A STUDY ON THE FEASIBILITY OF INTRODUCING TANDEM EXCHANGES

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Given various tandem candidate sets, this paper deals with a long-term planning of the optimal evolution of local network over some study period. The exact approach of such a problem involves too many variables. In the case of n tandem candidate sets and T periods, total nxT network dimensioning procedures are required. The number of alternatives for possible network evolutions increases exponentially as n and T increase. In this paper we develop a simplified method of determining the optimal evolution of a network. Examples of numerical results are presented.

1. INTRODUCTION

There have been many studies on the optimal dimensioning of junction groups which interconnect a network of telephone exchanges including tandem (local/transit combined or pure) exchanges for a fixed time or during planning horizon [3].

The advantages in using tandem exchanges mainly appear in the efficiency of junction groups, reduction of the circuits, and route safety. The network structure including tandem exchanges cannot be determined by the effect of only one time period. It should be determined considering the long-term planning of a telephone junction network.

Consider a network evolution over some study period and suppose we want to know the number and the locations of tandem exchanges and corresponding network structure at each time period during some study period with which the minimum of the total incremental cost over all time periods is attained.

The total incremental cost over all time periods depends on the dimensionings of the network at each time period, which are in turn the functions of the tandem exchanges. This nature makes our original problem very difficult to solve. Therefore we assume that the locations of tandem exchanges are determined in advance. Then the dimensioning of the network at each time period depends only on the number of tandem exchanges and by considering various combinations of the given locations of tandem exchanges, called tandem candidate sets, our original problem becomes simple and tractable.

Let n and T be the numbers of tandem candidate sets and time periods respectively. Let \( G_{ij}, i=1 \text{ to } n, j=1 \text{ to } T \) be the network structure at period j in the case of i-th tandem candidate set. Consider a typical evolution \( G_{11} \rightarrow G_{i2}, \ldots \rightarrow G_{iT} \), denoted by \( (i_1, i_2, \ldots, i_T) \) for short. Then we can find the approximate minimum of the total incremental cost over all time periods of this evolution using the method proposed by [3]. However in our problem, we cannot adopt the above method since the number of evolutions increases exponentially as n and T increase. Therefore we try another simplified approach using cross-sectional analysis: Irrespective of evolutions we determine \( G_j \) for each \( i, j \) using a particular dimensioning method (Berry’s chain flow model, Y. Rapp method, or other dimensioning methods). Since our approach requires nxT dimensioning procedures, the methods as simple as possible are recommended.
2. OVERALL PROCEDURE

The optimization of junction network and the determination of the number and the location of tandems can be obtained according to the procedure as follows.

- To investigate current number of the circuits which interconnect every two exchanges.
- To forecast the point--to--point traffic for several study years.
- To establish tandem candidate sets each of which is composed of one or more exchanges selected among the group of exchanges available as tandems, as shown in Table 1. A Tandem candidate set can be composed of only existing exchanges or only pure tandems which will be purchased in the future.
- To calculate the distances and the transmission costs between the two exchanges for the optimization of junction network.
- To determine the routing rule and identify the order of the routes which the offered traffic between the two exchanges can take.
- To optimize the junction network using the information which is imposed and obtained previously. This is done for every tandem candidate set and every study year. Hence we should repeat this procedure nxT times if the number of tandem candidate sets is n and the number of study years is T. The result of the optimization for a tandem candidate set and a study year is denoted by $G_{ij}$ ($i=1,2,\ldots,n$, $j=1,2,\ldots,T$) as shown in Fig. 1.
- To construct all the possible evolution chains each of which consists of a sequence reachable arcs from $G_{ij}$ to $G_{k,j+1}$. A reachable arc $G_{ij} \rightarrow G_{k,j+1}$ can be obtained in such a way that the tandem candidate set i is a subset of the set k. More detailed definitions are made in section 3.
- To calculate the total incremental cost over all time periods for each evolution chain and select the minimum cost evolution chain.

