Delay Analysis of an ATM Switch for Continuous-Bit-Rate Traffic

L. G. Dron  
M.I.T.  
Cambridge, MA, USA

G. Ramamurthy  
AT&T Bell Laboratories  
Holmdel, NJ, USA

B. Sengupta  
NEC Research Institute  
Princeton, NJ, USA

In this paper, we model a packet multiplexer for Continuous Bit Rate (CBR) traffic in an ATM network as an nD/D/1 queue. We compare the efficiency of various algorithms for finding the delay distribution. In particular, we propose a new algorithm whose time complexity is $O(n^2)$ where $n$ is the number of voice sources being multiplexed. Further, the use of the central limit theorem can reduce the time complexity to $O(n)$ for large $n$. We find an asymptotic formula whose time complexity is independent of $n$ and it works well (for practical purposes) over a wide range of parameter values.

We examine and comment on the use of the $M/D/1$ results as an approximation. In addition to comparing the performance of these algorithms, we show that the buffer requirements for such a queue is significantly less than the theoretical maximum (even when the requirement on the call disruption probability is very low). This result has important implications in the design of buffer size. Further, the buffer requirement is relatively insensitive to the design criterion (call disruption probability) so that inaccuracies in measurements and/or traffic forecasts will not lead to erroneous design.

1. INTRODUCTION

As we move into the broadband era, Asynchronous Transfer Mode (ATM) with fixed cell size is being proposed as the underlying mechanism to transport voice, video and data in a single integrated broadband system. Coding voice (without silence detection) and video with fixed rate coders results in Continuous Bit Rate (CBR) traffic streams where cells from individual voice or video sources arrive at fixed intervals. Such continuous bit oriented traffic also originate when traffic from existing circuit switched system is offered to a broadband network for transport. While the arrival process from individual streams may be regarded as a renewal process, the aggregate cell arrival process (at an ATM switch input) resulting from the superposition of many independent streams is a complex nonrenewal process. Because of the periodic nature of the traffic, if a cell from a given source experiences a long delay (or is blocked due to the finiteness of the buffer), then successive cells from the same source would experience the same delay (or will be blocked) until the superposed arrival pattern undergoes a change. Voice and video traffic have stringent requirements on the delay, delay jitter and loss they can incur in a network. This is all the more important in the case of circuit switched traffic (offered to broadband networks), where cell loss or delay can cause loss of synchronization between the end systems and consequent loss of session. Placing tight bounds on delay, delay jitter, cell loss, and sizing of buffers at each node are important design issues. It is worth noting that CBR traffic is also generated by data sources that are subject to rate control (see Ramamurthy and Dighe (1990A)).

In this paper, we model a multiplexer for CBR traffic streams and determine the exact cell waiting time distribution via the $nD/D/1$ queue. The assumptions of this model are as follows:

1. The arrivals to this queue are from a superposition of $n$ independent equilibrium renewal processes each with an interarrival time of exactly 1 time unit. This arrival mechanism represents the arrival of cells from $n$ simultaneously active sources. Each source (independently of others) generates one cell in (exactly) 1 time unit. The fact that there are $n$ independent equilibrium renewal processes implies that for each source, the first cell is generated uniformly in the interval (0,1) independently of other sources. If the time of arrival for the first cell is $t_1 (0 < t_1 < 1)$ for a source, the time of arrival for the $k$th cell is $(k-1) + t_1$, for $k \geq 2$.

2. This superposed traffic stream is fed to a multiplexer which serves the cells on a first come first served basis with a (deterministic) service time of $D$ units. The completion of a service by the multiplexer constitutes the insertion of a cell into the transmission medium.

3. The arrival rate is $n$ and the service time is $D$, so the utilization of the server is $nD$ which we refer to as $\rho$. Thus $n$ and $\rho$ are sufficient to characterize this problem completely.

Bhargava et al. (1989), Eckberg (1979, 1982), Gravey (1984), Ott and Shanbhikumar (1990), Ramamurthy and Sengupta (1988), Sengupta (1990) and Virtamo and Roberts (1989, 1990) have also solved similar problems. Slightly different modeling approaches are considered by Heffes and Lucantoni (1986) and Sriram and Whitt (1986). However, the time complexity of some of these algorithms is $O(n^2)$ or $O(n^3)$. For large $n$, this is a disadvantage and Ramamurthy and Sengupta (1988) have reported that their method runs into numerical difficulty when $n \geq 100$. In this paper, we provide an improvement in the method due to Ramamurthy and Sengupta (1988), and propose a new algorithm whose time complexity is $O(n^2)$. Further, the use of the central limit theorem can make the time complexity $O(n)$ for large $n$.

