OPTIMAL ROUTING IN VIRTUAL CIRCUIT COMMUNICATION NETWORKS

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The problem of optimal routing in virtual circuit (VC) communication networks such as the X.25 packet-switched networks, the frame-relay networks, or the ATM networks, is considered. To describe the related characteristics of the generally asymmetrical transmit and receive traffic in such networks, we propose a related multicommodity (RMC) flow model with two traffic matrices. For solving the optimal flow assignment problem with this model, we present an algorithm which incorporates the transmit-receive (TR) relation, an important practical constraint existing in most of the operational VC networks. We show that the optimal routing scheme resulting from the new model is now dependent on the TR ratio of the individual related commodities. For some special cases, the optimal routing is purely distination-oriented or even symmetrical. We also show the performance degradation caused by using the traditional multicommodity (MC) flow model and its dependency on the network topology.

I Introduction

Packet switching is emerging as the most efficient and economical way to move bursty information among geographically dispersed locations. It provides improved reliability and better performance than other forms of data transmission, and it reduces costs. As a result of these important advantages, both the number and the range of packet switching based communication systems have significantly increased [1].

In the process of designing a packet-switched network, routing or flow assignment forms an important step. The search for an optimal network topology, for example, may require huge amounts of optimal route computations [2][4].

The problem has drawn the attention of a large number of researchers, and various methods have been developed for its efficient solution [1]-[3]. As the first packet-switched network, the ARPANET adopted the datagram (DG) technique, most of the published methods were based on a framework that did not take into account further routing constraints in virtual circuit (VC) networks.

In a DG network, packets are routed independently of each other. The traffic from one node to another, say from \( n_1 \) to \( n_2 \), is not related to the traffic in the opposite direction of the same node pair, i.e. from \( n_2 \) to \( n_1 \). Consequently, only one traffic matrix is needed for describing the entire external traffic.

In a VC network, however, routing is carried out for each VC. After a VC has been established, all the packets (both data and control sequences) sent between two session endpoints can only be transmitted on a fixed path, the VC, for the entire duration of the session. Though the VC network has become a standard for data communication, to our knowledge no work has been published so far concerning the effect of this special characteristic on the optimal routing problem.

Recently, the concept of the VC has been extended beyond the scope of the traditional packet switching. The CCITT has proposed the provision of frame-relaying in ISDN [5], and accepted ATM (asynchronous transfer mode) as the final transfer mode for B-ISDN [6]. Compared with the X.25 packet switching which provides VC service at layer 3 of the OSI model, the frame-relaying offers VC multiplexing at layer 2 by using the LAPD protocol. In ATM, the information to be transferred is first divided into small, fixed size blocks called cells. These cells are then transmitted over a VC at layer 1 of the OSI model in a slotted operation with respect to the instantaneous need for information transfer. Though the VC service is not offered at the same layer within the protocol architecture of the three kinds of VC networks, from a network flow point of view, no essential distinctions have to be made among them. Therefore, though we shall use the terminology of packet switching throughout this paper, the model and method developed here can be easily extended for the design of the frame-relay and the ATM networks.

We shall focus on the problem of minimizing the average packet transfer delay in a VC network through optimal flow assignment. Briefly, our model is based on the following fact: for each node pair \( n_1 \) and \( n_2 \), call requests in one direction, say from \( n_1 \) to \( n_2 \), can be routed independently from the call requests in the opposite direction, but the packet flows of any VC in both directions are always related, and they are routed simultaneously. More precisely, let \( r_{1,2}^{(o,d)} \) denote the total transmit packet rate belonging to VCs between the origin-destination (OD) node pair \( o \) and \( d \), initiated by subscribers of node \( o \), and \( r_{2,1}^{(o,d)} \) denote the total receive packet rate belonging to the same VCs (Fig. 1). We shall refer to the requirement pair \( (r_{1,2}^{(o,d)}, r_{2,1}^{(o,d)}) \) as related-commodity\((o,d)\), or simply \( r_{cm}(o,d) \). The routing algorithm must always assign the transmit and the receive flows jointly. As a result of this constraint, a new kind of network flow problem arises.
To distinguish the traditional multicommodity (MC) flow problem from the problem considered here, we shall refer to the latter as a related multicommodity (RMC) flow problem.

