TRAFFIC STUDIES OF TRANSMISSION BIT RATE CONVERSION IN ATM NETWORKS

Karl Rothermel
SIEMENS AG
Munich, Germany

Inside an ATM network, both the cell length and the transmission bit rate may vary, therefore the cell cycle (ratio of cell length to transmission bit rate) may also vary. Special cycle conversion devices will be located at those points of the ATM network where the cell cycle changes. In converting from a small cell cycle $T_1$ to a larger cell cycle $T_2$ it may be that, in a given time interval, more cells arrive than can be removed. Suitable buffers must be used in this case to avoid frequent cell loss. In this paper a mathematical method is presented to dimension these buffers in the case of a Bernoulli arrival process. Closed form expressions for the cell loss probabilities are given.

1. INTRODUCTION

The future broadband telecommunication network will be based on the asynchronous transfer mode (ATM). In an ATM network, the user information sent from a terminal is segmented and combined with a header into fixed length ATM cells. These ATM cells are transmitted to the receiving terminal via ATM trunks and ATM switching nodes. (For an overview of the basic principles of ATM see e.g. [1], [2].)

On each pipe in the ATM network the cell length $L$ is constant. Thus, on each pipe also the quotient

$$T = \frac{L}{v}$$

is constant, where $v$ is the transmission bit rate. This quotient $T$ is called the cell cycle length $T$ (or more briefly: cell cycle or merely: cycle).

However, considering the whole ATM network the ATM cell length as well as the transmission bit rate and with that the cell cycle length may vary. (See section 2 for some examples of varying cycle lengths.) At those points in the network at which the cycle changes special cycle conversion devices are located. There is no problem in converting from a cycle $T_1$ to a cycle $T_2$ with $T_1 > T_2$. In converting from a cycle $T_1$ to a cycle $T_2$ where $T_1 < T_2$ it may happen that more cells are supplied in a time interval than can be removed by using the cycle $T_2$. Suitable buffers must be used in this case to avoid frequent cell loss.

Since the waiting buffer is finite, an overflow of the buffer cannot be excluded. The buffer must, however, be dimensioned so that such an overflow can occur with only a very low probability, e.g. $10^{-10}$.

In this paper a mathematical method is presented to dimension these buffers in the case of a Bernoulli arrival process.

2. DIFFERENT CYCLE LENGTHS IN AN ATM NETWORK

In this section, some examples are given which illustrate that different cell cycle lengths might occur in an ATM network.

- Different transmission systems

In ATM networks, preferably transmission systems with a payload of 149.76 Mbit/s according to the SONET digital hierarchy are used. However, transmission systems with other bit rates are also employed (e.g. systems according to CCITT recommendation G.703 with a bit rate of 139.264 Mbit/s).

- Signalling and control cells

Various internal signalling and control messages must be interchanged by the units of the ATM network (e.g. between terminals and switching nodes). For this purpose, semi-permanent virtual connections are installed and the signalling and control messages are sent as ATM cells via these virtual connections through the ATM network.
The processing of these cells is done by microprocessors. The microprocessors are not able to accept the signalling and control cells at the high transmission bit rate of the ATM pipe. Therefore, a transmission bit rate conversion is necessary. This is a conversion from the high bit rate (small cycle length) of the ATM pipes to the low bit rate (large cycle length) of the pipes being directly connected to the microprocessors. Thus, the cycle conversion device has to contain a waiting buffer.

3. CALCULATION OF THE CELL LOSS PROBABILITY DUE TO BUFFER OVERFLOW IN THE CYCLE CONVERSION DEVICE

To calculate the probability of a buffer overflow in the cycle conversion device, we define a Markov chain and from the state transition diagram of this Markov chain we calculate the stationary buffer occupancy probabilities by solving a system of linear equations.

3.1 Assumptions

In the following, we consider a cycle conversion device having a buffer with s buffer places at which cells of fixed length arrive with a cycle $T_1$. The cells are removed from the buffer with cycle $T_2$, where $T_1 < T_2$ (see fig. 1).

$$ q = p(T-1-B) \frac{T_2}{T_1} - \frac{T_2}{T_1} \quad (1) $$

where B is the probability for cell losses due to buffer overflow. The $\approx$-character is valid for small values of B (e.g. $10^{-10}$).

In the following we calculate B for given values of s and p (or q).

For this purpose the stochastic process $N(t)$ is defined as the number of ATM cells waiting at a time t in the buffer.

