An Improved Heuristic Approximation for the $GI/GI/1$ Queue with Bursty Arrivals*

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New transmission methods, protocols and services lead to an extended range of distributions, which may be of importance for the performance evaluation.

Many two/two-moment approximations have been proposed for the general, independent single server queue $GI/GI/1$. In reference [6] a diffusion approximation takes into account the complete arrival interval distribution, whereas reference [9] applies a heuristic approach to obtain a three/two-moment approximation.

In the present work a similar approach to that in [9] is applied. In both cases a hyperexponential distribution is used as a general three-moment model for bursty arrivals. However, as the former method uses an interpolation method to adjust to intermediate values between two known extremes, the present model applies an extension to the exact $H_2/M/1$ solution.

The results of the analytical approximation have been compared with extensive simulations, and a weighted measure of deviation for comparison of different approximations has been introduced.

1 Introduction

For any stationary queue there exists the simple relation given by Little's formula:

$$Q = \lambda \cdot W$$  \hspace{1cm} (1)

where

- $\lambda$ = arrival rate
- $Q$ = mean queue length
- $W$ = mean waiting time

The relation applies to any section or the whole of a linear chain of queues and servers. Thus, as long as the arrival rate is known, it is sufficient to calculate either $Q$ or $W$, and the other is given straightforward by relation (1).

The more complete solution is to determine the waiting time/queue length distributions, from which any secondary values like mean, variance, skewness etc. can be calculated.

There is no known explicit solution to the general queue $G/G/m$ or the general single server queue $G/G/1$, whereas many special cases are treated in the queueing literature. Most notable are those of the FCFS queues $M/M/m(1)$, $M/G/1, M^{*}/G/1, G/M/m(1)$, which have exact mean value solutions. For some of these even distributions can be calculated. When given as a transform, moments may be calculated even if explicit distribution functions cannot be found.

Normally, renewal properties are assumed, implying complete independence between an interval between two adjacent events and any previous interval in the same process.

Also, independence between arrival and service processes is assumed. To make this clear, often the $G$ in the Kendall notation is replaced by $GI$, and this applies to arrival as well as service processes. The importance of this assumption is clearly demonstrated by G. Lind [1].

The lack of an exact solution to the $GI/GI/1$ queue has led to numerous attempts to find a good approximation. Some of these attempts have been published in references [2]-[9]. References [2]-[6] present approximations based on diffusion differential equations, whereas references [7]-[9] may be classified as heuristic approximations, where a known exact solution is modified to cater for the assumptions differing from those of that exact solution.

In most cases the approximations are based on the moments of arrival and service distributions of the queue. We will introduce the notation $j/k$-approximation for one that applies $j$ moments of the arrival interval distribution and $k$ moments of the service time distribution. Invariably the lowest $j(k)$ moments are applied. In references [2]-[5] and [7]-[8] only 2/2 approximations are presented, meaning that first and second moments of both processes are applied. The Kimura approximation presented in [6] is an $\infty/2$ approximation, as it relies on an integral equation involving the complete arrival interval distribution with all moments.

The approximation presented in [9] is a 3/2-approximation, based on a hyperexponential arrival interval distribution to give three independent parameters, whereas the two moments of the service time distribution are assumed to be those of any arbitrary distribution.

The present paper proposes a new 3/2-approximation. The initial approach is similar to that of reference [9], in the sense that it applies the exact solution of the $H_2/M/1$ queue.

*Part of this work was carried out at Bond University, Gold Coast, Queensland
as a starting point. The main difference in approach is that the previous solution applies a rather arbitrary interpolation between exact extreme cases, whereas the present proposal uses a modification of the exact $H_2/M/1$-formula over the whole range, and such that the same extreme cases will still be exact.