Figure 1: Example of the RMC flow in a VC network

The algorithm we shall present for solution of the optimal RMC flow assignment problem is derived from the Frank-Wolfe’s algorithm [8] which has been applied to solving MC flow problems successfully.

In Section II we shall formulate the RMC flow problem mathematically. Methods for solving the problem will be described in Section III. In Section IV we give some numerical results and in Section V the conclusion.

II Model and Problem Formulation

We shall model the topology of the backbone network using a graph $G = (V, A)$ where $V = \{v_1, v_2, \cdots, v_N\}$ is the set of vertices (nodes) and $A = \{a_1, a_2, \cdots, a_M\}$ is the set of arcs (edges). Each switch node is represented by one vertex, and each trunk is represented by one arc (for each direction of transmission). If arc $a_m$ is incident out of $v_i$ and into $v_j$, it is denoted by $a_m(i, j)$ or simply $a(i, j)$. Each arc $a_m$ has a capacity $C_m$ (bits/second). Similarly, each vertex $v_n$ has a capacity $K_n$ (bits/second) equal to the switching capacity of the switch node represented by $v_n$. Let $C = (C_1, C_2, \cdots, C_M, K_1, K_2, \cdots, K_N)$.

As a brief description, the trunks are modeled as $M/M/1$ systems for calculating their queuing and transmission delay [1],[2]. Other trunk models are available: For networks with more than one line per trunk, $m - M/M/1$ or $M/M/m$ model is better suited; if the transmission speeds of parallel lines within the trunk are different, the flow distribution among them can be optimized [3],[4],[7]. To take node queuing and processing delay into account, we model the switch as a single $M/M/1$ queue, though more complex models could also be used. The reason for such choice is that we want to concentrate better on the new RCM flow problem. The relationship between the average delay of a packet traveling from origin to destination (the average is over time and over all pairs of nodes) and the average flows on the arcs and through the nodes is

$$T = \frac{1}{\gamma} \left\{ \sum_{m=1}^{M} \frac{f_m}{C_m - f_m} + \sum_{n=1}^{N} \frac{\varphi_n}{K_n - \varphi_n} \right\}$$

where

- $T$ total average delay per packet [seconds/packet].
- $\gamma = \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{\varphi_n}{K_n} + \sum_{n=1}^{N} \frac{\varphi_n}{K_n}$ total packet arrival rate from external sources [packets/second].
- $f_m$ total bit rate on arc $a_m$ [bits/second].
- $C_m$ capacity of arc $a_m$ [bits/second].
- $\varphi_n$ total bit rate through node $v_n$ [bits/second].
- $K_n$ capacity of node $v_n$ [bits/second].

The optimal routing problem for a VC communication network is now formulated as follows:

**Given:** Topology of a network $G(V, A)$, capacities of its arcs and nodes $C$, and offered traffic $R_t = (r_{t(o,d)})$ and $R_r = (r_{r(o,d)})$, called the transmit and receive traffic matrix respectively.

**Objective:** Minimize average packet transfer delay $T(f)$ as defined by Eq.(1) over $f$, where $f = (f_1, f_2, \cdots, f_M, \varphi_1, \varphi_2, \cdots, \varphi_N)$.

**Constraints:** a) $f$ is an RMC flow satisfying both traffic matrices $R_t$ and $R_r$. b) $f \leq C$.

We shall next investigate the properties of the set of feasible flows $F = F_a \cap F_b$, where $F_a = \{f|f$ satisfies related multicommodity constraint a$\}$, and $F_b = \{f|f$ satisfies capacity constraint b$\}$.

