A universal framework is presented for the approximate analysis of general queueing network models (QNMs) of packet-switched computer communication systems (CCSs) incorporating locally adaptive routing. Such networks can be viewed as arbitrary collections of nodal queueing systems each of which consists of a set of outgoing channel queues and a node dispatching the incoming traffic of packets to these queues according to a policy that depends on their status. Abstract product-form approximations for the joint equilibrium distribution of the queue lengths of open and closed QNMs are established, subject to entropy maximisation and nodal mean value constraints. Moreover, approximate flow conservation formulae are defined, based on the use of the Generalised Exponential (GE) distributional model, and iterative computational procedures are suggested. As a demonstration, the proposed methodology is applied to study QNMs under the semidynamic routing policy "Join the shortest queue with instantaneous jockeying (SQJ)". The GE/GE/c/SQJ nodal queueing model - with GE-type interarrival and service time distributions and c(c ≥ 2) heterogeneous single server queues under SQJ routing - plays the role of a building block in the solution process. Numerical validation results are given and favourable comparisons against simulations are made.

1. INTRODUCTION

Analytic studies involving queueing network models (QNMs) of packet-switched computer communication systems (CCSs) are most appropriate in predicting performance as opposed to time consuming simulations. The cornerstone in this field was set by Kleinrock [1] who introduced the first open QNM of a stored-and-forward (S/F) communication network. This model comprises from single server queues which represent channels operating under random alternate routing, where the incoming traffic at each node is distributed to one of several outgoing channels according to a fixed transition probability. The analysis uses the standard algorithm for open Jacksonian networks and is based on Kleinrock's "independence assumption" (i.e., each time a packet enters a node, a new length is drawn independently from its queue length distribution). However, to compensate for traffic and topological variations in many CCSs, several authors (c.f., [2-4]) have studied QNMs with some form of adaptive or dynamic routing, where the selection of the next channel at a given node is made on the basis of a state dependent policy. The solution of such QNMs is exceedingly difficult mainly due to (i) the strong statistical dependency amongst the queues attached to a single node, and (ii) the effect of adaptivity on the output stream of each queue.

In the majority of cases of interest, the practical implementation of the above solutions is heavily based on the assumption of the exponential distribution. However, in reality, the exponential distribution is not an appropriate model to describe, even approximately, the sizes of information packets in actual networks (e.g., X25) and, subsequently, the stochastic behaviour of channel transmission times.

This paper presents a universal analytical approach for the approximate study of S/F packet-switched CCSs under locally adaptive routing, where the selection of a channel at each node depends on the status of the outgoing channels. These systems are represented by general single class "node-oriented" QNMs (NO' QNMs) which can be viewed as arbitrary collections of nodal queueing systems corresponding to network nodes, where channels act as single servers of infinite capacity queues, while the nodes play the role of packet dispatchers. Such a nodal model represents a QNM of a single node with infinite capacity and is denoted by "G/G/c/P", where G/G stands for generally interarrival/transmission time distributions of packets and c is the number of the attached outgoing channel queues operating under an abstract routing policy P.

The approach is based on the principle of maximum entropy (ME), viewed as an inference procedure, subject to nodal mean value constraints. The use of entropy maximisation is motivated by earlier applications of the principle in analysing queueing systems and networks with random routing (c.f., Kouvatsoos [5, 6]). Moreover, the generalised exponential (GE) distributional model of the form (c.f., [5, 6])

\[ F(t) = 1 - \frac{2}{c+1} \exp \left( -\frac{2t}{c+1} \right), \quad t \geq 0, \] (1.1)

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where 1/\nu is the mean and C is the squared coefficient of variation (SCV) - is used to approximate general distributions with known first two moments. Although the GE distribution is improper for C < 1, nevertheless, it is still a useful and versatile tool in the field of systems modelling.

ME product-form approximations for open and closed 'NO' QNMs are established in Sect. 2. Iterative computational procedures and GE-type flow conservation formulae are proposed in Sect.3. As a demonstration, GE-type 'NO' QNMs under the semidynamic routing policy "Join the shortest queue with instantaneous jockeying (SQJ)" are analysed in Sect.4. Favourable comparisons against simulation follow in Sect.5 and concluding remarks are given in Sect.6.

