
<td>( a_0 \times C_s )</td>
<td>( a_0 \times C_s )</td>
<td>( a_0 \times C_s )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Direct T1 - T2</td>
<td>0</td>
<td>( a_1 \times C_s )</td>
<td>( a_0 \times C_s )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Trunk T1 - T2</td>
<td>0</td>
<td>( a_2 \times C_s )</td>
<td>( a_0 + a_2 \times C_s )</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig.1 Network model

* Switching cost is assumed to be proportional to switched traffic. For example, this cost function is suitable for building block structure switching systems.

** ECCS method: economical hundred call seconds method. This is called the LTC (last trunk capacity) method in Japan.
of traffic. However, for simplicity, only the mean value is used in the numerical examples in Section 3.

3. COMPARISON OF DESIGN METHODS

Let us examine the economic advantages (cost savings) of alternate routing (Method C) and skewer connection (Method D) over the method in which only direct final trunks are used (Method B). Since the difference among Methods B, C and D results from the different methods of trunk operation used (i.e., direct final, high usage and skewer connection), the advantage of Method C defined in this paper can be called cost saving by high usage trunking, and the advantage of Method D can be called cost saving by skewer connection. These cost savings are expressed as follows:

Cost saving by high usage trunking \( H = X^*_b - X^*_c \), (1)

and

Cost saving by skewer connection \( S = X^*_b - X^*_d \), (2)

where \( X^*_i \) = network cost designed by method \( i \).

3.1 Economic Advantages of High Usage Trunking (H)

Many papers have dealt with alternate routing (e.g., Refs [4] - [6]). This section focuses on the cost saving \( H \) achieved through using direct traverse trunks as high usage trunks, taking a cost ratio (defined here as direct route cost / alternate route cost) and traffic offered as parameters. The relation \( H > 0 \) is easily deduced, considering that high usage trunks are optimally engineered by the ECCS method.

Using terms in Table 1, cost saving \( H \) is calculated as:

\[
H = \sum_{i=1}^{2} H_i \tag{3}
\]

\[
H_i = \frac{N(a_i) - n_i}{\alpha} (C + c_1 + c_i + 2 \alpha C) - W_i \frac{K_i (N(a_i) - n_i)}{\alpha} \tag{4}
\]

where

\[
K_i = \frac{C(i)}{W_i},
\]

\[
W_i = C + c_1 + c_4 + 2\alpha C,
\]

\[
C(i) = c_0, \quad C(2) = c_2.
\]

The above value \( K_i \) (i=1,2) is the cost ratio of direct route / alternate route for traffic \( a_i \), and \( W_i \) corresponds to the alternate route cost. The term \( H_i \) is the cost saving for traffic \( a_i \) by high usage trunking. Since \( H_1 \) and \( H_2 \) have the same form, the characteristics of \( H \) can be clarified by examining either \( H_1 \) or \( H_2 \). For this purpose, the general expression \( H_i \) (Eq. (4)) is used.

Cost saving \( H_i \) is proportional to the cost of the alternate route \( W_i \). The term \( K_i \) depends on traffic \( a_i \) and cost ratio \( K \) when \( W_i \) is fixed.

The characteristics of \( H \) vs. \( a \) are shown in Fig. 2. The oscillation of cost curves is due to the discreteness of the numbers of trunks. From observing Fig. 2, it can be said that:

a) When traffic is fixed, the larger the cost ratio is, the larger the cost saving is. In other words, high usage trunking is effective in cases where the direct route cost is not much cheaper than the alternate route cost (see Appendix 1).

b) For a cost ratio in excess of roughly 1/2, the larger the offered traffic is, the more the cost saving is. In contrast, for a cost ratio of less than roughly 1/2, the cost saving disappears as the offered traffic increases (see Appendix 2).

3.2 Economic Advantages of Skewer Connection (S)

Using the factors in Table 1, cost saving by skewer connection \( S \) (Eq.(2)) is expressed as follows:

\[
S = N(a_2) c_5 + \{N(a_1) - N(a_1 + a_2)\} c_0 + \{N(a_0) - N(a_0 + a_2)\} c_4 - a_2 c s
\]

\[
= P c_0 + Q c_4 - a_2 c_s - N(a_2) \Delta c,
\]

where

\[
c_5 = c_0 + c_4 - \Delta c \quad (\Delta c > 0),
\]

\[
P = N(a_1) + N(a_2) - N(a_1 + a_2),
\]

\[
Q = N(a_0) + N(a_2) - N(a_0 + a_2).
\]

From the above equations, cost saving \( S \) is characterized as the trade-off between the positive terms \( P c_0 \) and \( Q c_4 \) and the negative terms \( a_2 c_s \) and \( N(a_2) \Delta c \).

Both of the two positive factors \( P c_0 \) and \( Q c_4 \) signify a reduction in the number of direct final trunks. This reduction is due to the increased trunk efficiency obtained from com-
bining traffic parcels which are too small to justify sending them by direct routes into a larger traffic parcel which may be sent by a direct route economically.

In order to analyze the above cost saving factors in detail, we first discuss $P$ and $Q$. The term $P$ is a function of traffic $a_1$ and $a_2$. Figure 3 shows a relationship between $a_2$ and $P$ when $a_1$ is fixed. The oscillation of the curves is due to the discreteness of the numbers of trunks, as mentioned in Sect. 3.1. From this figure, it is understood that the value of $P$ becomes larger as $a_2$ increases, but levels off when $a_2$ is large.

The former characteristic is analytically proven by:

$$P = \frac{1}{f(a_2)} - \frac{1}{f(a_1 + a_2)} > 0$$

where $f(a)$ means the ATC for traffic $a$.