2 A Common Basis for 3/2-approximations

In formulating the present problem, the most frequently used symbols are:

- **Index $a$** = arrival (arrival variables may be non-indexed for simplicity)
- **Index $s$** = service
- **$\lambda$** = arrival rate
- **$\mu$** = service rate
- **$p = \lambda/\mu$** = load, $0 \leq p \leq 1$
- **$\sigma$** = load at arrival instants, $0 \leq \sigma \leq 1$
- **$m_1$** = $i$-th ordinary moment
- **$m_2$** = mean
- **$c$** = coefficient of variation
- **$\epsilon = c^2 + 1$** = Palm's form factor
- **$r = \frac{c}{2m_2}$** = relative second moment
- **$q = \frac{m_4}{6m_2^2}$** = relative third moment
- **$W$** = mean waiting time
- **$R$** = relative mean waiting time
- **$Q$** = mean queue length

The quantities $c, \epsilon$ and $r$ are equivalent, with a very simple one-to-one relation. The reason for the choice of parameters $r$ and $q$ is simply that negative exponential distributions have moment relations $m_2 = \frac{\sigma}{2}$ and $m_3 = \frac{3\sigma^2}{2}$, in general $m_i = \mu^i \cdot m_1^i$. Hence, for that distribution we always have $r = q = 1$, and the mathematical expressions tend to be simple.

In the following two different 3/2-approximations will be discussed. The first is the one presented in [9]. For simplicity it will be termed My1. The new approximation presented in this paper will be termed My2. The common basis for My1 and My2 is the solution for the $H_2/M/1$ queue. This queue is solved as a special case of the $GI/M/1$ queue, where

$$ W = \frac{\sigma}{\mu(1 - \sigma)} $$

and

$$ \sigma = \int_0^\infty e^{-\mu(1-\sigma)t}dF(t) = F^*(\mu(1 - \sigma)) $$

$F^*(s)$ = Laplace-Stieltjes transform of the arrival interval distribution $F(t)$.

A crucial point in the development of the 3/2-approximation is the non-symmetry with respect to arrival interval and service time distributions, this being the very basis for applying three and two moments respectively. Concerning the service time distribution it is well known that only first and second moments are of significance for the $M/GI/1$ queue. It is by no means given that this property prevails for the $GI/GI/1$ case, but it will be assumed here to be sufficiently accurate for our purpose. The non-symmetry is, however, very clear from the fact that the $GI/M/1$ queue depends on all moments of the $GI$-distribution. On the other hand it is stated by several authors that three moments will be sufficient in most practical cases.

From the above our working hypothesis will be that a 3/2-approximation will be necessary and sufficient in the $GI/GI/1$ case, this being adverse to the usual 2/2-assumption. The necessity will be made clear in the following, whereas the sufficiency will not be substantiated in this paper, and since it is not exact, there will always have to be a practical judgement.

Returning now to the $H_2/M/1$ case, the distribution function and the corresponding density function are given in (4) and (5):

$$ F(t) = 1 - p e^{-\lambda t} - (1 - p) e^{-\lambda_2 t}, \quad 0 \leq p \leq 1 $$

$$ f(t) = p \lambda e^{-\lambda t} + (1 - p) \lambda_2 e^{-\lambda_2 t} $$

Introducing (4) in (3) leads to a third degree equation with the particular solution $\sigma = 1$, and two solutions $\sigma > 1$ and $\sigma < 1$, of which only the latter one is feasible:

$$ \sigma = \frac{1}{2\mu} \left\{ \mu + \lambda_1 + \lambda_2 - \sqrt{\mu (\mu + \lambda_1 - \lambda_2)^2 - 4\mu \mu (\lambda_1 - \lambda_2)} \right\} $$

This expression is also found in reference [10].