2. THE ME PRODUCT-FORM APPROXIMATIONS

2.1 Open 'NO' QNMs

Consider an open 'NO' QNM representing a packet-switched CCS (or subsystem of CCS) containing N nodes and M channels with general external interarrival time and service (transmission) time distributions. It is assumed that (i) each node operates under an arbitrary locally adaptive policy P, (ii) all jobs (packets) belong to the same class, and (iii) an abstract service discipline is associated with each channel queue. For each node i, i = 1,...,N, let

- \( O(i) \) be the number of outgoing channels,
- \( OQ(i) \) be the set of outgoing channels,
- \( I(i) \) be the number of incoming channels,
- \( IQ(i) \) be the set of incoming channels,
- \( \lambda_0(i), C_{a0}(i) \) be the rate and SCV of the external interarrival times,
- \( \lambda_{nd}(i), C_{nd}(i) \) be the rate and SCV of the overall (merged) interarrival process.

For each channel queue m, m = 1,...,M, let

- \( \mu(m), C_{sm}(m) \) be the rate and SCV of the service (transmission) time distribution,
- \( \lambda_q(m), C_{aq}(m) \) be the rate and SCV of the interarrival time distribution.

The state of the network at any given time is described by an integer vector \( \mathbf{k} = (k_1, \ldots, k_N) \), where \( k(i) \) is a vector describing the state of node i, i = 1,...,N, namely \( k(i) = (k_1(i), \ldots, k_0(i)) \), and \( k(i)_j, j = 1,2,\ldots,0(i) \), denotes the number of jobs (0) in the outgoing channel queue j, j = 1,2,...,0(i), associated with node i, i = 1,2,...,N. Let \( Q(i) \) be the set of all feasible states of node i, i = 1,2,...,N and Q be the Cartesian product of all \( Q(i) \)'s constituting the set of all feasible system states. Finally, under the stability condition \( \prod_i (\lambda_{nd}(i)/\mu(m)) < 1 \), where \( \mu = \sum_m \mu(m) \), let

\[
P(k(i)) = \text{be the joint probability of the system to be in state } k(i),
\]
\[
P_{nd}(i)(k(i)) = \text{be the marginal probability of node } i \text{ to be in state } k(i),
\]
\[
p_q(m)(k) = \text{be the probability of channel queue } m \text{ to have } k \text{ packets.}
\]

Suppose the available information about Q places a number of constraints on \( P(k) \). It is assumed that these take the form of mean values of several suitable nodal functions \( \{f_j(i)(k(i)), i = 1,2,\ldots,N\} \), where K is less than the number of possible states. The form of the ME solution \( P(k) \), \( k \in Q \), can be completely specified by maximising the system’s entropy functional

\[
H(P) = -\sum_{k \in Q} P(k) \log P(k),
\]

subject to the constraints

\[
\sum_{k \in Q} f_j(i)(k) P(k) = \langle f_j(i) \rangle,
\]

for \( j = 1,2,\ldots,K; i = 1,2,\ldots,N \), where \( \langle f_j(i) \rangle \) are the prescribed mean values defined on the set of functions \( \{f_j(i)(k(i)), k \in Q\} \).

The maximisation of (2.1), subject to constraints (2.2) - (2.3), can be carried out using the Lagrange method of undetermined multipliers leading to the equilibrium solution

\[
P(k) = \frac{1}{Z(k)} \prod_{i=1}^{N} \exp\left(-\sum_{j=1}^{K} \lambda_j(i) f_j(i)(k)\right), \quad k \in Q,
\]

where \( \lambda_j(i), j = 1,2,\ldots,K; i = 1,2,\ldots,N \), are the Lagrange multipliers corresponding to the set of constraints (2.3), and \( Z(k) = \exp(\lambda_0(i)) \) is the normalising constant, \( \lambda_0(i) \) being the Lagrange multiplier determined by (2.2). It can be verified (by interchanging the sum and product in (2.4), c.f., Kouvatsos [5], Georgatsos [7]), that the ME joint probability \( P(k) \), \( k \in Q \), can be expressed as

\[
P(k) = \prod_{i=1}^{N} P_{nd}(i)(k(i)),
\]

where \( P_{nd}(i)(k(i)), k(i) \in Q \) is the marginal ME solution of a stable G/G/0(i)/P nodal model, i = 1,2,...,N, subject to normalisation and constraints \( \langle f_j(i) \rangle \), i = 1,2,...,N, respectively.

