NETWORK CAPACITY PLANNING WITH UNCERTAIN SCENARIO

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The evolution of the existing telecommunication networks towards the Integrated Digital Networks and the Integrated Services Digital Networks imposes to telecommunication firms to consider network planning as a dynamic problem with several different evolutive scenarios. Inherent uncertainty of a specific evolutive scenario suggests to utilize stochastic models for the network planning. In this work we address the problem of (node) capacity planning with uncertain scenario and propose an approach based on lagrangean relaxation and dynamic programming techniques. Some computational experience for both randomly generated and real-world networks are presented.

1. INTRODUCTION

The evolution of the existing telecommunication network towards the Integrated Digital Networks (IDN) and the Integrated Services Digital Networks (ISDN) imposes to telecommunication firms (both manufacturing and operating companies) to consider network planning as a dynamic problem [1,2]. A further aspect to be considered is the existence of several different evolutive scenarios. These scenarios concern demand evolution, technological innovations, network management of the operating companies, competition among several manufacturing companies, etc. The occurrence of a specific evolutive scenario and its inherent uncertainty suggest to utilize stochastic models for the network planning [3,4].

In this work we address the problem of (node) capacity planning with uncertain scenario. In Section 2 the problem is formulated. In Section 3 a solution approach, based on lagrangean relaxation and dynamic programming techniques, is described. Section 4 reports some computational results on the performances of this approach. In Section 5 this method is applied to determine the times of introduction and the modalities of growth of the digital and ISDN systems in the switching nodes of a network.

2. PROBLEM FORMULATION

The problem of dynamically planning the node capacity updating of a telecommunication network, with uncertain demand/cost scenario, can be formulated as follows.

Given:
- the set of the N network nodes;
- the (discrete) time horizon of the planning 0, ..., T;
- the set of the L possible capacity levels of the nodes;
- the set of the 5 possible future demand/cost conditions (scenarios), each with probability of occurrence $p_s, s=1, ..., S$;

- the cost $c_{n,s,t}(i,j)$ of augmenting the capacity of node $n$ from level $i$ to level $j$ at time $t$ in scenario $s$, and the corresponding amount of required resources $s_{n,s,t}(i,j)$; at time 0 (present time) these quantities are simply denoted as $c_n(i), s_n(i)$, respectively;

- the budget of available resources $W_{s,t}$ and $W_0$, for scenario $s$, time $t$, and for present time, respectively.

Find:

The minimum total expected cost of the capacity planning, subject to the constraints on the budget of resources available at each time, for each scenario. More specifically, let $Y_n$ and $X_{n,s,t}$ be the capacity installed on node $n$ at present time, and at time $t$ for scenario $s$, respectively: the total expected cost of this planning choice is then given by

$$z = \sum_{n=1}^{N} \sum_{s=1}^{S} p_s [c_{n,s,1}(Y_n, X_{n,s,1}) + \sum_{t=2}^{T} c_{n,s,t}(X_{n,s,t-1}, X_{n,s,t})].$$  \hspace{1cm} (1)

The constraints are:

$$\sum_{n=1}^{N} s_{n}(Y_n) \leq W_0,$$

$$\sum_{n=1}^{N} s_{n,s,1}(Y_n, X_{n,s,1}) \leq W_{s,1}, \hspace{1cm} s=1, ..., S,$$

$$\sum_{n=1}^{N} s_{n,s,t}(X_{n,s,t-1}, X_{n,s,t}) \leq W_{s,t}, \hspace{1cm} s=1, ..., S, \hspace{0.5cm} t=2, ..., T.$$  \hspace{1cm} (2)

It is easy to see that this is a hierarchical planning model similar to the model utilized in [3] in a Packet Switching environment. This model has the additional feature of considering several time periods, instead of only two. Formally, this problem can be viewed as a particular multiconstrained matroidal knapsack problem [5], thus implying its NP-hardness, i.e. strong circumstantial evidence against the existence of a polynomial-time method for its solution [6]. This fact suggests the use of (partially) enumerative methods or of fast heuristics, where finding tight bounds to the value of the optimum plays a crucial role.

