EFFECTS OF LOAD VARIABILITY ON LINE ACCESS SYSTEM CAPACITY

Jay M. Bennett

Bellcore, 331 Newman Springs Road, Red Bank, New Jersey, 07701-7030, USA

Traffic engineers should always use the more limiting criterion in engineering a line access system. It has been recognized that Average Busy Season (ABS) criteria should be used when traffic variability is low and High Day (HD) criteria should be used when traffic variability is high. To be precise about which procedure produces the lower capacity is difficult in general since the conclusion depends on the basic load/service curve of the unit, the service criteria used, and the degree of traffic variability over units and days. For a variety of HD blocking criteria, this paper describes at which HD/ABS ratio ABS engineering should be replaced by HD engineering. The paper provides generic results for usage capacities of line access systems having sizes from 20 to 500 channels.

1. INTRODUCTION

Line access systems are the concentrators that switch lines to internal channels in a modern digital switching system. They must be engineered to meet both average and peak traffic loads. Two such load objectives in common use in the United States are Average Busy Season (ABS) and High Day (HD). Both are defined with respect to a Time Consistent Busy Hour (TCBH). The TCBH is a single daily peak traffic hour unique for each office which has been selected on the basis of past traffic studies. Average Busy Season Busy Hour (ABSHH) load is the average traffic level in the TCBH during the busy season (three months with the highest average traffic). High Day Busy Hour (HDBH) load is the highest traffic level in the TCBH for a single day in a whole year; special holidays and days with heavy traffic attributable to extraordinary events which are not expected to recur annually are excluded. The major difference between TCBH engineering and Extreme Value Engineering (EVE) is that EVE daily finds and uses the peak traffic hour (Bouncing Busy Hour (BBH)) while TCBH engineering uses the same preselected peak hour each day.[1]

Line access system usage capacity is obtained using load/service curves which give blocking for a given load. Using a given objective blocking fraction, the traffic engineer finds the capacity by using the curve to find the load associated with the objective level of blocking. However, the engineer must be careful that the load/service curve is applicable to the selected blocking criterion. In particular, a load/service curve developed for individual hour blocking on a specific component may be used for HD engineering, but should not be used against a criterion of average blocking. Such averaging always occurs when an ABS blocking criterion is used. In these cases, the load/service curve must be adjusted to account for variability in load. A standard indicator of day-to-day variability in load is the ratio of HD traffic load to ABS traffic load for the office. This ratio is called the HD/ABS ratio throughout this paper.

ABS and HD service criteria must be satisfied simultaneously. The traffic engineer should always use the more limiting criterion in engineering a line access system. It has been recognized that ABS engineering should be used when traffic variability is low (i.e. low HD/ABS ratio) and HD engineering should be used when traffic variability is high. For a variety of HD blocking criteria ranging from 0.05 to 0.20, this paper describes at which HD/ABS ratio ABS engineering for 0.015 blocking should be replaced by HD engineering.

The paper describes the procedure used to calculate the lower capacity. The calculation depends on the basic load/service curve of the line access system, the blocking criteria used, and the degree of traffic variability among load units and days. Generic results for usage capacities of line access systems having sizes from 20 to 500 channels are presented. These results will be of primary interest to traffic engineers involved in system engineering.

2. LOAD VERSUS SERVICE CURVES

Figure 1 shows load/service curves for line access systems with different numbers of channels. The
curves were calculated using a software package developed by Lederman.[2]

The following assumptions were made:

1. All attempts have full access to all channels. Virtually all modern switches have full-access time-slot interchange systems for the line concentrator. The results presented here apply to these systems and to others where an equivalent full access group size can be derived.

2. The arrival process is a mixture of originating attempts for dial tone and terminating attempts for the completion of calls.

3. Originating attempts are modeled as an Erlang C process (blocked calls delayed). 55% of offered load is assumed to be originating.

4. Terminating attempts are modeled as an Erlang B process (blocked calls cleared). 45% of offered load is assumed to be terminating.

3. ABS ENGINEERING CAPACITY

Each office has m line access systems. For each of n days in a month, the TCBH load on each unit is recorded. This creates an \( m \times n \) matrix of office loads for the month. The average of all loads in the matrix divided by the number of channels in each unit is the average occupancy of the units. Similarly, for each of n days in a month, the TCBH blocking on each unit is recorded. The average of all blocking values in this matrix is the average blocking experienced by the units. ABS capacity is the average occupancy when average blocking reaches a selected ABS blocking criterion. Throughout this paper, an ABS blocking criterion of 0.015 is used.

Since the ABS criterion is for average blocking, ABS capacity cannot be calculated by reading the occupancy associated with 0.015 blocking directly from the basic load/service curve. Instead an average hour load/service relationship must be established. This relationship is more complex since it must include an additional parameter for load variability.

