MULTICHANNEL SERVICES: PERFORMANCE OF SWITCHING NETWORKS

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We present a method for determining the traffic performance of large-scale T-S-T networks switching a mixture of traffic streams of different bandwidth requirements. The mutual dependence of the matching process and the states of the T-stages is taken into account. The performance is determined, for each stream, in terms of the end-to-end loss and the matching effort. Numerical considerations in treating large-scale networks, employing high-capacity T-stages, are discussed with emphasis on computational simplicity, efficiency, robustness, and accuracy. The analysis is verified by simulations. The T-T-T network performance is briefly discussed.

1. INTRODUCTION

Network sharing by a mixture of traffic streams of different bandwidth requirements introduces an interesting problem of traffic-capacity evaluation. In treating a single shared trunk group operated as a loss system, useful results were reported in [1], [2], and [3], and network applications were reported in [4], [5], and [6]. In this paper, we concentrate on the case of T-S-T networks. Our objective is to develop a high-precision method for the calculation of blocking performance and matching effort (the mean value of the number of slots scanned per call). Time-slot expansion, to reduce or eliminate the blocking difference between streams and reduce the overall matching effort, has been studied. A T-S-T network is constructed from a number of peripheral time switches terminated on a central time-multiplexed space switching stage (Figure 1). The space switch and each time switch are full-access internally-nonblocking switching units. The outer side of each time switch represents a network port, and the inner side is connected to the space switch by a high-capacity link.

The interdependence of the states of the outer stages and the matching process is modeled by the use of state-dependent transition probabilities in formulating the system's state equations. The high-precision realized by recognizing state-interdependence was demonstrated in [7] for a system of tandem trunk groups, and in [8] for a high-capacity multistage network where the path selection is based on state comparison of available routes. The analytical results presented here are verified by simulations. A high-speed simulation technique enabled us to obtain results based on millions of attempts, for tighter confidence intervals, in a relatively short computer time. Results for the case of T-T-T networks are also given, although, due to lack of space, the analysis has been omitted.

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2. NOTATION, DEFINITIONS, and ASSUMPTIONS

The link channels shall interchangeably be referred to as slots, and the outer T-stages will be referenced as peripherals. There are $N$ peripheral modules, each comprises a transmitting and a receiving time switch, and serves two-way traffic. $M$ denotes the number of channels per network port, and $m$ is the number of channels per link. If a peripheral module supports line units, then $M$ is normally much larger than $m$, since the occupancy per line is usually a small fraction of an Erlang. The trunks served by a peripheral module are assumed to form a single group. Time-slot expansion, to facilitate matching, may be introduced by limiting the number of used channels per link to a value which is less than that determined by the physical capacity of the link. The expansion may be hardware or software controlled. Software-controlled expansion is more flexible; it may be tuned in small increments and may vary on a per-stream basis and from one peripheral to another.

In a connection setup, $PM_1$ and $PM_2$ denote the originating and terminating peripherals respectively. Their links to the space switch are denoted $L_1$ and $L_2$. $PM_1$ and $PM_2$ are predefined by the physical location of the calling and called parties. A multi-channel call originating from $PM_1$ and destined for $PM_2$, and requiring $\omega$ channels, is blocked at $PM_1$ or $PM_2$ if the number of busy channels in either $L_1$ or $L_2$ exceeds $n - \omega$, where $n$ ($n \leq m$) is the maximum number of used channels per link. In the 'equalizing expansion' discussed in section 5, a new arrival, regardless of its channel requirement, is rejected if the number of free channels in either $L_1$ or $L_2$ is less than a prescribed value $\Delta$. Matching loss, for an $\omega$-channel stream, is defined as the unavailability of $\omega$ matching slots given that the numbers of busy channels in $L_1$ and $L_2$ are within the permissible limit. Unsuccessful connection attempts, due to peripheral blocking or matching loss, are assumed to be cleared. In a 4-wire-equivalent connection, there is a one-to-one correspondence between the channels assigned in the forward path ($PM_1$ to $PM_2$) and those assigned in the return path ($PM_2$ to $PM_1$). Thus, once a forward path is found, a free return path is guaranteed.