In order to generalize equation (6) with respect to distribution type, three independent moments of $F(t)$ may be introduced. The details of this is found in [9], and only the resulting expression is given here:

$$ \sigma = \frac{1}{2\mu} \left\{ 1 + p \frac{q-r}{q+r} - \sqrt{(1 - \rho^2 \frac{q^2 + 3r q + 2r}{q + r^2})^2 + \rho^2 \frac{q^4}{(q+r)^2}} \right\} $$

By introducing (7) into (2) the mean waiting time in queue is found to be

$$ W = \frac{\rho}{\mu(1 - \rho)} \left\{ 1 + \frac{1}{\rho} \left[ \sqrt{(1 - \theta^2)^2 + (r - 1) - (1 - \theta)} \right] \right\} $$

where

$$ \theta = \frac{\rho(q - r) - (q - r^2)}{2p(r - 1)} $$

The $M/M/1$ queue is a good reference for comparison, and since the corresponding expression is

$$ W(M/M/1) = \frac{\rho}{\mu(1 - \rho)}, $$

we may introduce the relative mean waiting time $R = \frac{W(H_2/M/1)}{W(M/M/1)}$

$$ = 1 + \frac{1}{\rho} \left\{ \sqrt{(1 - \theta^2)^2 + (r - 1) - (1 - \theta)} \right\} $$

Equation (10) is a key expression for development of approximation My1 as well as My2. It may be noted right away that the possible range of $q$ is $r^2 < q < \infty$ for the $H_2$ distribution. This lower limit of $q$ does not necessarily apply in general for other distributions.
\[ q = r^2 \text{ corresponds to } \lambda_2 \to \infty \text{ and represents a batch Poisson distribution with geometric batch size.} \]

In the formula for any \( q \to \infty \) with limited and constant \( m_1 \) and \( m_2 \) is obtained when \( p \to 0, \lambda_1 \to 0 \) and \( p/\lambda_1^2 = k \) = constant. This represents a pure Poisson distribution with parameter \( \lambda_2 \), containing long gaps, those gaps however being so infrequent that the mean value is not influenced. In realistic terms a large \( q \) implies an interrupted Poisson process.

Two other limits of interest for (10) are:

\[ \rho = 1 \Rightarrow R = r \text{ independent of } q \]

\[ \rho = 0 \Rightarrow R = \frac{q_0 - 2r + r^2}{q - r^2} \Rightarrow R \to \infty \text{ for } q \to r^2 \]

\[ R \to 1 \text{ for } q \to \infty \]

### 3 The My1 Approximation

The My1 approximation presented in [9] is based on the observation that the limits \( q = r^2 \) and \( q \to \infty \) correspond to batch Poisson and pure Poisson arrivals, respectively. However, for those arrival processes the mean value solutions are given for general, independent service, i.e. \( MB/GI/1 \) and \( M/GI/1 \), where the latter in this context may be considered a special case of the former.

The \( MB/GI/1 \) solution has the compact general form

\[ W = \frac{\rho}{2\mu(1-\rho)} \left\{ \varepsilon_s + \frac{1}{\rho} \left( m_B - 1 + \frac{\sigma_B^2}{m_B} \right) \right\} \]

(11)

where \( m_B \) and \( \sigma_B^2 \) are mean and variance of the batch size.

In the \( H_2 \) arrival case we obtain

\[ m_B = \frac{1}{\rho} \text{ and } \sigma_B^2 = \frac{1 - \rho}{\rho^2} \]

to give

\[ W = \frac{\rho}{2\mu(1-\rho)} \left\{ \varepsilon_s + \frac{1}{\rho} (\varepsilon_s - 2) \right\} \]

(12)

The pure Poisson case can be interpreted either as the extreme case of \( p \to 1 \Rightarrow m_B = 1, \sigma_B^2 = 0 \), or as the case of \( q \to \infty \), while \( p \to 0, \lambda_1 \to 0 \) and \( p/\lambda_1 \to 0 \). The implication is \( \varepsilon_s = 2 \), to give the Pollaczek-Khintchine formula

\[ W = \frac{\rho \varepsilon_s}{2\mu(1-\rho)} \]

(13)

Thus we have exact mean value solutions for the extreme cases \( q = q_{\text{min}} \) and \( q \to \infty \) of the \( H_2 \)-distribution for the \( H_2/GI/1 \) queue. With the assumption that 3 moments is sufficient for the description of the arrival interval distribution from a practical point of view, we could apply the same formula for any \( GI/GI/1 \) queue with \( \sigma_s^2 \geq 1 \).

