AN ALGORITHM FOR DIMENSIONING THE OVERFLOW TRUNK GROUP IN A SYSTEM WITH TIME-DEPENDENT ARRIVAL TRAFFIC

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We present an approximate method to dimension the overflow trunk group of a loss system with discrete, nonstationary offered traffic. The time-dependent expectation and variance of the overflow streams are first approximated. Using these two measures, the number of circuits in the overflow group is then approximated using Rapp's formula.

I. INTRODUCTION

In classical teletraffic theory, the study of overflow traffic is of utmost importance in the design of a telecommunication network. Poisson traffic is assumed to arrive at a loss system with negative exponential holding times. Normally the expectation of the holding times is assumed constant, which may not be the case in real-life (this will be discussed later). For stationary Poisson offered traffic, it is assumed that the overflow stream is fully characterized by its expectation and variance. In this paper we also make the same assumptions even when the offered traffic is a nonstationary Poisson process. Hence we will use the notion of time-dependent expectation, $m(t)$, and time-dependent variance, $v(t)$.

In the stationary system, Wilkinson's Equivalent Random Method (ERM) is used to calculate the overflow expectation and variance where the 'equivalent' number of circuits and offered traffic are obtained using Rapp's formula. This enables dimensioning of the overflow group.

Loss queuing models assuming nonstationary Poisson arrivals are becoming more important in modelling real-life systems. This is because existing stationary models are found to be inadequate in dealing with real-life situations where arriving traffic is highly nonstationary or time-dependent. Recent studies seem to support the application of nonstationary models in approximating real-life cases [4,5].

Significant theoretical progress has been made by Jagerman [2] and Akimaru, et.al. [1]. Jagerman derived integral equations for the time-dependent blocking probability due to nonstationary Poisson traffic arriving at a loss system of N circuits with negative exponential holding times. He also suggested procedures to approximate the blocking probability for both stationary and nonstationary offered traffic.

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Akmaru, et. al. [1] obtained

\[ a(t)p_N(t) = m(t) + \frac{dm(t)}{dt} \]  
(1)

\[ v(t) = M(t,2) + m(t) - [m(t)]^2 \]  
(2)

where \( p_N(t) \) is the blocking probability on the primary group and \( a(t) \) is the offered traffic (the parameter of the nonstationary Poisson process). \( M(t,2) \) is the second factorial moment for the number of busy circuits in the overflow group. Calculation of \( m(t) \) using (1) is impossible as no exact formula is yet known for \( p_N(t) \) with \( a = a(t) \). However, for \( a = c \), a constant, exact \( p_N(t) \) can be derived [6,9]. Consequently computation of \( v(t) \) using (2) is also a problem, but with the added complication in the form of \( M(t,2) \). Computation of \( M(t,2) \) involves the solution of \( (N+1) \) partial differential equations; not an easy task, especially when \( N \) is large, as the case in real-life systems.

We also assume that the expectation of the holding times is constant and is normalized to one. For the case when this expectation is time-dependent, rescaling of the time-axis using a simple transformation will enable the theory developed for unit mean holding time to be used [11].

II. APPROXIMATION OF REAL-LIFE TRAFFIC

Real-life offered traffic, as measured at switching centres, can be approximated by a sequence of step functions [5]. The height of each step corresponds to the mean offered traffic during the time interval represented by its width on the time-axis. This time interval is, optimally, about one quarter-hour. Therefore what we actually have is a sequence of transient systems operating one after the other, in accordance with the step functions. Rather than to develop models for piecewise, continuous \( a(t) \), which is extremely difficult, we need only to consider transient models, that is for \( a(t) = a \). Transient models are also difficult to treat, but not as difficult as nonstationary models with \( a(t) \).

III. APPROXIMATION OF TRANSIENT OVERFLOW EXPECTATION

The expectation of the transient overflow traffic can be approximated using (1). This is accomplished by substituting an approximate expression for \( p_N(t) \), the transient blocking probability, in (1) and numerically solving the resulting differential equation. The exact expression for \( p_N(t) \), though available [6,9], is too cumbersome to use. An approximate expression which we used in (1) is of the form [2,8].

\[ p_N(t) \approx B(N,a)[1 - e^{-(a/N+1)t} + \tilde{p}_1(o)e^{-(a/N+1)t}/B(1,a/N)]^N \]  
(4)

where \( B(N,a) \) is the Erlang loss formula and \( \tilde{p}_1(o) \) is the initial blocking probability for an M/M/1 loss system with offered traffic \( a/N \). The expression for \( \tilde{p}_1(t) \) is

\[ \tilde{p}_1(t) = B(1,a/N)[1 - e^{-(a/N+1)t}] + \tilde{p}_1(o)e^{-(a/N+1)t} \]  
(5)
IV. APPROXIMATION OF TRANSIENT OVERFLOW VARIANCE

As mentioned in Section I, calculation of the variance using (2) is extremely difficult if \( N \) is sufficiently large. We propose a simple method to approximate this variance when the expectation is known. Peakedness, \( z \), is defined as

\[
z = \frac{v}{m}
\]

(6)

Hence if \( m \) is known, then \( v \) can be calculated by using \( v = mz \).

