OPTIMAL ASSIGNMENT OF REMOTE SWITCHING UNITS (RSU’S) TO DIGITAL TELEPHONE EXCHANGES

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The development of RSU's as a means for restraining the increasing cost trend of the subscribers network has shown itself to be a solution for a variety of applications. Wide use of RSU's in urban areas raises an assignment problem. The paper presents this problem as a Transportation Problem with the constraints that each destination (RSU's location) should be connected only to a single source (digital exchange) and suggests an optimal solution for this problem in an efficient way.

1. INTRODUCTION

Network growth on one hand and technical improvements on the other lead to changes of cost-effectiveness in the various parts of the telephone network, including the subscribers network that connects the subscribers to the local exchanges. Analysis of CPM (Critical Path Method), as a tool for managing projects of network expansion, has shown that the critical path of these projects underwent significant changes with the operation of digital equipment.

The changeover to digital switching equipment and the replacement of many wiring tasks, undertaken in the past, by keyboard-entered instructions, have shortened the process of switching equipment installation. Economically as well, the cost of digital switching equipment is today significantly less than that of analog switching equipment.

These trends in the telephone system have highlighted the importance of the subscribers network for the following reasons:

- The development process of the subscribers network has not been shortened and is now on the critical path.
- The increase in the relative and absolute costs of the subscribers network, following the use of advanced techniques for geographical distribution of the cables, in order to ensure their long term reliable operation.
- Further prospects of increasing the average traffic per subscriber by utilizing the telephone network for digital services.

The above developments in the telephone system have naturally created a high incentive to reduce the cost of the subscribers network and the time for its preparation. These trends have led to focus, at first, on cases where the marginal cost of the subscribers network is particularly high.

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Two groups of such cases have been observed:

1. Subscribers located at large distances from local exchanges.
2. Geographical locations containing a relatively few number of subscribers.

A technical solution to reduce the cost of the first group, which is typical to rural areas, is the development of Remote Switching Unit (RSU).

This development is, in essence, a line module of the digital exchange which has been "uprooted" and moved to a location physically closer to the subscribers. The connection of the remote unit to a nearby local exchange, which is done by a small number of common channels, creates in fact an advanced concentrator. Further development of the remote unit led to an equipment that is also an exchange that communicates between subscribers connected to the same remote unit location (Stand Alone RSU's).

A technical solution to reduce the cost of the second group, which is typical in sparsely populated areas, is the digital subscriber PCM via carrier. Thus, with a basic transmission unit of 2 Mb/s, installed near the premises of the subscribers, 30 neighboring users may be connected to the exchange. A sub-basic unit of 0.7 Mb/s has recently been developed capable of connecting up to 10 subscribers only.

Other technical connecting options that are economical for cases which are between the above two groups are also available and represent a variety of concentrator types.

Each of the connecting options has relative advantages over the others, so it would be useful to present the application range of each alternative.

Fig. 1—Application range of subscriber connecting options.
Figure 1 presents the application range of the above technical solutions by considering two parameters:

W – No. of subscribers connected.

D – Average distance of subscribers from the digital exchange.

Exact break-even points of distances and number of subscribers, depend among other factors on the type of soil of the geographical area and on cable parameters (cable type, diameter of wires etc.).

From the above figure one can see that the application range of subscriber PCM is limited to cases where the number of subscribers is lower than the value of $W_1$, which is around of a few tens (less than 100 subscribers).

The application range of RSU's is limited to cases where the number of subscribers is larger than the value of $W_2$ (around 300).

Further information dealing with the above options are published by Chouinard [1], Fleming [2], Homayoun [3] and Harell [4].

2. APPLICATIONS OF RSU'S

A general observation indicates that there is a trade-off between the cost of the subscribers network and of the local network (the inter-local exchange network).

The lower the number of switching junctions in the local network (representing a low cost local network) the higher the subscribers network costs and vice versa.