The remaining problem is the interpolation between \( q = q_{\text{min}} = q_0 \) and \( q \to \infty \). Still assuming only 3 moments being of importance, the adjustment has to be some function \( f = f(\rho, r, q, r_s) \). In the My1 approximation the adjustment function is chosen in a heuristic manner as a multiplication factor to the \( 1/\rho \)-term of (12), to give

\[ W \approx \frac{\rho}{2\mu(1-\rho)} \left\{ \varepsilon_s + \left( \frac{q_0}{q} \right)^{1/\rho-r} \cdot \frac{1}{\rho} (\varepsilon_s - 2) \right\} \]

(14)

This form gives the correct values for \( q = q_0, q \to \infty \) and \( \rho = 1 \). It agrees with the general tendencies for \( q_0 \leq q \leq \infty \) and \( 0 \leq \rho \leq 1 \), but cannot be expected to be very accurate in the complete interval over a broad range of distributions. The poorest agreement tends to be near \( \rho = 0 \), which is of least importance. Also near \( \rho = 1 \) the formula tends to overestimate \( W \) for high \( q \)-values. However, to obtain any substantial improvement, it is deemed necessary to introduce some more involved expression.

### 4 The My2 Approximation

The My1 approximation is a distinct improvement over all 2/2-approximations, and it matches well with the Kimura approximation (Kim). The Kim may be used in an explicit way by applying (7) for determination of \( \sigma \), thus being equally simple as My1 in use.

The shortcomings of My1 and Kim nevertheless ask for an improvement. The remedy proposed here is to take advantage of the full expression (8) instead of only the extreme value expressions (12) and (13), with a rather arbitrary interpolation. The assumption is that the general variation of \( W \) with \( \rho, r \) and \( q \) for any service distribution is similar to that of (8). It only has to be “elevated” to match any value of \( \varepsilon_s \) (or \( r_s = r_0 = \sigma_s^2 + 1 \)). This means that the \( H_2/M/1 \) solution is adapted to give an \( H_2/GI/1 \) solution, which again is used as a \( GI/GI/1 \) model.

The requirements put on an improved approximation should be to

1) allow any value of \( \frac{1}{\rho} \leq r_s \leq \infty \),

and give the exact solution for

2) the \( H_2/M/1 \) case, equation (10)

3) the \( M/GI/1 \) case: \( R = r_s \)

4) the \( M^{a(\text{Geo})}/GI/1 \) case: \( R = r_s + \frac{1}{\rho}(r_a - 1) \)

5) all \( GI/GI/1 \) queues when \( \rho = 1 \): \( R = r_s + r_a - 1 \)

* Assuming geometric batch size.

Condition 2) implies \( R = r_0 = \frac{2r_0 + 1}{q_0 - r_0^2} \) for the \( H_2/M/1 \) when \( \rho = 0 \), giving then \( q_0 \to r_0^2 \Rightarrow R \to \infty \) and \( q_0 \to \infty \Rightarrow R \to 1 \). One might want to put a condition similar to 5) for \( \rho = 0 \). That has not been found feasible and it is much less important than condition 5). (In the following, for simplicity, indexes may be omitted for arrivals, i.e. \( r = r_a \) and \( q = q_a \).