3. BOUNDS AND APPROXIMATE SOLUTIONS

In this Section we summarize the results we have obtained, concerning the efficient computation of lower and upper bounds to the optimum value $\mathcal{Z}$ of the objective function (1), together with the corresponding feasible planning choices. These results have been derived through an appropriate use of lagrangean relaxation (as suggested in [5]) and of dynamic programming techniques. A detailed exposition can be found in [7].
The matroidal interpretation of the problem formulated in Section 2 consists of the following steps:

(i) Associate an element $e$ of a partition matroid [8] to each planning choice of each network node, i.e. to each triple of the form $(y,X,n)$, where $n$ is a network node, $y$ is the capacity installed on node $n$ at present time, and $X$ is a $S \times T$ matrix whose entry in row $s$ and column $t$, represents the capacity installed on node $n$ at time $t$, in scenario $s$.

(ii) A base $B$ of the matroid is a set of $N$ elements, corresponding to $N$ specific planning choices, one for each network node.

(iii) Minimizing (1) subject to constraints (2) is therefore the same as finding a base $B$ minimizing

$$z = \sum_{e \in B} \omega(e)$$

subject to

$$\sum_{e \in B} d(e) \leq c,$$

where for each element $e$, $\omega(e)$ and $d(e)$ is an appropriate scalar and $ST+1$-dimensional vector respectively, and $c$ is the vector whose components are the right hand sides of (2).

In order to obtain a lower bound to the optimum value $\bar{z}$ of the objective function, we suggest to introduce the constraints (4) into the objective function (3), with the appropriate vector $\mu$ of lagrangean multipliers. For each $\mu \geq 0$, the corresponding value $w(\mu)$ of this lower bound may be computed using the greedy algorithm, i.e. finding for each node $n$, an element $e$ with smallest modified weight $\omega(e) + \mu \cdot d(e)$. However, the number of elements corresponding to each node $n$ is $LST+1$, so that a brute force enumeration of such elements appears to be computationally inpracticable. A much more efficient approach is through the use of dynamic programming techniques [8], applied to each node $n$ and each scenario $s$, according to the scheme depicted in fig. 1: arc lengths are chosen appropriately, so that the total length of a set of $S$ paths (one for each scenario) from a node with $t=0$ to a node with $t=T$ equals the modified weight of the corresponding planning choice.

The best lower bound $\bar{w}(\mu) = \max \{\omega(\mu): \mu \geq 0\}$ may be obtained using either:

(i) a (modified) subgradient method [9],

(ii) a special LP formulation [5] solvable with row generation and parametric dynamic programming techniques, or

(iii) a simple dichotomic search, considering in turn each component of $\mu$ [10].

For what concerns the computation of upper bounds to $\bar{z}$ and corresponding approximate solutions, we have again utilized dynamic programming techniques to implement various heuristics, both during and after the search of $\bar{U}$. In order to obtain good feasible solutions, a successful approach has been that of performing successive modifications of planning choices, starting from the (unfeasible) solutions corresponding to the lower bounds.
Both theoretical [5] and experimental [7, 10] results indicate that the gap between upper and lower bounds to $\tilde{z}$ should become smaller and smaller as the network size $N$ increases.

4. PERFORMANCE EVALUATION

The methods described in the previous Section have been implemented on a PC HP9000 series 500, using the resident BASIC interpreter. The code obtained has been tested in order to collect evidence on the quality of the bounds and the approximate solutions computed; the results of different algorithms have been also compared to evaluate their performances.

For this analysis we have used a simple network model and 3 sets of randomly generated planning problems; each set consists of 5 different problems and is characterized by a particular network size (5, 10 or 15 nodes).

The model considers 4 possible future scenarios, 5 time periods and 8 different capacity levels; at the beginning of the planning period each node is equipped with the first level and we suppose that at each time the increase in node capacity demand ranges randomly from 0 up to 3 levels.