Hence, the ME product-form approximation \( P(k) \), \( k \in Q \), subject to nodal constraints (2.2)-(2.3), suggests a decomposition of the 'NO' QNM into its component G/G/0(i)/P nodal queues. Evidently, given the routing policy P, the implementation of solution (2.5) requires the computation for each node i, i=1,...,N, of

(i) The parameters \( \lambda_{nd}(i), C_{nd}(i) \) of the merged interarrival process, and

(ii) The marginal ME solution \( P_{nd}(i)(k(i)) \).
2.2 Closed ‘NO’ QNMs

Consider an arbitrary closed ‘NO’ QNM of a packet-switched CCS with N nodes operating under an arbitrary locally adaptive policy P, M channels and fixed population of packets L. The notation of Sect.2.1 applies with the exclusion of all parameters referring to an external arrival process.

Assuming the same type of nodal constraints, as the ones incorporated in the ME formalism for open ‘NO’ QNMs, the ME joint state probability distribution can be expressed by truncating and normalising the state space of ME solution (2.4), namely,

\[ P(k) = \frac{1}{Z(L,N)} \prod_{i=1}^{N} f_i(k), \quad k \in Q(L,N), \]  

(2.6)

where \( Z(L,N) \) is the normalising constant and \( f_i(k) \) is the so called state function of each node \( i = 1, 2, ... , N \), given by the ‘exp’ function of the product of eq. (2.4). Note that unlike the ME solution for open ‘NO’ QNMs (c.f., (2.4)), the ME solution of closed ‘NO’ QNMs (c.f., (2.6)) cannot be decomposed further due to the finite state space restriction.

Clearly, the implementation of ME product-form approximation (2.6) requires

(i) The priori estimation of the Lagrange multipliers of \( \{ f_i(k) \} \), \( i = 1, 2, ... , N \), and

(ii) A convolution type procedure for calculating \( P(k) \) (c.f., (2.4)) and other performance measures of interest.

3. FLOW FORMULAE AND COMPUTATIONAL PROCEDURES

3.1 The Flow Equations

Consider a general open ‘NO’ QNM in which for each node \( i, i = 1, 2, ... , N \), it is assumed that

(i) External arrivals constitute a renewal process of rate \( \lambda_q(i) \) and SCV of the interarrival time, \( C_{aq(i)} \).

(ii) The dispatching of packets by a node to the outgoing channels is based on an abstract routing policy \( P \) incorporating local information. Processing (switching) times and propagation delays are considered negligible.

(iii) The transmission times for successive packets at channel \( m, m = 1, 2, ... , M \), are independent and identically distributed with a common probability distribution and parameters \( \mu(m) \) and \( C_s(m) \), \( m = 1, 2, ... , M \). Transmission times are also independent from one channel to another (c.f., Kleinrock’s independence assumption).

(iv) The departure process from any channel queue in the open ‘NO’ QNM is a renewal process with rate \( \lambda_{dq}(m) = \lambda_{q}(m) \) and SCV of the interdeparture time, \( C_{dq}(m) \). This assumption is generally unjustified but it is made in order to approximate the parameters of the interdeparture process.

For each node \( i, i = 1, 2, ... , N \), at equilibrium, the following job flow rate equations must hold:

\[ \lambda_{nd}(i) = \sum_{m \in Q(i)} \lambda_{q}(m) + \lambda_{q}(i), \quad i = 1, 2, ... , N. \]  

(3.1)

Moreover, the following approximation is made: The external interarrival times, channel transmission times and all flow (depart, split, merge) processes conform with a GE-type distribution. To this end, the merging formula proposed by Kouvatsos [5] can be applied to approximate the SCV of the overall (merged) interarrival process of GE streams of packets generated by incoming channels, namely

\[ C_{nd}(i) = \left[ \sum_{m \in Q(i)} \frac{\lambda_{q}(m)}{\lambda_{nd}(i)} [C_{dq}(m) + 1]^{-1} + \frac{\lambda_{q}(i)}{\lambda_{nd}(i)} [1 + C_{a q(i)}]^{-1} \right]^{-1}. \]  

