DEMAND FORECASTING MODELS FOR MARKETS WITH COMPETITION

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This paper introduces a competitive Bass model efficient for competitive markets. This model takes into account innovative effect, imitative effect and competitive effect. It is extended in three ways, relating to market extension. The estimation procedures for regression coefficients are shown, based on linear least squares systems. In one model alternative least squares procedures are used. In addition, three models are compared in a market where three telecommunication companies compete. The third model, which takes the increase of potential adopters into account, is the most vital among the three.

1. INTRODUCTION

A monopoly over telecommunication services in Japan was held by the Nippon Telegraph and Telephone Corporation until 1985. Thereafter, competition in some areas of telecommunication services began emerging. That competition has led to an interest at NTT in forecasting future demand under such competitive circumstances.

There is a well-known model among biologists for analyzing the competition between two species. This model is a natural extension of the logistic model. Among the models which represent the innovation diffusion process, the Bass model is as well known as the logistic model. The Bass model has been discussed and extended by many authors because it is a very simple model which explains the diffusion process qualitatively. In this paper, an extension of the Bass model which simply takes competition into account is proposed. Moreover, three models are compared in a market where three telecommunication companies compete. The third model, which takes the increase of potential adopters into account, is the most vital among the three.

Bass [2] proposed a purchasing model in which the probability, \( P(T) \), that an initial purchase will be made at time \( T \) given that no purchase has yet been made is a linear function of the number of previous buyers, \( Y(T) \), that is,

\[
P(T) = a + (b/m)Y(T),
\]

where \( a \) and \( b/m \) are constants, \( a \) reflects the fraction of all adopters who are innovators, and the product \( b/m \) times \( Y(T) \) reflects the pressures operating on imitators as the number of buyers increases. Innovators decide whether to adopt an innovation independently of the decisions of other individuals and imitators are influenced in the timing of adoption by the decisions of others. Using the likelihood, \( f(T) \), for purchase at time \( T \),

\[
P(T) = f(T) / [1 - F(T)],
\]

\[
Y(T) = m \int_0^T f(t) \, dt = mF(T),
\]

where

\[
F(T) = \int_0^T f(t) \, dt.
\]

From Eqs. (2.4), (2.5) and (2.6),

\[
dY(t)/dt = (m - Y(t)) \{ a + (b/m) Y(t) \}.
\]

Various extensions of this model have been proposed. The parameter \( m \), which is the total amount of purchasing during the period considered in the Bass model, is a function of time \( t \) in [13] and a function of price in [8] and [11]. The second factor in Eq. (2.8) is a function of time and/or quantity relating to information about goods, such as advertising costs in [4],[10],[11] and [15]. Papers [14] and [5] introduce price, \( p(t) \), multiplicatively, that is,

\[
dY(t)/dt = \{ \exp\{ -k\cdot p(t) \} \} \{ m - Y(t) \} (a + (b/m)Y(t)).
\]

In these models there is no competition. In [6] and [7], competition between two companies is considered and the cumulative sales, \( X_i \), of company \( i \) is given by

\[
dX_i/dt = (m - X_1 - X_2) \times \{ a_i (1 - k) p_i + \gamma (p_j - p_i) \} \]

where \( p_i \) = price of company \( i \),

\( k = \) industry price-sensitive parameter,
3. MODELS WHICH CAN TREAT MARKET EXTENSION

This section proposes three models which take into account the extension of a market. For example, such telecommunication services as automobile telephone and radio paging services has been served only around metropolitan at first and service areas has gradually expanded and the ISDN services called INS-Net64 and INS-Net1500 in Japan increase its attraction by addition of packet services. Both expansion of service areas and increase of service attraction are considered as cause of market extension. Based on these cause, three models are proposed.

(1) A model in which market extensions have instantaneous effects on the innovative effect, $a_u$ in Eqs. (2.11) and (2.12) or (2.14):

$$du/dt = (m-S_t)(a_u + \sum_h d_{uh}\delta_{uh}(h) + b_{uu}u_t - \sum_{v\neq u} b_{uv}v_t)$$

(3.1)

$$S_t = \sum_v v_t$$

(3.2)

where $u_t$ and $v_t$ are the number of subscribers of company $U$ and $V$ at time $t$, $S_t$ is the total number of subscribers, and $\delta_{uh}(h)$ is a dummy variable that takes a value, one, only at the $h$-th time point when company $U$ begins to collect subscribers in a new area or when it adds new attractive functions for existing services. These special time points are called as extension time points.