For this definition, we make the following assumptions:

1) Let $T_{1,i}$ be the $i^{th}$ cycle on the input line with beginning time $t_{1,i}$. A cell arriving in this cycle is counted as a waiting cell immediately after the cycle starts, i.e. for all $t > t_{1,i}$.

2) Let $T_{2,j}$ be the $j^{th}$ cycle on the output line with beginning time $t_{2,j}$. A cell read out in this cycle is no longer counted as a waiting cell immediately after the start of this cycle, i.e. for all $t > t_{2,j}$.
Fig. 2 illustrates the process $N(t)$ by means of a random walk. Arrows pointing upwards symbolize arrivals, downwards pointing ones departures of cells. The number of waiting cells can be increased only at the start of a cycle on the input line. In contrast, the number of waiting cells can be reduced only at the beginning of a cycle at the output line. The relative phase of the clocks changes due to the different cycle lengths.

3.2 Setting up the state diagram

We consider a time-discrete Markov chain $(X_i)_{i=1,2,...}$ with state space $\{k | 0 \leq k \leq s\}$.

Let $X_i$ be defined as follows. Let $T_{1,i}$ be the $i^{th}$ clock on the input line with beginning time $t_{1,i}$. Then

$$X_i := \lim_{\delta \to 0^+} N(t_{1,i} + \delta) \quad (2).$$

$X_i$ is the number of waiting cells at an arbitrary small time after the beginning of the $i^{th}$ cycle.

Let us now consider 2 cycles $T_{1,i}$ and $T_{1,i+1}$ on the input line. A new cell arrives at cycle $T_{1,i}$ with a probability $p$. With probability $1-p$ no cells arrive during this cycle. If $X_i > 0$, then it depends on the relative phase of the cycles on the input and output lines whether a cell can be removed during the cycle $T_{1,i}$.

If a new output cycle starts during input cycle $T_{1,i}$ (Case (a) in Fig. 3), then a cell leaves the buffer during this input cycle. If during cycle $T_{1,i}$ no new cycle starts on the output line (case (b) in Fig. 3), then no ATM cell can be removed.

If we consider an arbitrary cycle on the input line, then case (b) occurs with probability

$$f = \frac{T_2 - T_1}{T_2} \quad (3)$$

and case (a) with probability

$$1 - f = \frac{T_1}{T_2} \quad (4).$$

If we know the phase shift in a cycle in a real system, then in all following cycles it is deterministically defined whether case (a) or (b) occurs.

In our calculation, however, we assume as an approximation that (a) or (b) occurs by random in every cycle. Case (b) occurs with probability $f$ and case (a) with probability $1-f$.

Case (b) has the most unfavorable effects on buffer occupancy. In the real system, case (b) occurs in nearly fixed distances. In our calculation, by random case (b) can occur in short succession. Thus, in our calculation a conservative approximation is used.
Fig. 4 shows the state transition probabilities belonging to the Markov chain:

\[
\begin{align*}
1-p & \quad \text{(1-p)(1-f)} \\
p & \quad \text{(1-p)(1-f)} \\
\text{pf} & \quad \text{(1-p)(1-f)} \\
\text{pf} & \quad \text{pf} \quad \text{...}
\end{align*}
\]

An example will illustrate how these transition probabilities are obtained: a transition from state \(k\) (\(k > 0\)) in cycle \(j\) to state \(k\) in cycle \(k+1\) will occur, if either

- a cell arrives in cycle \(j+1\) and case (a) has occurred during cycle \(j\),

or

- no cell arrives in cycle \(j+1\) and case (b) has occurred during cycle \(j\).

The probability for the first event is \(p(1-f)\), that for the second event is \((1-p)\). Therefore, the probability for the transition from state \(k\) to state \(k\) is \(p(1-f) + f(1-p)\).

### 3.3 Calculation of the stationary state probabilities and of the cell loss probabilities

Since the Markov chain \((X_j)_{j=1,2,...}\) from Section 3.2 has a finite state space, it can easily be seen that this is ergodic and irreducible. Consequently there exist (see [3]) stationary state probabilities \(Q_k\) for the occupancy of \(k\) buffer places and the \(Q_k\) are solutions of the following system of equations, which is obtained by the state diagram of fig. 4:

\[
\begin{align*}
\theta_0 &= (1-p)\theta_0 + c\theta_1 \\
\theta_1 &= p\theta_0 + b\theta_1 + c\theta_2 \\
\vdots &= \vdots \\
\theta_k &= a\theta_{k-1} + b\theta_k + c\theta_{k+1} \\
\theta_{k+1} &= (1-p)\theta_1 + f\theta_2
\end{align*}
\]

for \(1 < k < s\), where \(a:=pf\), \(b:=p(1-f) + f(1-p)\), \(c:=(1-p)(1-f)\).