3.1 Office Load Variability

Load variability is typically expressed as a coefficient of variation CV. The office load coefficient of variation is calculated by taking the standard deviation of all values in the load matrix and dividing by the average load. Office load variability is composed of two major sources: Day-to-Day Variation and Unit-to-Unit Variation. Assuming that these two sources of traffic variability are independent, the total CV may be approximated as the "sum" of day-to-day and unit-to-unit traffic variability

\[
CV^2 = CV_U^2 + CV_D^2
\]

where \( CV_U \) is the Unit-to-Unit Coefficient of Variation and \( CV_D \) is the Day-to-Day Coefficient of Variation. If we were working with data from an empirical load/service curve based on individual hour or half-hour data points, Equation (1) should also include "source load variation" of the finite time period data.[3] However, since we are assuming a theoretical load/service relation based on Poisson input, this variation is not included here. For large server groups, this variation is negligible in any case.

3.1.1 Unit-to-Unit Variation

Unit-to-unit variation is controllable to a great degree by load balance. It is measured by the coefficient of variation of the unit loads averaged over all days. In terms of the matrix described above, the average load in each of the m rows is calculated. The Unit-to-Unit Coefficient of Variation \( CV_U \) is the standard deviation
of these m averages divided by the overall average. Under the assumption of good load balance among units, the Unit-to-Unit CV for the ABS busy hour was fixed at 0.06 throughout the analysis. A data study of in-service line access units indicated that this level of unit-to-unit variation was achievable with good class-of-service balance.

3.1.2. Day-to-Day Variation

This uncontrollable variability is measured by the coefficient of variation of the daily loads averaged over all units. In terms of the matrix described above, the average load in each of the n columns is calculated. The Day-to-Day Coefficient of Variation \( CV_D \) is the standard deviation of these n averages divided by the overall average. Day-to-Day CV is related to the HD/ABS ratio according to the following quadratic equation:

\[
\text{HD/ABS Ratio} = 1 + 2.62 \cdot CV_D + 2.01 \cdot CV_D^2. \tag{2}
\]

This equation is the key to relating HD engineering to ABS engineering. It has its origins in empirical studies performed in the development of EVE.[4] These studies produced the following formula for expected HD load (in CCS):

\[
\text{ABS Load} + 2.62 \cdot \sigma_{ABS} + 54.4 \cdot CV_{ABS} \tag{3}
\]

where \( \sigma_{ABS} \) is the standard deviation of ABS load and \( CV_{ABS} \) is the coefficient of variation of ABS load. For small values of \( CV_{ABS} \), this formula approaches the expression for a quantile of the normal distribution. Similarly, the gamma distribution approaches a normal distribution as its CV approaches zero. This reasoning inspired the development of a unitless HD/ABS ratio formula as a function of CV only based on the gamma distribution.

Since there is no single standard time period valid for all offices to determine High Day, the coefficient 2.62 was used to infer the number \( n \) of candidate days in a year. For small CV values, expected High Day load is the mean of the \( n^{th} \) order statistic from a sample of size \( n \) from the normal distribution. Since 2.62 is the mean of the 138th order statistic from a sample of size 138 from the standard normal distribution, the number of candidate days was inferred to be 138. The residuals from a CV-fit of means of 138th gamma order statistics were regressed on \( CV^2 \) to obtain the \( CV^2 \) coefficient of 2.01.

3.2. Calculation of ABS Capacity

The average blocking \( B_s(\bar{L}, CV) \) for average load \( \bar{L} \) with load coefficient of variation CV is calculated by integrating the basic load/service curve \( B_s(x) \) for \( s \) channels over the carried load probability density function \( f(x) \) with mean \( \bar{L} \) and standard deviation

\[
\bar{L} \times CV:
\]

\[
\overline{B}_s(\bar{L}, CV) = \int_0^\infty B_s(x) f(x) \, dx. \tag{4}
\]

The ABS capacity \( C \) for a unit with \( s \) channels in an office with traffic variability CV satisfies the following equation

\[
\overline{B}_s(C, CV) = 0.015. \tag{5}
\]

In our analysis, the blocking functions \( B_s(x) \) are the basic load/service curves described earlier in Section 2. The traffic variability CV values were calculated using Equations (1) and (2) under the assumption that the line access systems had good load balance (i.e., \( CV_U = 0.06 \)). The gamma density function was used for \( f(x) \) since past studies by Wilkinson [5] have established it as the standard statistical model for TCBH load. Figure 2 demonstrates how ABS capacity decreases as load variability increases. The solid lines in Figure 2 plot ABS capacities (in terms of channel occupancy) for each system size versus HD/ABS ratio.

![Figure 2](image-url)
To find the comparable HD capacity using ABS engineering, ABS capacity is multiplied by the applicable HD/ABS ratio. The lines in Figure 3 plot the HD capacity using ABS engineering for systems having from 20 to 500 channels.