The cost of installing level $j$ ($j=1,\ldots,8$) on a node equipped with capacity $i$ and submitted to demand $k$ is the sum of two different components: the growth cost, proportional to the difference $j-i$, and the cost of unsatisfied demand, proportional to $k-j$.

The various proportionality constants depend on the network node, scenario and time and are assumed to be realizations of independent random variables uniformly distributed over the set [0, 100].

The planning constraints are imposed by establishing for each time period of each scenario a maximum network capacity growth; specifically, we suppose that the sum of the capacity growth of each node must not exceed a budget randomly chosen in the set [1.3N, 2.9N], where $N$ is the number of nodes.
Table 1 summarizes the computational results obtained so far.

<table>
<thead>
<tr>
<th>N</th>
<th>LB_1</th>
<th>UB_1</th>
<th>σ</th>
<th>LB_1</th>
<th>UB_1</th>
<th>σ_1</th>
<th>τ</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>419.79</td>
<td>444.74</td>
<td>15.61</td>
<td>377.63</td>
<td>361.75</td>
<td>87</td>
<td>40</td>
</tr>
<tr>
<td>10</td>
<td>1644.67</td>
<td>1669.85</td>
<td>42</td>
<td>1505.50</td>
<td>13900.09</td>
<td>86</td>
<td>64</td>
</tr>
<tr>
<td>15</td>
<td>2677.36</td>
<td>2683.19</td>
<td>1.42</td>
<td>2623.34</td>
<td>23158.85</td>
<td>87</td>
<td>89</td>
</tr>
</tbody>
</table>

**TABLE 1**

The rows of the table refer to the three sets of examples defined above, and correspond to networks of 5, 10 and 15 nodes, respectively; each entry of the table is averaged on the 5 problems of each set. The second, third and fourth column report the best lower bounds computed, the value of the best approximate solution and the corresponding relative difference

\[ UB - LB = \sigma \cdot 100, UB \]

respectively.

The reported bounds are computed executing separately the three algorithms implemented for the lagrangean problem and choosing in each instance the closest bounds. The next three columns report the value of the bounds and of the corresponding relative difference before the program execution; the bounds are obtained determining the minimum-cost (unfeasible) planning choice (LB_1) and computing the cost of the feasible planning choice obtainable without augmenting the node-capacity (UB_1). Finally, the last column shows the time (in minutes) necessary for the execution of all the algorithms.

The results in the table are quite satisfactory: it should be noted that the relative differences in the fourth column are quite small and tend to decrease when the problem size increases.

The performance of the different algorithms are reported in table 2.

<table>
<thead>
<tr>
<th>GRADIENT</th>
<th>DICHOTOMY</th>
<th>SIMPLEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>515.61</td>
<td>580</td>
<td>22515.3167</td>
</tr>
<tr>
<td>10115.1</td>
<td>13101</td>
<td>266411.7</td>
</tr>
<tr>
<td>151013.1</td>
<td>1361402010.2</td>
<td>111214010.4</td>
</tr>
</tbody>
</table>

**TABLE 2**

The table reports, for each network size and each algorithm, the value of σ, the number of iterations (i.e. the number of evaluation of the function \( w(\mu) \)) and the time necessary for the computation (in seconds).
Looking at the table we can notice two interesting facts: the good performances of the dichotomic search and the relatively bad quality of the bounds computed by the simplex-like procedure. The first fact is not surprising and has been verified also in a previous work [10]; the second fact is more interesting because the simplex-like procedure implemented should be able to compute in polynomial time the exact value of the best lower bound, at least in the case of no degeneration. However we have found that for large problems the probability of degeneration is very high.

5. APPLICATIONS

The evolution of existing telecommunication networks towards IDN and ISDN gives rise to many complex planning problems, especially when the uncertainty upon the evolutive scenario is considered. In this Section we describe the application of the model in order to solve two real planning problems with uncertain scenario:

- optimum strategies for the network evolution;
- optimum strategies for the introduction of new services.

In each of these problems there are many basic factors that are known only with uncertainty: demand evolution, trend of costs, budget constraints of operating companies, and competition among manufacturers.