Table 1. An example of tandem candidate set

<table>
<thead>
<tr>
<th>Set No.</th>
<th>Tandem Candidate Set</th>
<th>Tandem Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{1, 4}</td>
<td>1:A, 4:C</td>
</tr>
<tr>
<td>2</td>
<td>{1, 4, 7}</td>
<td>1:A, 4:G, 7:J</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>{4, 7, 8, 14}</td>
<td>4:G, 7:J, 8:C, 14:M</td>
</tr>
</tbody>
</table>

3. DEFINITION AND CONSTRUCTION OF EVOLUTION CHAIN

In the problem of studying the feasibility of introducing tandem exchanges, we should consider a time horizon and according to the evolution of the network, determine when to introduce tandem exchanges and the location and the number of tandem exchanges.

To solve this problem, suppose that we have tandem candidate sets which represent the locations and the numbers of tandems, and time periods 0,1,\ldots, T.
For example, time periods are given by 0, 1, 2, 3, and tandem candidate sets are given by \{1\}, \{2\}, \{1,2\}, \{2,3\}.

The period 0 represents the present time and the set \{1\} means the one tandem is located at the position 1.

Similarly the set \{2,3\} means that two tandems are located at the position 2 and the position 3. Figure 2 represents a case of network evolutions, where \(G_{00}\) is the present network and \(G_{ij}\) is the optimized network for the \(i\)-th candidate set at the period \(j\), \((i \neq 0, j \neq 0)\).

In this case, consider the evolution from \(G_{11}\) to \(G_{22}\). This evolution is practically undesirable since at present time, one tandem is located at the position 1 and this evolution means that at next period we must remove a tandem at the position 1 and reinstall a tandem at the position 2.

Thus, as indicated in figure 2, only the arrow directions are desirable evolutions. For example, \(G_{00} \rightarrow G_{11} \rightarrow G_{31} \rightarrow G_{33}\) is a desirable evolution.

This can be represented by a chain \(C = (0,0) (3,1) (3,2) (3,3)\) which is called an evolution chain. Let \(\Delta_i\) be the cost of changing \(G_{00}\) to \(G_{31}\.

Then \(\Delta_i\) is given by \(\sum_{i=A_{31}} C_{il} A_{il}\),

where \(A_{31}\) is the arc set of \(G_{31}\) and \(C_{il}\) is the present worth of the unit cost of junctions in arc \(i\), and \(A_{i}(i)_{31}\) is the increase in number of junctions in arc \(i\) when the evolution of \(G_{00}\) into \(G_{31}\) occurs.

Therefore the total incremental cost of the evolution chain \(C\) is given by \(\sum_{p=1}^{3} \Delta_{p}\).

The optimal planning of introducing tandems can be obtained by comparing the total incremental cost of the possible evolution chains.

To generalize the preceding discussions, let us introduce the following notations.

\[2.1A.4.3\]
\(L_i\) : the \(i\)-th tandem candidate set, \(i=1\) to \(n\).

\(G_{ij}\) : the optimized network of \(i\)-th candidate set at period \(j\), \(j=0, 1, \ldots, T\).

\(N_{ij}\) : the set of nodes of \(G_{ij}\) which represents the location of exchanges.

\(A_{ij}\) : the arc set of \(G_{ij}\) which represents the links between exchanges.

\(\mathcal{C}\) : the set of all evolution chains

\(\Delta^c_p\) : the incremental cost of changing the whole network from its state at \(P-1\) to its state \(p\) on the chain \(C\).

\(\Delta n(k)_{ip}\) : the incremental number of junctions in arc \(k\) in \(G_{ip}\) when a evolution from period \(P-1\) to period \(p\) occurs.

\(C_{ip}\) : the present worth of the unit cost of junctions in arc \(i\) at period \(p\).

\(n(k)_{ij}\) : the number of junctions in arc \(k\) in \(G_{ij}\).