The latter characteristic is confirmed by:

$$\lim_{a_2 \to \infty} P = N(a_1) - N(a_1 + a_2)$$

It is easily understood that a similar relationship holds between $a_1$ and $P$ when $a_2$ is fixed. Furthermore, the relationship between Q and $a_0$ or $a_1$ is similar to that between $P$ and $a_1$ or $a_2$. Therefore, the total value of the positive factors ($Pc_0 + Qc_4$) also increases as $a_2$ increases, but levels off in a very large $a_2$ region.

The negative factors have the following characteristics. One of the negative factors, $a_2^2c_s$, is the skewer connection switching cost. This is incurred by traffic $a_2$ switched at node $T_2$. This factor increases linearly as $a_2$ increases. The other negative factor, $N(a_2)^2c$, is the cost saving in Method B (positive factor for Method B), obtained from the reduction in $T_1$-12 route length or transmission facility cost at $T_2$ node.

Considering all the above positive and negative factors, cost saving by skewer connection is characterized as a convex function of traffic $a_2$ and disappears in a large $a_2$ region. These characteristics are illustrated in Fig.4.

3.3 Comparison between Methods C and D

Based on the discussions in Sections 3.1 and 3.2, in which cost parameters are fixed, this section compares the two cost savings $H$ and $S$, varying the values of cost parameters.

Let us suppose that $c_0 = c$, $c_3 = c + c_4$ and $c_1 = c_2 = c_3 = c_4 = c_f$ (set to a fixed value), then

$$A = W \sum_{i=1}^{2} \left( K_1 (N(a_i) - n_i) - a_1' / \alpha \right),$$

$$S = PC + Qc_f - a_2^2c_s,$$

where $W = c_4 + 2c_4c_4$, $K_1 = C/W$ and $K_2 = (Cc_f)/W$.

When the transmission cost $C$ is large, cost saving $H$ proportional to $W$ becomes large. Furthermore, $H$ does not level off no matter how large the traffic $a_2$ is, because large $C$ yields large cost ratios $K_1$ and $K_2$. On the other hand, when $C$ is small, cost saving $H$ becomes small and levels off in a large $a_2$ region. In the same manner, cost saving $S$ by skewer connection increases as $C$ increases, but it always disappears in a large $a_2$ region regardless of the value of cost parameter $C$. These relationships are illustrated in Fig.5.
From the above discussion, it can be said that the skewer connection method is more favorable than alternate routing when:

a) The transmission cost of \( R_1 - R_2 \) (or \( T_1 - T_2 \)) link (which is proportional to the node-to-node distance) is large, and the traffic from \( T_1 \) to \( T_2 \) is small, and

b) Cost/erl of a skewer tandem switching system is relatively small, and transmission multiplexing/demultiplexing cost located at a skewer tandem center is small.

4. CONCLUSION

A hierarchical mesh structure with a "skewer connection" design method was proposed as a favorable transit network structure for the future. The skewer connection method was found to be advantageous when: i) the node-to-node traffic is small, ii) the nodes are located far apart, and iii) skewer tandem node cost is small.

The trunk engineering process of the skewer connection method is easier than that of conventional alternate routing. However, if this method is applied throughout a large-scale network, the final result will be a kind of non-hierarchical network having a "chaotic" quasi-complete graph, whose design and operation would be quite complicated. Therefore, the skewer connection should be utilized in an economically feasible portion of a network, instead of being spread over an entire network, in conjunction with the alternate routing which has a wide application area.

REFERENCES


APPENDIX 1

Let $H(K)$ be the cost saving by high usage trunking when the cost ratio is $K$. Assuming $K_1 < K_2$,

$$H(K_2) - H(K_1) = \frac{(K_2 - K_1)N - \alpha E(n_2, a) - \alpha E(n_1, a)}{\alpha} - K_2 n_2 + K_1 n_1.$$  

(A-1)

Since $\alpha K = -\frac{\alpha E(n, a)}{\alpha}$,

$$\alpha K_1 (n_1 - n_2) < \alpha E(n_2, a) - \alpha E(n_1, a) < \alpha K_2 (n_1 - n_2).$$

Thus,

$$H(K_2) - H(K_1) > (K_2 - K_1)N - K_2 (n_1 - n_2) - K_1 (n_1 - n_2).$$

where $N$: the number of direct trunks required to carry traffic "a" within a given link blocking probability objective,

$n_1$: the number of high usage trunks required when cost ratio is $K_1$,

$E(n, a)$: link blocking probability when traffic "a" is offered to $n$ trunks.

APPENDIX 2

Cost saving by high usage trunking ($H$) is equal to zero iff no traffic is desired to be offered to basic trunks; i.e., the number of high usage trunks $n$ is equal to the value of $N$ in Appendix 1. This condition is:

$$\frac{\alpha E(n-1, a)}{\alpha} - \frac{\alpha E(n, a)}{\alpha} > K.$$  

(A-3)

and $E(N, a) = b_0$.  

(A-4)

From the above relationships,

$$K < \frac{b_0}{\alpha} \frac{N}{a}.$$  

(A-5)

By using the following approximate formula of Wilkinson for $b_0 = 0.01$,

$$N = 0.970a + 1.752a^2 + 2.277 (a > 6.2).$$  

(A-6)

Ineq.(A-5) is expressed as follows:

$$0.0202a - 1.770a^2 + (100aK - 2.3) \leq 0.$$  

(A-7)

There exists "a" which satisfies Ineq.(A-7) iff $\alpha K < 0.41$.  

(A-8)

Thus, when $\alpha = 0.83$,

$$K < 0.5.$$  

(A-9)