5.1 Network Evolution

An evolution strategy is defined by times and modalities of replacement of old analog switching systems with new digital exchanges in each node of the network, and by the updating of the switching capacities. These decisions depend on the trade-off between the maintenance cost of analog systems, versus the installation cost of new digital systems. Since we can not exactly forecast these costs, we have to consider them as uncertain factors in the planning process.

In the following table four possible trends of costs are reported.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Analog Maintenance</th>
<th>Digital growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>increasing</td>
<td>constant</td>
</tr>
<tr>
<td>2</td>
<td>constant</td>
<td>constant</td>
</tr>
<tr>
<td>3</td>
<td>increasing</td>
<td>decreasing</td>
</tr>
<tr>
<td>4</td>
<td>constant</td>
<td>decreasing</td>
</tr>
</tbody>
</table>

**TABLE 3**

Figure 2 shows a numerical example of these scenarios.

With these scenarios in mind we have applied the model in order to find the optimal modernization strategy for a telephone metropolitan area network. The results are shown at two levels. We refer first to the times and modalities of the modernization of a single node of the network, which have to specify how the installation of the digital system is implemented in the node: without replacement of old analog lines (overlay) or with replacement of some analog lines (replacement).
The successive updating of the capacity of the digital system is denoted as digital growth. In Table 4 a modernization strategy of a node is shown.

<table>
<thead>
<tr>
<th>Time</th>
<th>Scenario 1,3</th>
<th>Scenario 2</th>
<th>Scenario 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>Replacement</td>
<td>Growth</td>
<td>Replacement</td>
</tr>
<tr>
<td>1992</td>
<td>Replacement</td>
<td>Growth</td>
<td>Replacement</td>
</tr>
<tr>
<td>1994</td>
<td>Replacement</td>
<td>Growth</td>
<td>Growth</td>
</tr>
<tr>
<td>1996</td>
<td>Replacement</td>
<td>Growth</td>
<td>Growth</td>
</tr>
<tr>
<td>1998</td>
<td>Replacement</td>
<td>Growth</td>
<td>Growth</td>
</tr>
<tr>
<td>2000</td>
<td>Growth</td>
<td>Growth</td>
<td>Growth</td>
</tr>
</tbody>
</table>

TABLE 4

For each node of the network the model provides this kind of optimal modernization strategy.

At the second level the optimal modernization of the network is given. An example of this strategy for the telephone metropolitan area network is shown in fig. 3, where we consider the case of a single budget constraint for the network modernization: as a consequence the replacement of analog lines is a gradual one. If we remove this constraint, the modernization is much faster, implying a more rapid digital growth: this is shown in fig. 4.
5.2 Introduction of new services

In network planning a basic problem is the optimal strategy for the introduction of new services.

The demand of new services is growing: many systems already exist and many others will be available in the future. The trend of costs and prices is uncertain, and the optimal strategy of introduction has to be found.

A solution of this problem is possible by means of a planning model with uncertain scenario: in this case the uncertainty affects both demand and cost of new services.

Four scenarios are considered in table 5.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Demand of new services</th>
<th>Cost of new services</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>strongly increasing</td>
<td>increasing</td>
</tr>
<tr>
<td>2</td>
<td>weakly increasing</td>
<td>increasing</td>
</tr>
<tr>
<td>3</td>
<td>strongly increasing</td>
<td>decreasing</td>
</tr>
<tr>
<td>4</td>
<td>weakly increasing</td>
<td>decreasing</td>
</tr>
</tbody>
</table>

TABLE 5

Numerical examples are reported in figures 5 and 6.

We have studied the particular case of a metropolitan area network. The solution consists of suggesting an optimal strategy for the introduction of new communication services in the whole network: see fig. 7.

We believe that these results are very important, since they allow the operating companies to decide about the modernization strategies, and to forecast the network evolution in an uncertain scenario. Similar techniques also allow to evaluate the technical and economical features of the systems to be developed.
A sensitivity and robustness analysis is made [4] on these results as a further planning guide.

**Figure 5**

**Figure 6**

**Figure 7**

5.2B.4.9
REFERENCES