(2) A model in which market extensions have successive effects on innovation effect, where $\xi_{uh}(h)$ instead of $\delta_{uh}(h)$ is used as a dummy variable which takes a value, one, ever since the $h$-th time point.

(3) A model in which the number of potential adopters varies according to the market area extension, that is, $m_{uh}$, $S_{uh}$ and $v_{ut}$ are used instead of $m$, $S_t$ and $v_t$, where

$$m_{uh} = m_0 + \sum_h m_h Du(h); \ S_{uh} = \sum_v v_{uh}$$

(3.3)

If company $U$ begins to collect subscribers at the $h$-th area extension time $t_h$ in area $A$ and the other companies has begun the service at time $t_1$ (<$t_h$) in area $A$, $D_{uh}(h)$ and $D_{uh}(i)$ are dummy variables which takes a value, one, ever since $t_h$. Also when company $U$ serves at areas where the other companies begin to collect subscribers at time $t_i$ (> $t_h$), $D_{uh}(i)$ takes a value, one, ever since $t_i$. $v_{uh}$ is the number of subscribers of company $V$ in areas where company $U$ serves. If company $X, Y$ and $Z$ begin services at time $t_1$, $t_2$ and $t_3$ respectively in the same area, $D_{uh}(h)$ is given in Fig. 1.

4. ESTIMATION PROCEDURES OF REGRESSION COEFFICIENTS

The estimation procedures of regression coefficients are shown based on the linear least squares systems[12].

In model (1), let $(m-S_t), (m-S_t)d_{uh}(h)$, and $(m-S_t)u_t$ be explanatory variables and $(v_{uh} - u_{i-1} - u_t)$ be criterion variables. Regression coefficients $a_u$, $d_{uh}$ and $b_{uv}$ are estimated by the ordinary least squares (OLS) and nonlinear least squares (NLS), procedures where all regression coefficients are nonnegative[12]. Convergence
of NNLS is also shown in [12]. Note that the regression coefficients \( d_{uh} \) function as coordinators of estimation errors at time points when \( \delta_{uu}(h) = 1 \), and do not functions in forecasting. Of course, if \( d_{uh} \) is allowed to be constant regardless of \( h \), \( d_{uh} \) can also function in forecasting. However, in this paper different values of \( d_{uh} \) are estimated to compare the accuracy of models.

In model (2), regression coefficients can be estimated by the same method as in model (1).

In model (3), only three competing companies, X, Y, and Z are considered in order to simplify the explanation. Here, the following formulas are used.

\[
\begin{align*}
    w_{ut} &= f_{ut}(m) g_u(c_u), \\
    f_{ut}(m) &= m_0 + \sum_h m_h D_{uh}^{(k)} - S_{ut}, \\
    g_u(c_u) &= a_u + \sum_h d_{uh} \delta_{uh}(h) + b_{uv} u_t - \sum_{v \neq u} b_{uv} v_t, \\
    m &= (m_0, m_1, \ldots, m_K)^T; \\
    K &= \text{the number of area extention times},
\end{align*}
\]

Regression coefficients are estimated by the alternative least squares (ALS) procedures using sums of squared errors, \( e_{1}^{(i)} \) and \( e_{2}^{(i)} \).

\[
\begin{align*}
    e_{1}^{(i)} &= \sum_u \left[ u_{tt+1} - \left( u_{tt} + f_{ut}(m^{(i)}) g_u(c_u^{(1-i)}) \right) \right]^2, \\
    e_{2}^{(i)} &= \sum_u \left[ u_{tt+1} - \left( u_{tt} + f_{ut}(m^{(i)}) g_u(c_u^{(i)}) \right) \right]^2,
\end{align*}
\]

where the following procedure converges because

\( 0 \leq e_{2}^{(i+1)} \leq e_{1}^{(i+1)} \leq e_{2}^{(i)} \leq e_{1}^{(i)} \).

\[
\begin{align*}
    D_{x1}^{(1)} &= 1, \\
    D_{x1}^{(2)} &= 0, \\
    D_{x1}^{(3)} &= 0, \\
    D_{x2}^{(1)} &= 0, \\
    D_{x2}^{(2)} &= 1, \\
    D_{x2}^{(3)} &= 0, \\
    D_{x3}^{(1)} &= 0, \\
    D_{x3}^{(2)} &= 1, \\
    D_{x3}^{(3)} &= 0,
\end{align*}
\]

Fig. 1 Values of \( D_{xu}^{(i)} \)

\[\text{ALS procedures}\]