Using RSU's has the potential to reduce local network costs, without necessarily increasing the costs of the subscribers network.

Beyond the original use of RSU's for saving costs of subscribers networks, this equipment has shown itself to be a technical solution for a large variety of applications in the local network.

RSU's can be used in each of the following ways:

- Promotion of Digital Services
  RSU's can be a means for promoting the digital services of the network for locations that still are served by analog exchanges which are not planned to be replaced in the near future. Introducing RSU's in these locations, which serve as a digital overlay, constitutes a solution for supplying digital services to selected customers that will be connected to nearby digital exchanges via RSU's. This might be an attractive solution for developed countries, whose analog equipment, especially in the local network, is expected to remain the major part for many years to come.

- Substitution and Replacement for Local Exchanges
  RSU's are, in essence, a substitutive equipment for local exchanges and should be taken into account especially in cases where the required capacity is relatively small. This application is valid for replacing existing analog local exchanges and also for network expanding.

- Changes in Service Area Boundaries
  RSU's are a means through which it is possible to change service area boundaries of local exchanges.
Service area changes are likely to lead to savings in the cost of local transmission equipment. This can be done by merging groups of different traffic characteristics, e.g. connecting subscribers from residential areas via RSU's to digital local commercial exchanges.

-Substitution and Replacement for Final Exchanges
RSU's are naturally a substitutive equipment for final exchanges, having small capacities for providing telephone services to small communities. In such cases the RSU's are connected to a larger mother exchange located in a nearby community.

-Promotion of the Digital Replacement Process
RSU's can be used as a solution for various cases when carrying out the inevitable digital replacement process. This solution may arise especially for locations having various constraints such as space and time, which impede implementing the replacement process.

The use of RSU's in the local network is therefore a factor which today constitutes an important component for designing and implementing both the expansion and replacement digital processes.

The potential use of RSU's invokes establishing technical standards which will make it possible to connect RSU's to digital exchanges of different manufacturers. At present each type of RSU can function only with digital exchanges from the same manufacturer.

3. PRESENTATION OF THE MODEL

The wide use of RSU's creates an assignment problem which is presented in this section by using the following notations:

N - The number of digital local exchanges in the area tested, numbered 1,2,...,N.
M - The number of RSU locations in the area, numbered 1,2,...,M.
Ci - The unused capacity of digital exchange i that is free for serving RSU locations.
Kj - The capacity of location j planned to be served by RSU.
Qij - The total cost for connecting location j to exchange i. These values depend on several factors such as:
- The geographical area and the distance between i and j
- The capacity of Kj
- The traffic characteristics of location j and junction i
The last factor is introduced in order to decide on priorities between locations with different traffic characteristics. Qij = ∞ if it is not intended, for whatever reason, to connect location j to junction i.

Xij - Assignment decision variable for connecting location j to junction i.
Xij = 1, i.e. location j will be connected to junction i.
Xij = 0, i.e. location j will not be connected to junction i.

For practical considerations it is assumed that every site of RSU's will be served only by a single digital exchange. Assigning RSU's of the same location to different digital exchanges is technically possible but impractical. The penalty for splitting the capacity of RSU's is in the cost of remote control equipment as well as in the cost of transmission.
equipment (causing loss of scale advantages).
Furthermore, the communication between the separated parts, between which there is usually a close relation, will be "clumsy" and inefficient.

Using the above notations we get the following model:

\[
Z = \min \left\{ \sum_{i=1}^{N} \sum_{j=1}^{M} Q_{ij} X_{ij} \right\} \\
\text{s.t.} \quad \\
\begin{align*}
I. & \quad \sum_{j=1}^{M} K_j X_{ij} \leq C_i & \forall i=1,2,\ldots,N \\
II. & \quad \sum_{i=1}^{N} X_{ij} = 1 & \forall j=1,2,\ldots,M \\
III. & \quad X_{ij} = 0 \text{ or } 1 & \forall i,j
\end{align*}
\]

With the above presentation the model is, at first sight, a multi-dimensional knapsack problem (Constraints I & III) having multiple choice constraints (Constraints II), which has no efficient solution. See for example, Salkin [5].