(3.2)

Equations (3.1)-(3.2) hold for any type of packet switched CCS (under analogous GE-type assumptions). However, for open adaptive ‘NO’ QNMs the splitting at each node is made dynamically and, therefore, the throughput of each queue, in general, is expected to be a non-linear function of the nodal traffic characteristics depending on distributional assumptions about the nodal arrival and channel transmission patterns. It can be observed that an arbitrary node \( i \) in a ‘NO’ QNM behaves exactly the same as if it was under random routing policy with transition probabilities \( \{ \lambda_{q}(m)/\lambda_{nd}(i), m \in Q(i) \} \) for dispatching the incoming traffic. Therefore, for each queue \( m \in Q(i) \) and using the splitting formula for calculating \( C_{aq}(m) \) (c.f., [5, 6]) it is implied that

\[ C_{aq}(m) = 1 + \frac{\lambda_{q}(m)}{\lambda_{nd}(i)} [C_{nd}(i) - 1]. \]  

(3.3)

Finally, treating each channel queue as a stable GE/GE/1 queueing system and applying the GE-type interdeparture formula (c.f., [5, 6]), the following expression for \( C_{dq}(m) \) can be obtained

\[ C_{dq}(m) = \rho(m)[1 - p(m)] + p(m)C_s(m) + (1 - p(m))C_{aq}(m), \]  

(3.4)

where \( \rho(m) = \lambda_{q}(m)/\mu(m) \).

Note that flow formulæ (3.2)-(3.4) are universal in the sense that they do not depend explicitly on the adaptive policy \( P \) employed at each node \( i, i = 1, 2, ... , M \). The effect of this policy is reflected via the expressions for the throughput of the outgoing channel queues.

3.2 The ME Computational Procedures

3.2.1 Open ‘NO’ QNMs
The ME algorithm for single class general open 'NO' QNMs under a locally adaptive policy \( P \) can be summarised in a step-wise fashion as follows:

Begin
Step 1 [Input data]

\[ N, M, OQ(i), IO(i), \lambda_0(i), C_{a0}(i), i = 1,2,...,N; \]
\[ \mu(m), C_s(m), m = 1,2,...,M; \]

Step 2 [Compute flows]

Step 2.1 Initiate traffic at each node;
Step 2.2 Solve each G/G/O(i)/P nodal queueing system \( i, i = 1,2,...,N; \)
Step 2.3 For all \( m \in OQ(i) \)
   Step 2.3.1 Obtain \( \lambda_q(m), m=1,2,...,M \). i.e.,
   \[ \lambda_q(m) = (1 - P_q(m))(0)\mu(m); \]
   Step 2.3.2 Obtain \( C_{dq}(m) \) (c.f., eq. (3.4));
Step 2.4 Compute new nodal traffic parameters \( \lambda_{nd}(i), \)
   \( C_{and}(i), i = 1,2,...,N \) (c.f., eqs. (3.1)-(3.2));
Step 2.5 Check convergence of \( \lambda_{nd}(i), C_{and}(i), i = \)
   \( 1,2,...,N \). If convergence, continue form step 3,
   else repeat form step 2.2;
Step 3 [Obtain performance measures]

End

The implementation of this algorithm requires a priori the knowledge of the ME solution of the G/G/O(i)/P nodal queueing system. This solution depends on the choice of the \( G \)-type distributions and the routing policy \( P \). Note that the proposed algorithm will generally involve the solution of complex non-linear equations, and thus the convergence of parameters \( \lambda_{nd}(i), C_{and}(i), i = 1,2,...,N \), should be experimentally tested for each particular routing policy \( P \).