(i) Use estimates in model (1) as initial values \( m^{(0)} \) and \( c_u^{(0)} \) of \( m \) and \( c_u \), where

\[
\begin{align*}
    m_0 &= m_1 = m_2 = \ldots = m_K = 0 \quad \text{and} \quad e_{1}^{(0)} = e_{2}^{(0)} = \infty.
\end{align*}
\]

(ii) Calculate \( h_{ut}^{(i)} \) using \( c_u^{(i)} \), where

\[
\begin{align*}
    h_{ut}^{(i)} &= u_{tt+1} - u_{tt} + g_u(c_u^{(i)}) S_{ut}, \\
    h_u^{(i)} &= (h_{u1}^{(i)}, h_{u2}^{(i)}, \ldots, h_{ue}^{(i)})^T; \quad E \text{ is a time point before the present time.}
\end{align*}
\]

Let \( A_u \) be exploratory variables and \( h_u^{(i)} \) be criterion variables.

We can obtain \( m^{(i+1)} \) by NNLS procedures, where

\[
A_u = (g, 1, x_1, y_1, z_1, \ldots, x_E, y_E, z_E)^T;
\]

\[
D_{x1} = (x_1, x_2, \ldots, x_E)^T, \quad D_{x2} = (y_1, y_2, \ldots, y_E)^T, \quad D_{x3} = (z_1, z_2, \ldots, z_E)^T.
\]

\[
\begin{align*}
    \text{Table 1. Total Squared Errors in model (1) [x10^{-6}]} \\
    m & \quad e_x & \quad e_y & \quad e_z & \quad e_T \\
    \hline
    600 & 26.41 & 37.36 & 9.26 & 73.03 \\
    700 & 25.62 & 36.58 & 9.25 & 71.45 \\
    800 & 25.34 & 36.20 & 9.25 & 70.79 \\
    900 & 25.25 & 36.12 & 9.25 & 70.63 \\
    1000 & 25.22 & 36.15 & 9.26 & 70.63 \\
    1100 & 25.19 & 36.19 & 9.26 & 70.64 \\
    1200 & 25.17 & 36.22 & 9.26 & 70.66 \\
    \hline
    600 & 24.54 & 32.24 & 8.70 & 65.48 \\
    700 & 24.67 & 32.50 & 8.88 & 66.01 \\
    800 & 24.75 & 32.64 & 8.91 & 66.30 \\
    900 & 24.81 & 32.72 & 8.96 & 66.49 \\
    1000 & 24.84 & 32.77 & 8.99 & 66.61 \\
    1100 & 24.87 & 32.80 & 9.02 & 66.69 \\
    1200 & 24.89 & 32.83 & 9.04 & 66.75 \\
    \end{align*}
\]

\[
\begin{align*}
    \text{Table 2. Regression Coefficients in model (1) [x10^{-4}]} \\
    & \quad X & \quad Y & \quad Z \\
    a_u & 0 & 34.9 & 0 \quad 0 & 33.3 \\
    d_{u1} & 107 & 44.3 & -92.7 & 38.3 & - \\
    d_{u2} & 100 & 94.0 & -82.1 & 74.5 & - \\
    b_{ux} & 1.48 & .079 & .924 & 1.42 & 0 \quad .959 \\
    b_{uy} & 0 & 1.51 & 0 & 0 \quad 1.18 & 0 \\
    b_{uz} & .545 & 0 & 1.99 & .729 & 0 \quad 1.85 \\
    \hline
    a_u & 12.6 & -72.6 & 51.0 & 13.9 & -56.4 \quad 45.6 \\
    d_{u1} & 111 & 60.5 & -93.1 & 49.7 & - \\
    d_{u2} & 95.3 & 80.4 & -78.4 & 65.1 & - \\
    b_{ux} & -.671 & 7.78 & -3.04 & -.551 & 6.41 \quad -2.57 \\
    b_{uy} & 1.97 & -1.82 & 1.41 & 1.43 & -1.76 \quad 1.07 \\
    b_{uz} & -.032 & -6.02 & 3.08 & .146 & -4.70 \quad 2.67 \\
    \end{align*}
\]
Table 3. Total Squared Errors in model (2) \([\times 10^{-6}]\)

<table>
<thead>
<tr>
<th>m</th>
<th>(e_x)</th>
<th>(e_y)</th>
<th>(e_z)</th>
<th>(e_T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>49.03</td>
<td>33.20</td>
<td>9.25</td>
<td>91.48</td>
</tr>
<tr>
<td>800</td>
<td>49.37</td>
<td>33.32</td>
<td>9.26</td>
<td>91.94</td>
</tr>
<tr>
<td>1000</td>
<td>49.70</td>
<td>33.42</td>
<td>9.28</td>
<td>92.38</td>
</tr>
</tbody>
</table>