The traffic originating from each peripheral is assumed to be Poissonian. This assumption is based on a significant level of concentration within or preceding each peripheral, so that the traffic is generated by a larger number of independent sources. In modeling peaked or smooth traffic, state-dependent external arrivals may be specified [3]. Let $S$ be the number of streams, and let $\lambda_i$ and $v_i$ be the mean external arrival rate, originating and terminating, per peripheral and the per-call channel requirement of stream $i$. The weighted offered load for stream $i$ is defined as $a_i = v_i \lambda_i / \mu$, $\mu$ being the mean service rate. The streams are identified by the parameters $(a_i, v_i)$, $i=1..S$.

Traditionally, the traffic performance of a switching network is defined in terms of its (internal) matching-loss. Different configurations, path-selection methods, and connection rearrangement schemes are evaluated on the basis of their effectiveness in reducing the matching loss. The reduction of matching loss, by one artifice or another, does not necessarily result in a proportionate reduction in the end-to-end loss. Under a given traffic condition, a reduction in matching loss results in increased peripheral blocking, due to their mutual dependence. It is more appropriate then to evaluate the network loss by comparing its end-to-end blocking with that which would result if the same peripherals were interconnected by an ideal internally-nonblocking network.
3. ANALYSIS

Consider an \( n \)-channel trunk group offered \( S \) Poissonian streams, \((a_i, v_i), i=1..S\). The \( S \)-dimensional set of equations describing the system satisfies the local balance condition and, hence, has an exact recursive solution. A significant simplification, derived in [1] and [2], defines the aggregate state of the trunk group as the number \( x \) of its busy channels and the state-probability vector \( P \) is determined by the recursion:

\[
\sum_{j=1}^{S} a_i p_{x-v_i} = x p_x, \quad x=1..n, \quad p_x = 0 \text{ for } x<0, \quad \sum_{x=0}^{n} p_x = 1
\]

(1)

If the trunk group was offered state-dependent arrivals of mean rates \( \Lambda_{i,x}, i=1..S, x=0..n \), the corresponding \( S \)-dimensional set of equations may neither satisfy the local balance condition nor lend itself to simple recursive solutions. Fortunately, when the mean arrival rates vary slowly with \( x \), i.e.

\[
\left| \Lambda_{i,x} - \Lambda_{i,x-1} \right| / \Lambda_{i,x-1} \ll 1, \quad i=1..S, \quad x=1..(n-v_i),
\]

(2)

then, over a wide range of parameters, the exact state-probability vector \( P \) as determined from the \( S \)-dimensional set of equations deviates slightly from the vector \( \Pi \) determined from:

\[
\sum_{i=1}^{S} \alpha_{i,x-v_i} \pi_{x-v_i} = x \pi_x, \quad x=1..n, \quad \pi_x = 0 \text{ for } x<0, \quad \sum_{x=0}^{n} \pi_x = 1,
\]

(3)

where \( \alpha_{i,x} = v_i \Lambda_{i,x} / \mu \),

and \( \Pi \) can be used to approximate \( P \). This has been verified by comparison with the exact \( S \)-dimensional solutions for various values of \( m \) with different traffic mixtures.

This approximation may be used to model the mutual impact of the peripheral states and the matching process. Let \( P \) and \( Q \) represent the aggregate-state probability vectors of \( PM_1 \) and \( PM_2 \). The mean external arrival rates at each peripheral are assumed to be equal with a balanced community of interest among all peripheral pairs. Hence, \( P \) and \( Q \) are identical. Let \( f_{\omega,x,y} \) be the probability of connection attempt failure when an \( \omega \)-channel arrival finds \( PM_1, PM_2 \) in state \((x,y)\). A transition to pair-state \((x+c_\omega,y+c_\omega)\) takes place only if \( x\leq n-\omega, y\leq n-\omega \), and at least \( c_\omega \) matching slots in \( L_1 \) and \( L_2 \) are found. When \( PM_1 \) is in state \( x \), the probability of a transition to state \((x+c_\omega)\) is:

\[
F_{\omega,x} = 1 - \sum_{y=0}^{n} f_{\omega,x,y} q_y, \quad x=0..(n-\omega), \quad \text{where } q_y = p_y
\]