3.2.2 Closed 'NO' QNMs

The Lagrange coefficients of the ME solutions (2.6) can be approximately determined by employing the so-called pseudo-open 'NO' QNM (c.f., Kouvatsos [5, 6]) having the same topology as the original closed network, but with infinite capacity channel queues satisfying the job rate equations of flow conservation (c.f., (3.1)) and the fixed population mean constraint, i.e.,

\[ L = \sum_{m=1}^{M} <m>_o, \quad (3.5) \]

where \( <m>_o \) is the mean queue length of channel queue \( m \) in the pseudo-open 'NO' QNM. To this end, the implementation of the ME solution (2.6) can proceed in two stages (as for QNMs with random routing (c.f., [6])):

Stage 1. Solve the pseudo-open 'NO' QNM approximately by applying the ME computational procedure of Sect.3.2.1 (without external arrival traffic) iteratively until the fixed population mean constraint (3.5) is satisfied;

Stage 2. Incorporate into the state function \( f_i(n_i) \) of (2.6)
the estimates of the Lagrange coefficients \( \beta_j(i), j = 1,2,...,K; i = 1,2,...,N \), from stage 1 and apply a convolution based technique to compute the ME joint state probability (2.6) and various performance metrics. This procedure is performed iteratively by modifying the Lagrange coefficients until the flow balance equations (3.1) - as applied to the original closed 'NO' QNM-are satisfied;

Note that due to the complexity of the expressions involved it is not generally feasible to produce a proof of convergence of the iterative convolution procedure of Stage 2. Given a particular policy \( P \), this convergence, as in the case of open networks, should be experimentally tested.

4. APPLICATION TO GENERAL 'NO' QNMs UNDER SQJ ROUTING

In this section the proposed methodology is applied, as a demonstration, for the approximate analysis of general 'NO' QNMs under SQJ routing. Under this policy arriving packets at each node join the end of the shortest outgoing channel queue in a FCFS fashion and are permitted to jockey from one queue to another whenever departures take place and the difference amongst their queue lengths becomes equal to 2. In cases where more than two channel queues qualify either to accept an arrival or to generate a jockey, a fixed probabilistic rule is applied to resolve the conflict.

Various simulation studies (e.g., Chow and Kohler [3]), confirmed improvement in the performance of the network under SQJ routing when compared to other adaptive routing schemes. Although the overhead of jockeying may not be tolerated by very fast packet-switched CCSS, nevertheless for more moderate systems the SQJ policy has some desirable properties, namely

(i) Imitates on aggregate a multiple sever operation at each node.
(ii) Facilitates load balancing amongst the attached channel queues at each node.
(iii) Provides optimistic performance bounds (e.g., on mean queue lengths for open or throughputs for closed 'NO' QNMs, respectively) over all other locally adaptive routing policies (e.g., the shortest queue policy).

It is properties such as the above (and especially property (iii)) that make the study of networks with SQJ routing of particular interest.

The implementation of the computational procedures of Sect.3 clearly requires the determination of the ME state probability for the G/G/c/SQJ nodal system. However, due to the strong stochastic dependency that exists amongst the outgoing channel queues, only a few analytic studies, based on exponential assumptions, have been reported so far in the literature (c.f., [8, 9]).

4.1 The G/G/c/SQJ Nodal Queueing System

Consider a stable G/G/c/SQJ nodal queueing system with \( c \) (\( \geq 2 \)) heterogeneous servers complying with the underlying assumptions of Sect.3.1. For notational simplicity let \( \lambda, C_a \) be the arrival rate and the SCV of the interarrival
times,
\( \mu_i, C_{si} \) be the service rate and the SCV of the general service time at server \( i, i=1,2,...,c \).

Moreover, the state of the nodal system at any given time is described by an integer vector \( k = (k_1,...,k_c) \), where \( k_i \geq 0 \), \( i=1,2,...,c \), is the number of packets at the channel queue \( i \), \( i=1,2,...,c \). The set of all feasible states is denoted by \( Q \).

Because of the jockeying mechanism employed the system.