Recently, Sengupta (1989) has found the Laplace-Stieljes transform (LST) of the delay distribution of the $nD/G/1$
queue and has also done an asymptotic analysis of the nD/D/1 queue. Both of these results have an important bearing on the solution of the problem that we are studying. First, the moments of the delay distribution can be written down easily by taking derivatives of the LST of the delay distribution. Second, the asymptotic solution (as n→∞ and ρ→1) in such a way that \(\sqrt{n}(1-\rho) \to c\), where c>0 can be used to derive a simple approximation whose time complexity is independent of n. We also show (by numerical examples) that this approximation works very well over a wide range of parameter values. A practical implication of this result is that calculations for the delay distribution can be carried out in real time. This is of importance if this problem is a part of a larger optimization problem or if real time decisions are needed to be made about admission policy into the ATM network. Such decisions are common in congestion avoidance mechanisms such as Distributed Source Control (see Ramamurthy and Dighe (1990A and 1990B)). Also, a simple (and approximate) form of the result enables one to "see" the interrelationships between the parameters more easily than a complicated (and exact) form. In a similar spirit, the M/D/1 result can also be used as an approximation. We show that the M/D/1 result has limited usefulness, i.e., it works well for moderate values of ρ only.

Our main interest is the calculation of delay distribution which is important in the sizing and engineering of buffers. We show that even though there is a theoretical upper bound on the delay, this upper bound is rarely reached. In other words, the buffer requirement for achieving a call disruption probability of 10^-6 is an order of magnitude less than the theoretical maximum (corresponding to a call disruption probability of zero). Further, the buffer size required is not very sensitive to the design criterion (for example, call disruption probability). These facts have an important implication in the design of buffer size.

This paper is divided into two more sections. In section 2, we give an exact algorithm and approximations based on the asymptotic analysis and the M/D/1 results. In section 3, we show numerical results and their implications in the buffer design problem.

2. SOLUTION METHODS

In this section we characterize the delay distribution for a randomly chosen cell in the multiplexer described in section 1. Let W be the wait associated with a randomly chosen cell. Let \(P_a(x) = P(W \leq x)\). In the next three subsections, we characterize this distribution in three different ways. The first is an exact method (section 2.1). The second is an approximation based on an asymptotic analysis (section 2.2). The third is an approximation based on the M/D/1 results (sections 2.3). We also show in section 2.4 how to find the delay distribution if the number of calls in progress (n) is varying randomly in time.

2.1 EXACT METHOD

The following theorem shows how to calculate the waiting time distribution function.

Theorem 1: For 0 < x < (n-1)D,

\[
P_a(x) = \sum_{k=0}^{n-1} \frac{(n-1)!}{(n-1-k)!} (1-(n-1-k)D)D^kJkF_k(x)
\]  

(7)

where

\[
F_k(x) = \sum_{j=0}^{k} (-1)^j \frac{1}{j!(k-j)!} \left\{ \left[ \frac{x}{D} - j \right]^+ \right\}^k - j^k.
\]  

(8)

Proof: From theorem 1 of Sengupta (1990), the Laplace-Stieltjes transform (LST) of \(P_a(x)\) is given by

\[
\sum_{k=0}^{n-1} \frac{(n-1)!}{(n-1-k)!} (1-(n-1-k)D)D^j \left[ 1 - e^{-\beta D} \right]^k.
\]  

(9)

We observe that \((1 - e^{-\beta D})/sD)^k\) is the LST of the sum of k independent uniform random variables in \((0,D)\). Let \(f_k(x)\) denote its density function. Then from page 261 of Kendall and Stuart (1969), we have

\[
f_k(x) = \frac{1}{D(k-1)!} \sum_{j=0}^{k} (-1)^j \left\{ \left[ \frac{x}{D} - j \right]^+ \right\}^{k-j}.
\]

By integrating this, we obtain the distribution function,

\[
F_k(x) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^j \left\{ \left[ \frac{x}{D} - j \right]^+ \right\} - j^k.
\]

and observe that it is the same as (8). This completes the proof.