(4)

and \( P \) is determined from:

\[
\sum_{i=1}^{S} a_i F_{v_i, x-v_i} p_{x-v_i} = x p_x, \quad x=1..n, \quad p_j = F_{.,j} = 0 \text{ for } j<0, \quad \sum_{x=0}^{n} p_x = 1
\]

(5)

Let \( c_i \) be the carried traffic per peripheral, and \( \beta_i \) the end-to-end loss, for stream \( i \). Let \( C \) be the total carried traffic per peripheral and \( B \) the overall weighted mean blocking. Then:

\[
c_i = a_i \sum_{x=0}^{n-v_i} F_{v_i, x} p_x, \quad \beta_i = 1 - c_i / a_i
\]

\[
C = \sum_{x=1}^{n} x p_x = \sum_{i=1}^{S} c_i = (1-B) \sum_{i=1}^{S} a_i
\]

(6)
Due to the mutual interaction of the peripheral states and the matching process, the decomposition of $\beta_i$ into a peripheral blocking component and a matching loss component would be meaningless. Instead, we evaluate the network's internal blocking by comparing $\beta_i$ with the reference blocking $b_i$, where $b_i$ is the end-to-end loss seen by stream $i$ if the peripheral modules were connected to an ideal network. The value of $b_i$ can accurately be determined by the iterative solution of:

$$
\sum_{j=1}^{S} a_i (1 - b_i) p_{x-v_i} = x p_x, \quad x=1..n, \quad p_j = 0 \text{ for } j < 0, \text{ and } b_i = \sum_{x=n-v_i+1}^{n} p_{x} \quad (7)
$$

We now turn to the calculation of the mismatch probability. When the number of links, $N$, is sufficiently large, and assuming the traffic carried by each link to be split among several destinations, the states of the links become virtually mutually independent and the link pairs may be treated separately. In pair-state $(x,y)$, the matching problem is equivalent to drawing $(m-x)$ random samples, without replacement, from a population of size $m$ containing '$y'$ busy members and '$m-y'$ free members. The probability of drawing $k$ free members (the probability of finding $k$ matching slots) is determined by the hypergeometric density function, which is exact under the assumptions of link independence and the randomness of the occupied slots in each link:

$$
\eta_{k,x,y} = \binom{m-x}{k} \binom{x}{m-y-k} / \binom{m}{y}, \quad x \leq m-k, \quad x+y \geq m-k \quad (8)
$$

The probability of an $\omega$-channel attempt failure, at pair-state $(x,y)$, is then:

$$
f_{\omega,x,y} = \begin{cases} 
1 & n-\omega < x \leq n, \ n-\omega < y \leq n \\
\sum_{k=0}^{x+y > m-\omega} \eta_{k,x,y} & x+y \geq m-\omega, \ x \leq n-\omega, \ y \leq n-\omega \\
0 & \text{otherwise}
\end{cases} \quad (9)
$$

To compute the matching effort, we first determine the conditional distribution, given pair-state $(x,y)$, of the number of slot-matching attempts required to find the first $\omega$ matching slots, if any. We then derive an expression for the expected value. The mean value of the matching attempts is then obtained by averaging over all pair states. For each new (PM1, PM2) arrival, the initial slot is selected at random. For pair-state $(x,y)$, the probability of finding $k$ matching slots is given by (8). It is plausible that the $k$ matching slots, when $k > 0$, be uniformly spread in the sample space $m$, i.e., the position of a matching slot, relative to any reference slot, is uniformly distributed. Let $\phi_{\omega,k,j}$ denote the probability of finding the first $\omega$ matching slots after scanning $j$ slots. This is equal to the probability of finding exactly $\omega-1$ matching slots after scanning $j-1$ slots multiplied by the probability that the $j^\text{th}$ slot is a matching one. Thus:

$$
\phi_{\omega,k,j} = \binom{k}{\omega-1} \binom{m-k}{j-\omega} / \binom{m}{j-1} \binom{k-\omega+1}{m-j+1}, \quad k \geq \omega, \ \omega \leq j \leq m+\omega-k \quad (10)
$$