Table I. HD Occupancy Capacity (HD Engineering)

<table>
<thead>
<tr>
<th>HD</th>
<th># of Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blocking</td>
<td>20</td>
</tr>
<tr>
<td>0.05</td>
<td>0.688</td>
</tr>
<tr>
<td>0.07</td>
<td>0.718</td>
</tr>
<tr>
<td>0.10</td>
<td>0.752</td>
</tr>
<tr>
<td>0.15</td>
<td>0.793</td>
</tr>
<tr>
<td>0.20</td>
<td>0.823</td>
</tr>
</tbody>
</table>

5. HD/ABS RATIO BREAKPOINTS

The appropriate procedure (ABS or HD) for engineering is the one which estimates the lower capacity. In this way, both ABS and HD blocking criteria are satisfied. The points at which ABS and HD engineering both estimate the same capacity are the engineering breakpoints. Table II shows the HD/ABS ratio breakpoints for a broad range of system sizes. These breakpoints are plotted in Figures 2 and 3 using the alphanumeric codings "S" (5% HD blocking), "D" (7%), "F" (10%), "A" (15%), and "T" (20%). These figures demonstrate that HD engineering should be done whenever the HD/ABS ratio is greater than the breakpoint and ABS engineering should be done when the ratio is less.

Table II. HD/ABS Ratio Breakpoints

<table>
<thead>
<tr>
<th>HD</th>
<th># of Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blocking</td>
<td>20</td>
</tr>
<tr>
<td>Objective</td>
<td>1.180</td>
</tr>
<tr>
<td>0.05</td>
<td>1.246</td>
</tr>
<tr>
<td>0.07</td>
<td>1.330</td>
</tr>
<tr>
<td>0.10</td>
<td>1.457</td>
</tr>
<tr>
<td>0.15</td>
<td>1.577</td>
</tr>
</tbody>
</table>

As an example consider a line access system with 100 channels. If a 7% HD blocking criterion is applied, Table II shows that ABS and HD engineering produce the same capacity estimate when the HD/ABS ratio is 1.148. At this breakpoint, Table I says that the HD capacity is 0.893 and the ABS capacity is 0.893/1.148=0.778. These are the "7%" points plotted on the 100 channel curves in Figures 2 and 3. Figure 3 shows that ABS engineering should be used when the HD/ABS ratio is less than 1.148 since ABS engineering produces a lower HD capacity below the breakpoint. Conversely, Figure 3 shows that HD engineering should be used when the HD/ABS ratio is greater than 1.148 since ABS engineering produces a higher HD capacity above the breakpoint.

4. HD ENGINEERING CAPACITY

The HD capacity using HD engineering is calculated by simply reading the occupancy from the appropriate basic load/service curve for the objective blocking criterion desired. This calculation assumes virtually no traffic variability among line access systems in the HD busy hour, since this once-a-year variability cannot be measured. It is recognized that the observed HD blocking will be somewhat higher than the selected criterion. Therefore, the traffic engineer should select a conservative HD criterion.

Table I presents the HD occupancy capacities from HD engineering for line access systems with 20 to 500 channels. The comparable ABS capacity using HD engineering is calculated by dividing the office HD/ABS ratio into the HD capacity.
6. CAPACITY ESTIMATION PROCEDURES

Given the number of channels in the access system and a selected HD blocking criterion, the traffic engineer should compare the Office HD/ABS traffic load ratio with the appropriate HD/ABS breakpoint in Table II.

If the ratio is lower,
• the ABS capacity is found using the appropriate curve in Figure 2, and
• the HD capacity is found using the appropriate curve in Figure 3 (or multiplying the ABS capacity by the HD/ABS ratio).

If the ratio is greater,
• The HD capacity is found in Table I, and
• The ABS capacity is calculated by dividing the HD/ABS ratio into the HD capacity.

As an example, Figure 4 shows ABS capacities for a 100-channel system when different HD blocking criteria are applied. The solid line shows ABS capacity using ABS engineering. The various dotted and dashed lines show the ABS capacity when HD engineering is limiting. This figure is identical to the 100-channel curve in Figure 2 with the addition of the HD engineering lines, which were calculated using the simple procedure described above.

If the access system size is between the numbers of channels analyzed in the study, HD and ABS capacities can be interpolated from the tables and figures provided. Linear interpolation between the logs of the system size is preferred.

7. CONCLUSION

The selection of the proper technique for engineering line access systems depends on
• The size of the line access system (e.g. number of channels),
• The blocking criteria to be met (e.g. ABS and HD blocking objectives), and
• The degree of traffic variability across units and days (e.g. HD/ABS ratio).

This paper presents quantitative results and a generic methodology to insure that the more conservative engineering constraint is applied. Future work on this problem should focus on the sensitivity of these results to different ABS blocking criteria, different levels of load balance, and different load distribution assumptions.

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REFERENCES