It is possible to write down the first two moments of the distribution \(P_a(x)\) by differentiating (9). They are

\[
w_a^1 = \frac{D}{2} \sum_{k=1}^{n-1} \frac{(n-1)!D^k}{(n-1-k)!}
\]

and

\[
w_a^2 = \frac{D^2}{4} \sum_{k=1}^{n-1} \frac{(n-1)!D^k}{(n-1-k)!} (2k-2/3).
\]

Note that the LST of \(P_a(x)\) is expressed as a mixture of sums of uniform random variables in (9). By the central limit theorem, the distribution \(F_k(x)\) for large k can be approximated by the Normal distribution with the same mean and variance. The mean and variance of \(F_k(x)\) are \(kD/2\) and \(kD^2/12\). The algorithm of theorem 1 has a time complexity of \(O(n^2)\). However, for large n, the time complexity can be reduced to \(O(n)\) if one uses the central limit theorem approximation.
2.2 ASYMPTOTIC METHOD

Sengupta (1990) has shown that if \( n \to \infty \) and \( \rho \to 1 \) in such a way that \( \sqrt{n(1-\rho)} \to c \) where \( c > 0 \),

\[
P_n(\sqrt{n}) \to 1 - e^{-2x^3/D^2} - 2\phi(D).
\]

This result is based on a general heavy traffic limit theorem of Whitt (1985). Replacing \( \sqrt{n} \) by \( x, c \) by \( \sqrt{n(1-\rho)} \) and noting that \( \rho = nD \), we obtain

\[
P_n(x) = 1 - e^{-2x^3/D^2} - 2\phi(D),
\]

and this should work well when \( n \) is large and \( \rho \) is close to one. The first two moments of the distribution in (10) are

\[
w_1 = \frac{\sqrt{nD}}{2}, \quad \frac{1 - \Phi((1-\rho)c/n)}{\Phi((1-\rho)c/n)}
\]

\[
w_2 = \frac{\rho D}{2} - \rho(1-\rho)w_1^2
\]

where \( \phi \) and \( \Phi \) are the standard normal density and distribution functions respectively. Note that the time for calculations is independent of \( n \).

2.3 M/D/1 RESULTS

It is well known that if \( n \to \infty \) and \( \rho \) is kept fixed, the distribution \( P_n(x/D) \) behaves like the waiting time distribution of the M/D/1 queue with a utilization of \( \rho \) and a service time of 1. We have used a method due to Jageman (1987) to find the distribution \( P_n(x) \) approximately. To find this distribution, let \( \Theta \) be the positive number satisfying

\[
e^\Theta = 1 + \Theta/\rho
\]

and define \( \hat{K}_+(s) = \rho(e^{-s} - e^{-\Theta s})/(\rho - s) \) with

\[
\Gamma = \frac{2(\hat{K}_+(0) - \hat{K}_+(-\Theta))}{\hat{K}_+(-\Theta) - \hat{K}_+(-\Theta)}.
\]

Then,

\[
P_n(x) = \begin{cases} 
1 - \frac{\Gamma \rho}{\rho + \Theta} e^{\Theta(D-1)} & \text{if } x < D \\
1 - e^{-\Theta x/D} & \text{if } x \geq D.
\end{cases}
\]

The mean and the second moment of M/D/1 distribution are well known (see p. 256 of Cohen (1982)). For our problem they are

\[
w_1^r = \frac{\rho D}{2(1-\rho)} \quad \text{and} \quad w_2^r = \frac{\rho D^2(2+r)}{6(1-\rho)^2}.
\]

Like the asymptotic method, the time for calculations is independent of \( n \).

2.4 VARYING NUMBER OF CALLS IN PROGRESS

The results of sections 2.1-2.3 are based on the assumption that the number calls in progress \( (n) \) is fixed. In reality, this is not quite true. Actually, all calls have finite durations and they are offered to a finite trunk group. So, let us assume that calls arrive according to a Poisson process with rate \( \lambda \) to a finite trunk group of \( c \) servers. The call holding times have general distribution and they are independent with mean \( 1/\mu \). We assume that calls that arrive when all trunks are busy are lost. Let \( \pi_n \) denote the steady state probability of \( n \) calls in progress. Then, from section 5.2.2 of Gross and Harris (1985),

\[
\pi_n = \frac{(\lambda/\mu)^n}{n!} \sum_{i=0}^c \frac{(\lambda/\mu)^i}{i!}
\]