Combot, Tsui & Wiehmayer [6] solved a similar model by a heuristic iterative way. In each iteration a sequence of the one-dimensional knapsack problem has to be solved. For solving the one-dimensional knapsack problem, the algorithm developed by Greenberg & Hegerich [7] has been used.

A closer look into the model presented indicates that it is only a particular case of the general approach of the multi-dimensional knapsack problem having multiple choice constraints, since the coefficients $K_j$ in the model are not a function of the variable $i$.

We would like to use this specific feature in order to find an efficient solution to the model. To do so, we define the following converted variables and coefficients.

\[
X_{ij} = K_j X_{ij} \quad \text{and} \quad Q_{ij} = Q_{ij}/K_j \quad \forall i,j
\]

Introducing these terms and multiplying constraints II, for every $j$ by the denominator $K_j$, we get the following approach.

\[
Z = \min \left\{ \sum_{i=1}^{N} \sum_{j=1}^{M} Q_{ij} X_{ij} \right\} \\
\text{s.t.} \quad \\
\begin{align*}
I. & \quad \sum_{j=1}^{M} X_{ij} \leq C_i & \forall i=1,2,\ldots,N \\
II. & \quad \sum_{i=1}^{N} X_{ij} = K_j & \forall j=1,2,\ldots,M \\
III. & \quad X_{ij} = 0 \text{ or } K_j & \forall i,j
\end{align*}
\]

In the last approach we get a transportation problem (Constraints I & II) with the requirement that each destination is served by a single supplier (Constraints III).
4. WAYS OF SOLUTION

Constraints III limit the space of variable states to $2^{NM}$, while adding constraints II reduce the space of variable states to $N^M$. For real dimensional values of N and M we get quite a large combinatorical problem; so full enumeration cannot be considered.

Two possible ways for solving the last approach can be considered.

Solution No. 1
Solving a sequence of regular transportation problems, that satisfy constraints I and II, creating a search tree and using the B&B (Branch and Bound) technique until constraints III are satisfied.

This way is very much similar to one of the classical ways for solving the Traveling Salesman Problem (TSP) by solving instead a sequence of assignment problems, using a B&B search until a full tour is found.

The branching step, from a particular state in the search tree is done by considering different single suppliers to destinations for which constraints III have not been satisfied.

Solution No. 2
Solving a sequence of problems that satisfy constraints II and III, creating a search tree and using the B&B search until constraints I are satisfied.

The algorithm based on this way, has been developed by De-Maio & Reveda [8] and is fully covered in the reference.

By defining a potential solution as a solution satisfying constraints II & III and a feasible solution as a potential solution satisfying also constraints I, the algorithm creates a sequence of potential solutions satisfying two conditions:

i. the associated objective values obtained from the potential solutions are in order which is not descending.

ii. a potential solution is tested for feasibility only if all potential solutions having lower objective values have been tested.

Satisfying these two conditions makes it possible to obtain an optimal solution when a feasible solution has been found for the first time.

This is based on the fact that getting potential solutions, derived directly from a certain unfeasible solution, is straightforward, so it is possible to calculate in advance the objective values of the derived solutions.

Of the two solutions mentioned, Solution No. 2 is much faster, and even problems with a few hundred integer variables, representing real size problems, can be solved very efficiently.

5. SUMMARY

In order to restrain the increasing cost trend of the subscribers network a variety of technical solutions have been developed, among which is the RSU. A large number of applications for RSU's have been described. The wide range of applications in the local
network creates an assignment problem which has been defined and presented. The problem has been converted to a special transportation model which has an efficient solution, so real dimensional problems can be solved very efficiently.

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