From this system of equations, \(Q_1, \ldots, Q_k\) can be calculated recursively:

\[
\begin{align*}
\theta_0 &= \frac{1}{1 + \frac{p(\theta_0/c)^{s-1} - 1}{c(\theta_0/c)^{s-1} - 1}} \\
\theta_k &= \frac{\theta_{k-1}}{1 + \frac{p\theta_{k-1} + \theta_{k-2}}{c\theta_{k-1} + \theta_{k-2}}}
\end{align*}
\]

By use of the equation \(a + b + c = 1\) and (6) it can be shown by induction, that:

\[
\begin{align*}
Q_k &= \frac{\theta_{k-1}}{1 + \frac{p\theta_{k-1} + \theta_{k-2}}{c\theta_{k-1} + \theta_{k-2}}} \\
\theta_0 + \ldots + \theta_s &= 1
\end{align*}
\]

we get:

\[
\begin{align*}
\theta_0 &= \frac{1}{1 + \frac{p(\theta_0/c)^{s-1} - 1}{c(\theta_0/c)^{s-1} - 1}} \\
\theta_k &= \frac{\theta_{k-1}}{1 + \frac{p\theta_{k-1} + \theta_{k-2}}{c\theta_{k-1} + \theta_{k-2}}}
\end{align*}
\]

Equation (1) specifies the load \(q\) on the departure line. The probability that a cycle on the departure line is empty, is thus

\[
1-q = 1 - \frac{pt_2 + pT_2}{T_1 + T_1}
\]

On the other hand, in a cycle \(T_2\) with beginning time \(t_2\) on the output line, no cell is transmitted exactly when the buffer is empty in the cycle running at a time \(t_2\) on the input line. The probability for this is \(Q_0\).

It follows that:

\[
\begin{align*}
Q_0 &= 1 - \frac{pt_2 + pT_2}{T_1 + T_1} \\
B &= \frac{Q_0 - 1 + \frac{pt_2 + pT_2}{T_1 + T_1}}{pT_2/T_1}
\end{align*}
\]

4. NUMERICAL EVALUATION

4.1 Buffer dimensioning

Equations (8) and (9) show that the cell loss probability \(B\) is a function of the load on the inlet pipe \(p\), the buffer size \(S\) and the ratio of the cycle lengths \(T_1/T_2\). Thus, for given \(p\) and \(T_2/T_1\) these equations can be used to calculate the buffer size which is necessary for a cell loss probability less than a certain threshold, e.g. \(10^{-10}\).
In figure 5, the necessary buffer size is plotted versus $T_2/T_1$ for a loss probability of $10^{-10}$ for different values of the load $q$ on the outlet line. (Recall that $p$ and $q$ are connected via equation (1).)

![Figure 5](image)

It can be seen that the required buffer size initially increases relatively quickly, then changes to an almost linear rise. The slope of this linear section depends on $q$.

4.2 Example: Conversion from SONET pipes to G.703 pipes

Fig. 5 can be used for dimensioning buffers in cycle conversion devices. For example, we consider the case of a cycle conversion device which converts from a SONET pipe to a G.703 pipe. We assume a cell length of 53 bytes (incl. header) on both pipes.(see [4]). This is the cell length which will be recommended by CCITT on the pipes between ATM nodes. The cycle length $T_1$ on the SONET pipe (transmission bit rate: 149.76 Mbit/s) is $2.831 \mu s$; the length $T_2$ on the G.703 pipe (transmission bit rate: 139.264 Mbit/s) is $3.045 \mu s$. The ratio $T_2/T_1$ is $1.076$. It can be seen from figure 5 that the buffer of the cycle conversion device has to contain 18 buffer places if a load $q=0.85$ is assumed.

5. CONCLUSION

The conversion of different transmission bit rates and cell lengths and thus different cell cycles is a problem which often occurs in ATM networks. In the transition from a short cycle to a longer cycle, a buffer is required. An analytical method has been presented to dimension this buffer in the case of a Bernoulli arrival process. Numerical results were also presented.

6. REFERENCES