As far as arrivals of cells to the multiplexer are concerned, we assume that each call in progress generates cells in the manner described in section 1. Let \( \sigma_n \) be the customer average probability that there are \( n \) calls in progress at an arrival instant of a cell. Then, \( \sigma_n \) is a random modification of \( \pi_n \), i.e.,

\[
\sigma_n = \frac{n\pi_n}{\sum_{i=0}^c i\pi_i} \quad \text{for } n = 1, ..., c.
\]

Note that it is more likely that a higher number, rather than a lower number of calls are in progress when a cell is chosen randomly. Let \( W \) denote the steady state delay for a randomly chosen cell. If \( cD < 1 \) and \( 0 < x < 1 \), our approximation for the delay distribution is

\[
P(W \leq x) \approx \sum_{n=1}^c \sigma_n P_n(x)
\]

and

\[
E(W) = \sum_{n=1}^c \sigma_n w_n^r
\]

for \( r = 1, 2 \), where \( w_n^r \) is the \( r \)th moment of \( P_n(x) \). The rationale for this approximation is as follows. Call durations are typically of the order of minutes and therefore, the frequency with which new calls are generated or old calls are torn down is significantly less than the packetization interval (assumed to be 1 unit of time here). Let us divide the interval of time between two successive state changes into two subsets A & B. The interval A is exactly the first packetization interval and B is the remaining time until the next state change. Since the nD/D/1 queue reaches steady state in one packetization
interval (see lemma 1 of Ramamurthy and Sengupta (1988)), the actual delay distribution on the set $B$ is exactly $P_r(x)$ introduced earlier. Further, the duration of the set $B$ is substantially larger than that of the set $A$, which means that a very small error is introduced if we assume that the delay distribution is $P_r(x)$ on the set $A$ also. Indeed most of these intuitive arguments can be formalized and it can be proved that the maximum error in $P(W \leq x)$ in (11) is bounded by $2\lambda(1-\pi_2)$ where $\lambda$ is measured in call arrivals per packetization interval. In the case of voice for example, the packetization interval is 0.008 seconds (if we assume an ATM payload of 64 bytes), and so the value of $\lambda$ is extremely small. This means that the approximation for $P(W \leq x)$ in (11) is very good.

3. NUMERICAL RESULTS AND THE BUFFER DESIGN PROBLEM

In figures 1 and 2, we show a comparison of results for the exact, $M/D/1$ and asymptotic methods for $n = 24$ and 2000; $p = 0.99$. In these figures, we show the delay distribution expressed in number of cells or packets. Hence, $H(k) = P_r(x)$, where $k = x/p$. Note that this transformation gives us (approximately), the queue size at arrival instants. Expressing the delay distribution in number of cells has the advantage that this quantity is directly related to the buffer sizing problem. Two observations that can be made from these figures are: (1) the $M/D/1$ queue underestimates the delay distribution significantly and (2) the asymptotic method is very close to the exact answer (specially at the tail of the distribution).

In table 1, we report the mean and standard deviations of delay expressed in number of cells the $n = 24$ and 2000 but for values of $p$ that range from 0.3 to 0.99. From this table, it can be seen that the $M/D/1$ result is of limited use when the utilization is larger than 80%. However, the $M/D/1$ result is good for low to moderate utilizations. On the other hand, the asymptotic method works well for utilization greater than 80%.

In tables 2 and 3, we show the $p^{th}$ percentile of delay as a function of $p$ (for $n = 24$ and 2000) where $p = 99.999$ and 99,9999 and $p$ is varied from 0.3 to 0.99. We express the $p^{th}$ percentile in number of cells. There are two important observations that we can make. At light to moderate loads, the percentage error in the asymptotic method is not very small. Indeed the $M/D/1$ approximation may work better at these loads. However, we notice that the asymptotic method is rarely off by more than 3 cells for all values of $p$, $n$ and $p$. This implies that a buffer design based on the asymptotic method gives an answer within 3 cells of the correct answer. This feature together with the simplicity of the result makes the asymptotic method particularly attractive. The second observation has to do with the rapidity with which the function $(1-H(k))$ falls off as a function of $k$. For example, for $n = 2000$ and $p = 0.99$, a call disruption probability of $10^{-5}$ can be achieved with 98 buffers. To improve the call disruption probability by an order of magnitude to $10^{-6}$, one needs 108 buffers, i.e. only 10% more than the previous example. This implies that the design parameter (buffer size) is relatively insensitive to the design criterion (call disruption probability) and this property is inherent to the problem that we are studying. Needless to say, this is a very nice feature of the problem as inaccuracies in measurements and/or traffic forecasts will not make the design too far wrong.