Using above notations, our main problem is to find \(C^* \in \mathcal{C}\) such that

\[
\sum_{p=1}^{T} \Delta^c_p = \min \left\{ \sum_{p=1}^{T} \Delta^c_p \right\}, \quad C \in \mathcal{C}
\]

where if \(C\) is given by \((i_0, 0) (i_1, 1) \ldots (i_T, T)\) with \(i_0 = 0\) then \(\Delta^c_p = \sum_{k \in \mathcal{A}_{ip}} \Delta n(k)_{ip}\), and \(\Delta n(k)_{ip} = \max \{n(k)_{i, q} \} - \max \{n(k)_{i, q} \}\).

Next we present a constructive method of finding \(C\) using graph-theoretic concepts.

Any evolution chain can be written as

\[C = (i_0, 0) (i_1, 1) \ldots (i_T, T), \text{ where } i_0 = 0.\]

This can be simplified if we write

\[C = (i_0, i_1, \ldots, i_T), \text{ with } (i_1, \ldots, i_T) \in \mathbb{N}^T\]

where \(\mathbb{N} = \{1, 2, \ldots, n\}\) and \(\mathbb{N}^T\) is a Cartesian product. First we define a matrix \(Q\) which represents the inclusion relations between \(L_i\)'s as follows;

\[q_{ij} = \begin{cases} 1, & \text{if } L_i \subseteq L_j \\ 0, & \text{otherwise} \end{cases}\]

And assume \(i < j\) implies \(|L_i| < |L_j|\), where \(|S|\) represents the cardinality of a set \(S\).

This \(nxn\) upper triangular matrix \(Q\) is called the evolution matrix. Using \(Q\), for each \(i \in \mathbb{N}\), we can construct inductively a rooted tree \(T_i\) whose root and height are \(i\) and \(T-1\) respectively, as follows.

The sons of \(i\) are all \(j\) such that \(q_{ij} = 1\). Then the current tree is of height \(1\). Assuming that the tree of height \(k\) is given, the tree of height \(k+1\) is obtained as follows; for each leaf \(j\) of the tree of height \(k\), the sons of \(j\) are all \(k\) with \(q_{jk} = 1\).

Then the set of all evolution chains is given by

\[C = \bigcup_{i=1}^{T} C_i, \quad \text{where }\]

\[C_i = \{(0, i, j, \ldots, k) \mid (i, j, \ldots, k) \in \mathbb{N}^T, i+j+\ldots+k = \text{a chain from the root } i \text{ to a left } k \text{ in } T_i \}.\]
4. AN ALGORITHM FOR THE SHORTEST CHAIN PROBLEM WITH VARIABLE ARC COSTS.

In figure 2, consider the incremental cost of the evolution from $G_{12}$ to $G_n$. This cost can be changed according to the history of the evolution. That is, the evolution from $G_{12}$ to $G_n$ involves three evolution chains $C_1 = (0,0)(1,1)(3,2)(3,3)$, $C_2 = (0,0)(2,1)(3,2)(3,3)$, and $C_3 = (0,0)(3,1)(3,2)(3,3)$. Thus usual algorithms for the shortest chain problem can not be applied to our problem, and general algorithms for solving shortest chain problems with variable arc costs do not exist except enumeration. In our problems, total enumeration requires many redundant computations.