Table 4. Regression Coefficients, \(c_u\) and \(u+20\) in model (3)

<table>
<thead>
<tr>
<th>(X)</th>
<th>(Y)</th>
<th>(Z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_u)</td>
<td>(56.2)</td>
<td>(46.4)</td>
</tr>
<tr>
<td>(b_{ux})</td>
<td>(0.223)</td>
<td>(0)</td>
</tr>
<tr>
<td>(b_{uy})</td>
<td>(0)</td>
<td>(0.508)</td>
</tr>
<tr>
<td>(b_{uz})</td>
<td>(0)</td>
<td>(2.75)</td>
</tr>
<tr>
<td>(u+20)</td>
<td>(225.6)</td>
<td>(262.4)</td>
</tr>
<tr>
<td>(p_{u20})</td>
<td>(35.5)</td>
<td>(41.3)</td>
</tr>
</tbody>
</table>

Table 5. Regression Coefficients, \(c_u\), \(P_{u20}\) and \(u+20\) in model (3)

<table>
<thead>
<tr>
<th>(X)</th>
<th>(Y)</th>
<th>(Z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_u)</td>
<td>(65.8)</td>
<td>(50.6)</td>
</tr>
<tr>
<td>(b_{ux})</td>
<td>(0.708)</td>
<td>(0)</td>
</tr>
<tr>
<td>(b_{uy})</td>
<td>(0)</td>
<td>(1.24)</td>
</tr>
<tr>
<td>(b_{uz})</td>
<td>(0)</td>
<td>(2.51)</td>
</tr>
<tr>
<td>(u+20)</td>
<td>(222.7)</td>
<td>(257.8)</td>
</tr>
<tr>
<td>(p_{u20})</td>
<td>(36.7)</td>
<td>(42.5)</td>
</tr>
</tbody>
</table>

Table 6. Regression Coefficients, \(m\), in model (3)

<table>
<thead>
<tr>
<th>(m)</th>
<th>(m_0)</th>
<th>(m_1)</th>
<th>(m_2)</th>
<th>(m_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>378</td>
<td>205</td>
<td>0</td>
<td>56.2</td>
</tr>
<tr>
<td>560</td>
<td>391</td>
<td>218</td>
<td>0</td>
<td>58.5</td>
</tr>
<tr>
<td>610</td>
<td>415</td>
<td>244</td>
<td>0.2</td>
<td>63.9</td>
</tr>
<tr>
<td>800</td>
<td>501</td>
<td>330</td>
<td>35.2</td>
<td>52.0</td>
</tr>
</tbody>
</table>

5. APPLICATION EXAMPLE

Using these formula, obtain \(c_u(i+1)\) from the NNLS problem

\[ B_u(i+1) c_u(i+1) = w_u. \]

If \(e_u(i) \leq e_u(i+1)\), adopt \(m(i+1)\) and \(c_u(i)\) as regression coefficients; otherwise, go back to (ii).

If \(e_u(i+1) \leq e_u(i)\), let \(m(i+1)\) and go back to (ii).

In a market where three telecommunication companies compete, the accuracy of three models are compared. Table 1 shows total squared errors \(e_u\) in model (1), where

\[ e_u = \sum_{i} \left[ (u_{i+1} - (u_i + (m_i - S_i)) g_1(c_u)) \right]^2. \]  
(5.1)
Therefore, model (1) is better than model (2).

(ii) Market shares, \( p_{20} \), are stable. Therefore, model (3) is a useful model for estimating market shares.

(iii) Regression coefficients, \( b_{x0} \), which present the competitive effects are zero. This means that competitive effects are negligible when compared with the market extension effects.

6. CONCLUSION

As the number of subscribers grows, traffic demand increases. Therefore, forecasting the number of subscribers is very important for telecommunication companies who compete for these subscribers. For the case where three telecommunication companies compete, it was shown that model (3), in which the number of potential adopters varies according to market extension, is the most important among the three. We can create another model, for example, one in which \( h = \) constant regardless of \( h \). Various models which are extensions of our models should be compared in accordance with the market conditions.

Knowing the exact numbers of subscribers in rival companies is usually difficult and we can only approximate them through sampling surveys. Regression coefficients in the proposed models were estimated by nonnegative least squares procedures in single precision arithmetic. In single precision arithmetic, some local minimum estimators may be obtained. However, we should not use double precision arithmetic considering the accuracy of data, but should compare some estimators using various a priori knowledge.

REFERENCES