Because of the jockeying mechanism employed the possible system states \( \{ k \} \) are restricted to be of the equivalent form \( \{ k_{1..c} \} \), \( k \in [1,c] \), where \( (i_{1..c}) \) is a sequence of queues attached to node, each of which having \( k \) (\( \geq 0 \)) packets while the rest of the queues contain \( k+1 \) packets (ie, channel queues \( i_{1..c} \) have an 'advantage' over the others in accepting new arrivals). Finally, the set of all states of the node, where queues \( (i_{1..c}) \) are open for arrivals, is more explicitly denoted by \( Q_{1..c} \). Suppose the following mean value constraints of the unknown state probability \( P(k) \), \( k \in Q \) are known to exist:

(i) The normalisation,
\[ k \notin Q \quad P(k) = 1, \quad (4.1) \]
(ii) The mean queue length constraint, \(<n>\),
\[ k \notin Q \quad a(k) P(k) = <n>, \quad (4.2) \]
where \( a(k) \) denotes the total number of packets in the system,
(iii) The full node constraint, \( U, 0 < U < <n> \),
\[ \sum_{k \in Q} a(k) P(k) = U, \quad (4.3) \]
and for all \( (i_{1..c}) \in C(1..c; 1), 1 = 1,...,c-1 \), where \( C(1..c; 1) \) is the set of feasible combinations of symbols \( 1,...,c \) over \( 1 \),
(iv) The zero side constraints, \( y_{1..c} \in (0,1) \),
\[ P(Q_{1..c} = y_{1..c} \), \quad (4.4) \]
(v) The positive side constraints, \( \delta_{1..c} \in (0,1) \),
\[ k \in Q^{*}_{1..c} \quad P(k) = \delta_{1..c}, \quad (4.5) \]
where \( Q^{*}_{1..c} = Q_{1..c} \) \( \setminus \{ 1..c \} \).

Notice that the choice of constraints (4.1)-(4.3) is inspired from the ME analysis of a stable G/G/c queue (c.f., [7]), while constraints (4.4)-(4.5) are introduced in order to differentiate the states that belong to different categories (where certain queues have 'advantage' over the others). By applying the Lagrange method of undetermined multipliers the form of the ME solution \( P(k) \), \( k \in Q \), can be similarly determined, as in the case of (2.4), subject to constraints (4.1)-(4.5). The normalising constant \( Z \) and the Lagrange coefficients \( X, W, q's \) and \( g's \) \( (2(2-c-1)+3 \) in total) corresponding to constraints (4.2)-(4.5), respectively, can be computed by approximating each G/G/c/SQJ nodal queueing system by a GE/GE/c/SQJ nodal model (having the same rate and SCV for the interarrival and service times, respectively). As a result, the state transition rates of a GE/GE/c/SQJ nodal system and the corresponding global balance equations can be established. The probabilistic arguments behind their derivation can be found in Georgatsos [7].

By substituting the form of the proposed ME solution \( P(k) \) into the GE-type global balance equations, the introduced Lagrange coefficients can be calculated by (i) establishing \( X \) as the unique real root \( e(0,1) \) of a non-linear equation, and (ii) calculating \( W \), \( q's \) and \( g's \) by solving two systems of linear equations, each one of order \((2^c-1) \times (2^c-1)\). It can be verified that this ME solution is identical to the exact global balance solution of the GE/GE/c/SQJ nodal queueing system and thus, it generalises earlier results in the area (c.f., [8, 9]). Note that the marginal probability of having \( k \) packets at the outgoing channel queue \( m \), \( P_Q^{(m)}(k) \), can be obtained by marginalising the joint state probability \( P(k) \), \( k \in Q \). It can also be shown (c.f., [7]) that the aggregate queue length distribution of a stable GE/GE/c/SQJ nodal queueing system is equivalent to that of the corresponding GE/GE/c queue at equilibrium with \( c \) heterogeneous servers.

Note that due to the complexity of the global balance equations relating to the GE/GE/c/SQJ queue, conventional algebraic methods for solving such equations (eg, method of \( z \)-transforms) either would fail or they may require a tremendous amount of calculations. However, determining the form of the ME solution, \( P(k) \), subject to an appropriate set of constraints, and substituting into the GE-type global balance equations, it has been possible to obtain directly an exact product form expression for \( P(k) \), without employing, in comparison, a great deal of calculations.