In this section, we develop some economic computational method for solving our main problem. First we construct a rooted tree $G$ of height $T$ whose root is 0 by letting 1, 2, ..., $n$, which are roots of $T_1$, $T_2$, ..., $T_n$, respectively, be the sons of the root 0. Thus all $T_i$ are subtrees of $G$. Then any chain from the root 0 to a leaf of $G$ is an evolution chain. Let $C = (0,i_1, ..., i_T)$ be a chain from the root 0 to a leaf $i_T$ in $G$. Then $C$ is also in $C$. We define the cost of an arc $(i_{p-1}, i_p)$ in the chain $C$ as the incremental cost of the evolution from $G_{i_{p-1}i_{p-1}}$ to $G_{i_{p-1}i_p}$ denoted by $d(p)_{i_{p-1}i_p}$. Clearly $d(p)_{i_{p-1}i_p} = \Delta_i^p$. Consider another chain $C' = (0,j_1, ..., j_{p-2}, i_{p-1}, i_p, ..., i_T)$ in $G$. Notice that $C$ and $C'$ have the arc $(i_{p-1}, i_p)$ in common. Then we must have $i_1 = j_1, i_2 = j_2, ..., i_{p-2} = j_{p-2}$. In this case, we have $\Delta_i^p = \Delta_j^p = d(p)_{i_{p-1}i_p}$. Therefore each arc in $G$ has the unique arc cost and usual shortest chain algorithm can be applied to $G$ to find the shortest chain from the root 0 to any leaf of $G$. We recommend Dijkstra's algorithm which can be modified to meet our problem. That is, each arc cost need not be calculated in advance. Let $W$ be a certain labeled set in the process of Dijkstra's algorithm.

Then we need only the costs of arcs in the cut generated by $W$. Termination occurs when any of the leaves of $G$ is labeled. This modification saves many redundant calculations and makes easy the computations of arc costs from period to period.

5. COMPUTATIONAL RESULTS

Our model was applied to a sample network. Junction costs are supplied by KTA RESEARCH CENTER for 4 time periods, i.e., 1986, 1991, 1996, 2001. The basic time period is 1986. The junction costs for 1986 were based on capital costs and those for 1991, 1996, 2001 were calculated by discounting the 1986 costs by factors of 0.79, 0.63, 0.50, respectively. The discounting factor is $(1+r)/(1+i)$, where $r$ is an inflation rate and $i$ is an interest rate.

The numbers of exchange and exchange offices at each time period are summarized in Table 3.

<table>
<thead>
<tr>
<th>Table 3. The numbers of exchanges and exchange offices</th>
</tr>
</thead>
<tbody>
<tr>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>the number of exchange offices</td>
</tr>
<tr>
<td>the number of exchanges</td>
</tr>
</tbody>
</table>

In all cases any two distinct exchanges can be an O-D pair having 1 overflow route unless the origin is EMD. The kinds of exchanges are NO1A, MI0CN, NO4, and EMD. If the origin is EMD, then either direct or overflow route is permitted.
The number of tandem candidate sets are 8 and the maximum cardinality of them is 4. The number of evolution chains generated was 65. Table 4 summarizes the results of computations with respect to the number of tandem exchanges.

The optimal evolution chain falls into the category (0, 2, 2) in Table 4. The minimum evolution cost is $9,231,240 and total cpu time is 858 seconds. We used Y.Rapp's approximation method in dimensioning each $G_i$. Notice that users can choose another dimensioning technique according to the available computer systems, the required accuracy, and the limitation of cpu time.

Table 4. The summary of computations

<table>
<thead>
<tr>
<th>the number of tandems</th>
<th>the average total cost ($)</th>
<th>the number of tandems</th>
<th>the average total cost ($)</th>
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<td>1 1 4</td>
<td>10,217,408</td>
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<td>9,367,305</td>
<td>1 2 2</td>
<td>9,715,361</td>
</tr>
<tr>
<td>0 0 3</td>
<td>9,720,314</td>
<td>1 2 3</td>
<td>10,278,461</td>
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<td>10,363,772</td>
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<td>9,966,976</td>
<td>1 3 3</td>
<td>9,846,303</td>
</tr>
<tr>
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<td>1 3 4</td>
<td>10,100,094</td>
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<td>1 1 3</td>
<td>10,055,163</td>
<td>4 4 4</td>
<td>10,653,705</td>
</tr>
</tbody>
</table>

6. CONCLUSIONS

In this paper, we have developed a simplified method for studying the feasibility of introducing tandem exchanges. We simplified our original problem by assuming that tandem candidate sets were given in advance and the dimensionings of $G_i$ on a evolution chain were carried out using cross sectional analysis. This method can reduce cpu time by a large amount. Further work is required to tackle our original problem in more reliable ways.
REFERENCES