The expected value of this distribution, averaging over $j$, is:

$$
\Phi_{\omega,k} = \sum_{j=\omega}^{m+\omega-k} j \phi_{\omega,k,j} = \omega (m+1) / (k+1), \quad k \geq \omega \quad (11)
$$

The mean value, given pair-state $(x,y)$, is then obtained by averaging over $k$, noting that when $k < \omega$, $m$ slots are inspected before the call is rejected:
\[ \Psi_{\omega,x,y} = \sum_{k=0}^{m} \omega(m+1) \eta_{k,x,y} / (k+1) + m \sum_{k=0}^{\omega-1} \eta_{k,x,y} \] (12)

\[ = \frac{\omega(m+1) \{ m+1 - (x+y-m) \eta_{0,x,y} \}}{(m-x+1)(m-y+1)} + \sum_{i=0}^{\omega-1} \left( m - \frac{\omega(m+1)}{i+1} \right) \eta_{i,x,y} \]

The overall mean search effort, expressed as the mean number of slot matching attempts, is then:

\[ E_\omega = \sum_{x=0}^{n-\omega} p_x \sum_{y=0}^{n-\omega} p_y \Psi_{\omega,x,y} \] (13)

The upper limit in the summation is \((n-\omega)\), not \(n\), because no matching attempt is made when either \(x\) or \(y\) is \(>n-\omega\).

### 4. NUMERICAL EVALUATION

The solution of (5) is carried out iteratively. Let \(P^{(i)}\) denote the approximation of state vector \(P\) obtained after iteration \(i\), \(i=1..I\), where \(|P^{(i)}| - P^{(i-1)}| = O(\epsilon), \epsilon << 1; \) typically \(\epsilon = 10^{-6}\).

Noting that the convergence takes an oscillatory course, it can be accelerated by replacing \(P^{(i)}\) with \(0.5 \left( P^{(i)} + P^{(i-1)} \right)\), for \(i > 1\), in calculating the transition probabilities in (4). Initially, the coefficient \(F_{..}\) in (5), may arbitrarily be set equal to unity.

The computation of the matching probabilities in (8) for large peripherals (\(m = 4096\) for example) is potentially a time-consuming process. However, fast and numerically robust calculations are possible by making use of the recursions:

\[ \eta_{k+1,x,y} = \eta_{k,x,y} (m-y+1)(m-x-k) / (y(k+1)) , \]

and

\[ \eta_{0,x,y} = \eta_{0,x,y} (x+y-m) / y . \] (14)

In the pair-state diagram of Figure 2, the domain of interest, where the mismatch probability is positive, is shown by the shaded area. Since \(\eta_{k,x,y} = \eta_{k,y,x}\), the calculations are restricted to pair states \((x,y)\), where \(x \geq y\). An \(\Omega\)-element array \(U\), with elements \(u_0..u_{\Omega-1}\), where \(\Omega\) is the largest per-call channel requirement, stores intermediate results in the procedure below.

For \(m \geq x \geq 0\); set \(u_0 = 1, u_k = 0\) for \(0 < k < \Omega\)

For \(m \geq y \geq m-x; \ r = (m-y+1)/y\)

For \(\Omega-1 > k \geq 0\);

\[ u_{k+1} = r \left( u_k (m-x-k) / (k+1) \right) \]

then

\[ u_0 = \frac{u_0 (x+y-m)}{y} \]

and

\[ \eta_{\nu_j,x,y} = \sum_{k=0}^{\nu_j-1} u_k , \ \nu_j = 1..S \]

Note that \(y\) and \(k\) must vary in a descending order in their respective domains. This procedure is executed only once for a given \(m\). For large values of \(m\), \(\eta_{..}\) vanishes rapidly as we move towards low-occupancy states, and only a small fraction of the mismatch domain need be visited. An efficient
data structure is used to save computer memory; the storage required is much less than the apparent S.m^2 elements. For m = 4096, for example, the storage is of the order of 0.01 S.m^2.