For the steady-state case, we have

\[
z = 1 - m + \frac{a}{(m + N + 1 - a)}
\]

(7)

Unfortunately (7) cannot be used for approximating the transient \( z(t) \) as division by zero could occur \[10\]. However, approximation of \( z(t) \) via the interrupted Poisson process \[3\] seems to remove such a possibility. In this case we have

\[
z \approx 1 + \left( \frac{\beta \gamma}{\gamma + \omega} \right) \left( \frac{1}{\gamma + \omega + 1} \right)
\]

(8)

where

\[
\gamma = \frac{\omega}{a} \frac{\beta - a\delta_0}{\delta_0}, \quad \omega = \frac{\delta_0}{\beta} \frac{\beta - a\delta_1}{\delta_1 - \delta_0}
\]

\[
\beta = a \frac{\delta_2((\delta_1 - \delta_0) - \delta_0(\delta_2 - \delta_1))}{2\delta_1 - \delta_0 - \delta_2}
\]

\[
\delta_2 = \frac{2}{a^2} \frac{(\delta_0\delta_1)^{-1}}{B_2 - 2B_1^{-1} + B_1^{-1}}
\]

\[
\delta_1 = \frac{1}{a} \frac{\delta_0^{-1}}{B_1^{-1} - B_1^{-1}}, \quad \delta_0 = B
\]

\[
B = B(n, a), \quad B_1 = B(n + 1, a), \quad B_2 = B(N + 2, a)
\]

\[
B(N, a) = \frac{a^N}{N!} \sum_{i=0}^{N} \frac{a^i}{i!}
\]

Once \( m \) is obtained from (1) and \( z \) is approximated using (8), \( v \) can be approximated using (6).
V. COMPARISON WITH SIMULATIONS

The above approximations were applied to two examples of loss systems with offered traffics representing real-life cases:

(a) $N = 15$

(b) $N = 15$

Figure 1

Figure 2
Tables 1 and 2 show the comparison between simulated values and values obtained by the above approximations.

<table>
<thead>
<tr>
<th>TIME</th>
<th>EXPECTATION</th>
<th>VARIANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SIMULATION</td>
<td>APPROXN.</td>
</tr>
<tr>
<td>5</td>
<td>(0.0000, 0.0000)</td>
<td>0.0008</td>
</tr>
<tr>
<td>10</td>
<td>(2.7939, 2.9681)</td>
<td>2.7048</td>
</tr>
<tr>
<td>20</td>
<td>(2.1414, 2.2438)</td>
<td>2.0690</td>
</tr>
<tr>
<td>25</td>
<td>(0.3371, 0.4466)</td>
<td>0.3650</td>
</tr>
</tbody>
</table>

Table 1: Case (a) with confidence limit for simulations set at 0.1

<table>
<thead>
<tr>
<th>TIME</th>
<th>EXPECTATION</th>
<th>VARIANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SIMULATION</td>
<td>APPROXN.</td>
</tr>
<tr>
<td>5</td>
<td>(3.2682, 3.4531)</td>
<td>3.3995</td>
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<tr>
<td>10</td>
<td>(0.2894, 0.3693)</td>
<td>0.3650</td>
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<tr>
<td>15</td>
<td>(8.3549, 8.5938)</td>
<td>8.3542</td>
</tr>
<tr>
<td>20</td>
<td>(3.2977, 3.4323)</td>
<td>3.4012</td>
</tr>
<tr>
<td>25</td>
<td>(0.0329, 0.0521)</td>
<td>0.0728</td>
</tr>
</tbody>
</table>

Table 2: Case (b) with confidence limit for simulations set at 0.1

IV. EQUIVALENT TRAFFIC AND CIRCUITS

Once the time-dependent expectations and the variances of the overflow streams are obtained, the equivalent offered traffic and circuits need to be calculated. An approximation would be by using formulae used for the stationary case, namely,

\[ a_{eq}(t) = v(t) + 3 \frac{v(t)}{m(t)} \left[ \frac{v(t)}{m(t)} - 1 \right] \]  

\[ N_{eq}(t) = \frac{a_{eq}(t)[m(t) + v(t)/m(t)]}{m(t) - 1 + v(t)/m(t)} - m(t) - 1 \]
where \( m(t) \) and \( v(t) \) are the sums of the expectations and variances, respectively. Equation (9) is Rapp's formula. Using \( a_{eq}(t) \) and \( N_{eq}(t) \), the overflow group could be dimensioned accordingly for each quarter-hour.

VII. CONCLUSION

The above method is a rough approximation. It is assumed that the equations used in the forms of (9) & (10) for the stationary case can be similarly used for the nonstationary case. More research need to be done in this aspect. Such research will concentrate on deriving more accurate dimensioning procedures. Such procedures will be important in networks with dynamic routing techniques, where circuit allocations are continually revised for optimization.

REFERENCES