4. SUMMARY

Continuous bit rate traffic generated by existing circuit switched systems as well as packetized voice and video sources using fixed rate coders will be a dominant traffic in evolving broadband networks. Such traffic types are characterized by stringent performance requirement. In this paper we develop an analytic model for a packet (cell) multiplexer for CBR traffic in an ATM network, and find its delay distribution. We develop an approximation based on an asymptotic analysis whose time complexity is independent of the number of sources. The approximation works well over a range of parameter values and can be used in real time calculations. We also show that for small call disruption probability, the number of buffers required is significantly less than the theoretical maximum, and the buffer size is fairly robust to variations in traffic parameters.

REFERENCES


Table 1: Mean and Standard Deviation of Delay for n=24 and 2000, expressed in number of cells.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Exact mean</th>
<th>std.dev</th>
<th>M/D/1 mean</th>
<th>std.dev</th>
<th>Asymptotic mean</th>
<th>std.dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>2.32446</td>
<td>1.56015</td>
<td>49.49975</td>
<td>49.83197</td>
<td>2.95357</td>
<td>1.60236</td>
</tr>
<tr>
<td>0.98</td>
<td>2.23933</td>
<td>1.55010</td>
<td>24.49995</td>
<td>24.83105</td>
<td>2.84398</td>
<td>1.59582</td>
</tr>
<tr>
<td>0.95</td>
<td>2.00481</td>
<td>1.50840</td>
<td>9.50000</td>
<td>9.82768</td>
<td>2.55133</td>
<td>1.55856</td>
</tr>
<tr>
<td>0.90</td>
<td>1.67404</td>
<td>1.41472</td>
<td>4.50000</td>
<td>4.82182</td>
<td>2.16062</td>
<td>1.46500</td>
</tr>
<tr>
<td>0.80</td>
<td>1.18216</td>
<td>1.19719</td>
<td>2.00000</td>
<td>2.30940</td>
<td>1.62328</td>
<td>1.25429</td>
</tr>
<tr>
<td>0.70</td>
<td>0.84434</td>
<td>0.98825</td>
<td>1.16667</td>
<td>1.46249</td>
<td>1.28048</td>
<td>1.06812</td>
</tr>
<tr>
<td>0.60</td>
<td>0.60487</td>
<td>0.80605</td>
<td>0.75000</td>
<td>1.03078</td>
<td>1.04792</td>
<td>0.91750</td>
</tr>
<tr>
<td>0.50</td>
<td>0.42990</td>
<td>0.65141</td>
<td>0.50000</td>
<td>0.76376</td>
<td>0.88213</td>
<td>0.79766</td>
</tr>
<tr>
<td>0.40</td>
<td>0.29841</td>
<td>0.51975</td>
<td>0.33333</td>
<td>0.57735</td>
<td>0.75910</td>
<td>0.70191</td>
</tr>
<tr>
<td>0.30</td>
<td>0.19704</td>
<td>0.40496</td>
<td>0.21429</td>
<td>0.43448</td>
<td>0.66476</td>
<td>0.62457</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Exact mean</th>
<th>std.dev</th>
<th>M/D/1 mean</th>
<th>std.dev</th>
<th>Asymptotic mean</th>
<th>std.dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>19.77569</td>
<td>13.51087</td>
<td>49.49975</td>
<td>49.83197</td>
<td>20.27822</td>
<td>13.53614</td>
</tr>
<tr>
<td>0.98</td>
<td>15.05853</td>
<td>11.74851</td>
<td>24.49995</td>
<td>24.83105</td>
<td>15.51478</td>
<td>11.77707</td>
</tr>
<tr>
<td>0.95</td>
<td>8.19661</td>
<td>7.65411</td>
<td>9.50000</td>
<td>9.82768</td>
<td>8.65392</td>
<td>7.72768</td>
</tr>
<tr>
<td>0.90</td>
<td>4.29940</td>
<td>4.45654</td>
<td>4.50000</td>
<td>4.82182</td>
<td>4.78043</td>
<td>4.58921</td>
</tr>
<tr>
<td>0.80</td>
<td>1.97567</td>
<td>2.26447</td>
<td>2.00000</td>
<td>2.30940</td>
<td>2.46985</td>
<td>2.44091</td>
</tr>
<tr>
<td>0.70</td>
<td>1.16025</td>
<td>1.45087</td>
<td>1.16667</td>
<td>1.46249</td>
<td>1.65714</td>
<td>1.65714</td>
</tr>
<tr>
<td>0.60</td>
<td>0.74767</td>
<td>1.02664</td>
<td>0.75000</td>
<td>1.03078</td>
<td>1.23333</td>
<td>1.23333</td>
</tr>
<tr>
<td>0.50</td>
<td>0.49900</td>
<td>0.76202</td>
<td>0.50000</td>
<td>0.76376</td>
<td>1.00000</td>
<td>1.00000</td>
</tr>
<tr>
<td>0.40</td>
<td>0.33287</td>
<td>0.57655</td>
<td>0.33333</td>
<td>0.57735</td>
<td>0.85000</td>
<td>0.85000</td>
</tr>
<tr>
<td>0.30</td>
<td>0.21407</td>
<td>0.43410</td>
<td>0.21429</td>
<td>0.43448</td>
<td>0.73333</td>
<td>0.73333</td>
</tr>
</tbody>
</table>
Table 2: pth percentile of Delay Distribution expressed in number of cells, for $p = 99.999$ and $99.9999$ and $n = 24$.