5. PERFORMANCE CONSIDERATIONS

The performance difference between streams can be controlled by appropriately admitting arrivals on the basis of the states of the originating and terminating peripherals. The expected value of the number of matching slots, at pair state (x,y), is determined from (8) as: <k> = (m-x) (m-y) / m, and the variance is smaller than the mean. If we choose an equalizing expansion A such that:

\[ \Delta^2 = \Omega m, \]

then in the worst case, at pair state (m-\Omega, m-\Omega), an \Omega-channel call has a 50% chance of successful matching. The overall matching effort decreases, and the end-to-end blocking difference between streams is significantly reduced, if a new arrival of any stream is rejected when:

\[ x > m - \sqrt{\Omega m} \text{ or } y > m - \sqrt{\Omega m} \quad (m > \Omega, \text{ normally } m >> \Omega). \]  

This desirable behavior is realized at the expense of a slight reduction of network utilization. We note that the condition in (2) is uniformly valid under indiscriminate expansion. With the equalizing expansion, the boundaries in (9) are modified and the condition in (2) is violated in the range:

\[ m - \Delta < x, \ y \leq m - \Delta + \Omega - v_i, \ i=1..S \]  \[ v_i < \Omega. \]  

This slightly affects the accuracy of the solution.

6. The T-T-T NETWORK

In the T-T-T network, the space switch of Figure 1 is replaced by \( k \) time switches, 1\( \leq k \leq m \), and each of the m-channel links is demultiplexed into \( k \) bands, each being assigned to one of the middle switches. A single-channel connection is established between any pair of free channels in likewise-numbered channel bands (of m/k channels each) in \( L_1 \) and \( L_2 \). For \( k < m \), the matching opportunity is higher than that of a T-S-T network with identical peripherals and under similar traffic conditions. The analysis is similar to that of the T-S-T case, although the derivation of the mismatch probability for multi-channel connections is more involved. Due to lack of space, we only give numerical results based on the analytical solution, which were also verified by simulation.

7. RESULTS

We used three traffic streams (S=3), with \( v_1 = 1 \), \( v_2 = 6 \), and \( v_3 = 24 \), with equal weighted offered traffic: \( a_1 = a_2 = a_3 \). Both the analytical and simulation results are presented. The number of peripherals, N, used in the simulation is 64. Figure 3 shows the load-service characteristics of a T-S-T network with \( m = 1024 \). Without expansion, the blocking seen by the streams differ significantly. We show the load-service curves for the 24-channel and 6-channel streams; the blocking of the single-channel stream is insignificantly small. When an equalizing expansion of 160 slots is used, the grade-of-service of the three streams is almost equalized, and we show the results for the 24-channel stream only. Figure 4 shows the reduction in matching effort when the 160-slot equalizing expansion is used. The effort per single-channel call is about 1/6 of that of a 6-channel call in the load range shown and is omitted in Figure 4. Figure 5 shows results for large peripherals,
with \( m = 4096 \). The T-S-T blocking performance is compared with that of an ideal network. The blocking of the 1-channel and 6-channel streams in the T-S-T network is negligible and is not shown in the figure. Figure 6 shows the performance of a T-T-T network with \( m=1024 \) and 32 middle time switches \((\kappa = 32)\). Comparing with the performance of an ideal network, it is seen that, in effect, the internal loss of the T-T-T network is insignificant.

![Figure 1](image1.png)

**Figure 1**

T-S-T Network

![Figure 2](image2.png)

**Figure 2**

Pair-state Diagram: Mismatch Domain

![Figure 3](image3.png)

**Figure 3**

Load-Service Characteristics

T-S-T, \( m = 1024 \)

![Figure 4](image4.png)

**Figure 4**

Matching Effort: T-S-T, \( m = 1024 \)
8. CONCLUSION

We presented an accurate method for traffic analysis of T-S-T networks serving a mixture of different-bandwidth traffic streams. The end-to-end blocking and the matching effort were determined and an appropriate expansion factor, to reduce the matching effort and equalize the grade-of-service, was discussed. The principles are applicable to other internally-blocking networks, and we have shown results for the T-T-T network.

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REFERENCES