4.2 Numerical Results

The credibility of the proposed ME methodology for the approximate analysis of general 'NO' QNMs is illustrated against simulation (with 95% confidence intervals) using two typical numerical tests regarding open and closed 'NO' QNMs with three nodes and two outgoing channels per node under the SQJ routing policy. Ties amongst queues at arriving instants are resolved with fixed probabilities \( \mu(i)/[\mu(1)+\mu(2)] \) in favour of queue 1, \( i=1,2 \). For both ME algorithm and simulation, the GE distribution is used to approximate general external interarrival time and service time distributions with known rate and SCV (taken here as greater than one).

Tables 1 and 2 display marginal utilisations (UTILs) and mean queue lengths (MQLs) per channel queue, together with maximum and average tolerance errors MAX.TOL, AV.TOL, respectively. Note that the time complexity for computing the normalising constant of the ME solution for the closed 'NO' QNM is of \( o(ML) \).

It can be seen (c.f., Tables 1 and 2) that the ME results are very comparable to those obtained by simulation. It has been generally observed over many other numerical tests (c.f., [7]) that the average error tolerance for UTILs is less than 0.03 and for the MQLs is less than 0.05. The best performance is achieved under exponential assumptions, where the discrepancy between SIM and ME is
virtually zero. The performance begins to deteriorate for cases where there is a large difference in the value of SCVs amongst the channel queues of the network, particularly when SCVs much greater than one are involved. This may be attributed to the fact that in such cases the renewal underlying assumption for the overall arrival process at each node is further violated.

### TABLE 1: OPEN 'NO' QNM

**Input Data:** M=6, N=3, C<sub>01</sub>=2, C<sub>03</sub>=1, C<sub>0(1)=25</sub>, C<sub>0(3)=13</sub>, \( \mu=(1, 2, 0.5, 1, 0.5, 3) \), \( C_s=(11, 5, 9, 15, 11, 9) \)

<table>
<thead>
<tr>
<th>Queues</th>
<th>UTIL SIM</th>
<th>ME</th>
<th>MQL SIM</th>
<th>ME</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0.71</td>
<td>0.70</td>
<td>14.30</td>
<td>14.40</td>
</tr>
<tr>
<td>2</td>
<td>0.65</td>
<td>0.65</td>
<td>14.12</td>
<td>14.12</td>
</tr>
<tr>
<td>3</td>
<td>0.77</td>
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</tr>
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<td>4</td>
<td>0.72</td>
<td>0.72</td>
<td>13.03</td>
<td>11.91</td>
</tr>
<tr>
<td>5</td>
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<td>0.80</td>
<td>8.25</td>
<td>7.46</td>
</tr>
<tr>
<td>6</td>
<td>0.63</td>
<td>0.63</td>
<td>8.12</td>
<td>7.30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MAX.TOL</th>
<th>AV.TOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>UTIL</td>
<td>0.011</td>
</tr>
<tr>
<td>MQL</td>
<td>0.005</td>
</tr>
</tbody>
</table>

### TABLE 2: CLOSED 'NO' QNM

**Input Data:** M=6, N=3, L=8, \( \mu=(1, 2, 1, 2, 1, 2.5) \), \( C_s=(40, 50, 60, 70, 20, 30) \)

<table>
<thead>
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<th>Queues</th>
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5. **CONCLUSIONS**

A new analytic framework is proposed, based on the principle of ME, for the approximate analysis of general open and closed 'NO' QNMs of packet-switched CCSs incorporating locally adaptive routing. Abstract product-from approximations for the state probability distribution of open and closed 'NO' QNMs are established, subject to nodal mean value constraints. Moreover, universal approximations for the flow conservation are defined, based on GE-type renewal assumptions, and iterative computational procedures are suggested.

The utility of the methodology is demonstrated in the approximate analysis of 'NO' QNMs under SQJ routing policy. The ME solution of the GE/GE/c/SQJ nodal system plays the role of a building block in the computational process. The credibility of the ME algorithm is illustrated against simulation for both open and closed 'NO' QNMs and moreover, conjectures on GE-type performance bounds over two-phase distributions are made.

The work is currently extended towards the approximate study of 'NO' QNMs with finite capacities (and thus, blocking) involving different types of adaptive routing policies.

### REFERENCES