<table>
<thead>
<tr>
<th>$p$</th>
<th>Exact # cells</th>
<th>M/D/1 # cells</th>
<th>Asymptotic # cells</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>10.67</td>
<td>573.15</td>
<td>11.63</td>
</tr>
<tr>
<td>0.98</td>
<td>10.59</td>
<td>285.44</td>
<td>11.52</td>
</tr>
<tr>
<td>0.95</td>
<td>10.35</td>
<td>112.79</td>
<td>11.17</td>
</tr>
<tr>
<td>0.90</td>
<td>9.95</td>
<td>55.02</td>
<td>10.61</td>
</tr>
<tr>
<td>0.80</td>
<td>9.10</td>
<td>26.34</td>
<td>9.60</td>
</tr>
<tr>
<td>0.70</td>
<td>8.21</td>
<td>16.65</td>
<td>8.69</td>
</tr>
<tr>
<td>0.60</td>
<td>7.30</td>
<td>11.74</td>
<td>7.90</td>
</tr>
<tr>
<td>0.50</td>
<td>6.38</td>
<td>8.74</td>
<td>7.20</td>
</tr>
<tr>
<td>0.40</td>
<td>5.47</td>
<td>6.67</td>
<td>6.58</td>
</tr>
<tr>
<td>0.30</td>
<td>4.57</td>
<td>5.11</td>
<td>6.05</td>
</tr>
</tbody>
</table>

Table 3: pth percentile of Delay Distribution expressed in number of cells, for $p = 99.999$ and $99.9999$ and $n = 2000$.

<table>
<thead>
<tr>
<th>$p$</th>
<th>Exact # cells</th>
<th>M/D/1 # cells</th>
<th>Asymptotic # cells</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>97.35</td>
<td>573.11</td>
<td>97.76</td>
</tr>
<tr>
<td>0.98</td>
<td>88.87</td>
<td>285.43</td>
<td>89.14</td>
</tr>
<tr>
<td>0.95</td>
<td>67.98</td>
<td>112.80</td>
<td>68.37</td>
</tr>
<tr>
<td>0.90</td>
<td>45.58</td>
<td>55.20</td>
<td>46.67</td>
</tr>
<tr>
<td>0.80</td>
<td>25.05</td>
<td>26.34</td>
<td>26.96</td>
</tr>
<tr>
<td>0.70</td>
<td>16.36</td>
<td>16.65</td>
<td>18.61</td>
</tr>
<tr>
<td>0.60</td>
<td>11.70</td>
<td>11.74</td>
<td>14.14</td>
</tr>
<tr>
<td>0.50</td>
<td>8.78</td>
<td>8.74</td>
<td>11.38</td>
</tr>
<tr>
<td>0.40</td>
<td>6.76</td>
<td>6.67</td>
<td>9.52</td>
</tr>
<tr>
<td>0.30</td>
<td>5.24</td>
<td>5.11</td>
<td>8.17</td>
</tr>
</tbody>
</table>

Fig. 1: Delay distribution for $n = 24$, $p = .99$ expressed in number of cells.

Fig. 2: Delay Distribution for $n = 2000$, $p = .99$ expressed in number of cells.