The variables that will go into a new approximation are \( \rho, r, q \) and \( r_s \). In the \( H_2/M/1 \) case \( r_s = 1 \), so formula (10) should be the resulting form in that case. By studying (9) and (10), as rewritten here, there are several options of re-
placing a constant 1 by the variable $r_s$, the most obvious ones being those indicated.

$$R = 1 + \frac{1}{\rho} \left\{ \sqrt{(1-\theta)^2 + (r-1) - (1-\theta)} \right\}$$

$$r_s \quad r_s \quad \uparrow \quad \uparrow \quad R$$

There is also the possibility of terms to have vanished due to degeneration when $r_s = 1$. It turns out that the conditions 4) and 5) require that a term $y = 2r_s - 1$ is introduced. This does not conflict with any of the other conditions. The resulting formula is:

$$R \approx r_s + \frac{1}{\rho} \left\{ \sqrt{(r_s - \theta)^2 + (2r_s - 1)(r-1) - (r_s - \theta)} \right\}$$  \hspace{1cm} (15)

By comparing corresponding simulations, it turns out that the calculated values of $R$ are consistently too high when $r_s > 1$ and too low when $\frac{1}{2} < r_s < 1$. Furthermore, there are dependencies of $\rho,q$ and $r$ that are rather difficult to track. It was chosen to study possible modifications to the term $(2r_s - 1)$. Various forms like $(2r_s - 1)^d,(2r_s - 1)d$ and $(2r_s - 1 + d)$ have been studied, where $d = d(\rho,r,q,r_s)$. An otherwise feasible adjustment by means of the exponential form gave unstable conditions near $r_s = \frac{3}{4}$, and similar problems arose with the product form, so the addend form was settled for. The final adjusted approximation is the one given in (16) and (17).

$$R \approx r_s + \frac{1}{\rho} \left\{ \sqrt{(r_s - \theta)^2 + (2r_s - 1 + d)(r-1) - (r_s - \theta)} \right\}$$  \hspace{1cm} (16)

$$d = \left( 1 + \frac{1}{r} \right) (1-r_s) \left( 1 - \left( \frac{\rho}{q} \right)^3 \right) (1-\rho^3)$$  \hspace{1cm} (17)

Each of the conditions $r_s = 1,q = q_0$ and $\rho = 1$ makes $d = 0$. The exponents of value 3 are heuristic adaptations to mutually conflicting requirements, and so is the multiplier term $(1 + \frac{1}{r})$.

This new proposal for an approximation can be compared with the selection of previously published proposals given in [9]. Before turning to numerical results, a tabular comparison is presented in Table 1 to show in which cases the various approximations give exact values.

5 Simulations

In ref [9] a set of coefficients of variation was selected to give a broad range of different cases. The same set has been used in the present work:

$c^2_a = 1.0,1.2,1.5,2.0,4.0,8.0,16.0$
$c^2_s = 0.0,0.2,0.5,1.0,2.0,4.0,16.0$

This combined with

8 $q$-values and
11 $\rho$-values $(0.0,0.1,\ldots,0.9,1.0)$

leads to

$$7 \cdot 7 \cdot 8 \cdot 11 = 4312 R\text{-values}$$

for each 3/2-approximation (Kim, My1 and My2).

(For 2/2-approximations 539 $R$-values.)

The simulated loads were $\rho = 0.1,0.2,\ldots,0.9$, and $q$-values were identical to those used in the calculation. Altogether approximately 450 mill calls have been simulated.

Since simulations, because of their stochastic nature, cannot be expected to give exact target values for the parameters, a correction is used to give more correct comparison with calculated target values. Thus a correction factor

$$h = \frac{\rho \cdot \mu'(1-\rho')}{\mu'(1-\rho)} = \frac{\lambda \cdot \mu^2 \cdot (1-\lambda'/\mu')}{\lambda' \cdot \mu^2 \cdot (1-\lambda/\mu)}$$  \hspace{1cm} (18)

is applied to the simulated waiting time values, where $\lambda,\mu$ are target values and $\lambda',\mu'$ are the values obtained from the simulations.

6 Evaluation of Obtained Results

The accuracy of the various approximations vary widely, and the question arises how to evaluate results in a fair manner.

It is clear that 3/2-approximations have an adaptation possibility above that of 2/2-approximations. Comparisons between Kim and My1 came out in favour of Kim for small values of $c_a$ and $c_s$, whereas the opposite is the case for large values. A specification of the relevant conditions must therefore be requested when an approximation is to be evaluated. On the other hand it is highly desirable to find some overall figure of merit. Obviously there can be no indisputable standard quality measure, except by ranking in case one approximation turns out to be best in all imaginable cases.

It seems reasonable to select a set of feasible values of each parameter, combine these in all possible ways, calculate the $R$-values for various loads and compare with simulations of each point. Since the number of calls affected is proportional to the load, the deviation of each point should be given a weight proportional to the load. A further weighting according to some load distribution, might be chosen. However, since this must be rather arbitrary, a uniform distribution is assumed.

Since deviation at each point may be positive or negative, as well real values as absolute values are applied in the evaluation.

The basic measures will be

$$a_1 = \frac{R - R_{\text{sim}}}{R_{\text{sim}}} \quad \text{(real value)} \quad (19)$$

and

$$a_2 = \frac{|R - R_{\text{sim}}|}{R_{\text{sim}}} \quad \text{(absolute value)} \quad (20)$$

The load weighted measures for each value of subset $\{r_s,q_a,r_s\}$ or $\{c^2_a,q_a,c^2_s\}$ will be
In a rank ordering evaluation the following statements can be made:

- In any rank ordering the My2 approximation comes out far superior to any other approximation.
- In the single point count of \( a_2 \)-values and the load weighted \( A_2 \)-values there are examples where other approximations come out closer to simulations than My2 does. (Remember, though, that simulations do not give exact values). However, in the vast majority My2 comes out better. Only 3 out of 392 entries of \( A_2 \) exceed 0.1 (10%).
- For all aggregated \( A_{2k} \)-values and hence for the \( A_{2t} \) My2 comes out best. The highest value \( A_{2g} = 0.065 \) occurs for \( c_2^2 = 16(r_0 = 8, 5) \) and \( c_2^2 = 0(r_0 = 8) \).
- The Kim approximation comes second in ranking ahead of My1. With increased sampling of high \( c_0 \), \( c_0 \)-values the two would change order. They are both well ahead of the others, of which KL definitely comes next.

Calculation of the aggregated deviations \( A_{1t} \) and \( A_{2t} \) yields the results shown in Table 2, where the approximations are rank ordered. Kingman (King) is omitted since it is not really an approximation, but rather an upper bound.

The following comments are made on Table 2:

- The \( A_{1t} \) entry of My2 is virtually zero, indicating symmetric deviations relative to simulations. This is what would be expected for an exact formula. The positive entries of \( A_{1t} \) for all other approximations indicate varying degrees of over-estimation on average.
- My2 is the only approximation that is fairly close to an average deviation of 2-3% from simulated results, which would be expected for an exact solution.

7 Conclusion

The new heuristic approximation for the GI/GI/1 queue presented here is demonstrated to be a radical improvement compared to earlier models for bursty arrivals. The model is not adapted to smooth arrivals, and other models, like that by Kramer/Langenbach-Belz or that by Kimura may be recommended for such cases. A key to the improvement is that of applying 3 moments instead of only 2 for the arrival interval distribution.

So far only mean values of queue length and waiting time have been studied. Since the tail of the waiting time or queue length distributions are of particular interest, a future study concentrated on this extended target might turn out to be of great value.
References


<table>
<thead>
<tr>
<th>Case Approx</th>
<th>M/M/1</th>
<th>M/GI/1</th>
<th>H2/M/1</th>
<th>GI/M/1</th>
<th>M²/GI/1</th>
<th>GI/GI/1</th>
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Table 1: Exactness indication. Approximation exact when marked with X.

1. Batch size geometric

2. Provided exact numerical solution of $\sigma$. No explicit solution.

<table>
<thead>
<tr>
<th>Approximation</th>
<th>Aggr. real values $A_{1\ell}$</th>
<th>Aggr. absolute values $A_{2\ell}$</th>
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Table 2: Relative deviation of aggregated load weighted values of waiting time.