An RMC flow $f$ must satisfy the conservation equations, the nonnegativity and the transmit-receive (TR) relation constraints. Let

$$f_{t(o,d)}(i,j) + f_{r(o,d)}(i,j)$$

be the flow on arc $a(i,j), \forall a(i,j) \in A$ and

$$\varphi_{t(o,d)}(n) = (\varphi_{t(o,d)}(n) + \varphi_{r(o,d)}(n))$$

be the flow through node $v_n, \forall v_n \in V$, both due to the $rcm(o,d)$. For the conservation of flow we have

$$\sum_{j=1}^{N} f_{t(o,d)}(j,i) - \varphi_{t(o,d)}(i) = \begin{cases} -r_{t(o,d)}/\mu, & \text{if } i = o \\ 0, & \text{otherwise,} \end{cases}$$

for the transmit traffic, and

$$\sum_{j=1}^{N} f_{r(o,d)}(j,i) - \varphi_{r(o,d)}(i) = \begin{cases} -r_{r(o,d)}/\mu, & \text{if } i = d \\ 0, & \text{otherwise,} \end{cases}$$

for the receive traffic.
for the receive traffic. For nonnegativity of the flow we have
\[ f_{i,r}^{(o,d)}(i,j) \geq 0, \quad \forall a(i,j) \in A \] (8)
and
\[ \varphi_{i,r}^{(o,d)}(n) \geq 0, \quad \forall v_n \in V. \] (9)

For the TR relation of the flow we have
\[ f_{i,r}^{(o,d)}(j,i) = f_{i,r}^{(o,d)}(i,j)\varphi_{i,r}^{(o,d)}, \] \[ \forall a(i,j), a(j,i) \in A \] (10)
and
\[ \varphi_{i,r}^{(o,d)}(n)\varphi_{i,r}^{(o,d)}(n) = \varphi_{i,r}^{(o,d)}(n)\varphi_{i,r}^{(o,d)}, \] (11)
\[ \forall v_n \in V. \]

Let
\[ f^{(o,d)} = \left( f_1^{(o,d)}, f_2^{(o,d)}, \ldots, f_M^{(o,d)}, \varphi_1^{(o,d)}, \varphi_2^{(o,d)}, \ldots, \varphi_N^{(o,d)} \right) \]
where \( f_m^{(o,d)} \) and \( \varphi_q^{(o,d)} \) are flows on arc \( a_m \) and node \( v_n \), due to \( rcm(o,d) \), and let
\[ F^{(o,d)} = \{ f^{(o,d)} | f^{(o,d)} satisfies (4) - (11) \} . \]

Then \( F^{(o,d)} \) defined by the linear equations (4)-(7), (10)-(11) and inequations (8)-(9) is a convex polyhedron. If we only consider flows represented as a convex combination of loopless paths, then \( F^{(o,d)} \) is also closed and bounded. The RCM flow \( f \) satisfying both traffic matrices \( R_t = (r_1^{(o,d)}) \) and \( R_r = (r_2^{(o,d)}) \) can be expressed as follows
\[ f = \sum_{o=1}^{N} \sum_{d=1}^{N} f^{(o,d)} \] (12)
or
\[ f_{i,j} = \sum_{o=1}^{N} \sum_{d=1}^{N} f^{(o,d)}(i,j), \] \[ \forall a(i,j) \in A \] (13)
and
\[ \varphi_n = \sum_{o=1}^{N} \sum_{d=1}^{N} \varphi^{(o,d)}(n), \] \[ \forall v_n \in V \] (14)

where \( f_{i,j} \) is the total flow on arc \( a(i,j) \) and \( \varphi_n \) the total flow through node \( v_n \). Therefore, the set of RCM flows \( F_{RCM} \) is also a convex, closed, and bounded polyhedron. The set \( F_b = \{ f | f \leq C \} \) is a convex set; hence the feasible set \( F = F_{RCM} \cap F_b \) is also convex. Consequently, as the objective function is convex, any downhill search algorithm can be used for the optimization.

III Solution of the RMC Flow Problem

The algorithm we propose for the optimal routing problem is a variation of Franke-Wolfe's steepest descent algorithm [8]. Based on this algorithm, Gerla has proposed the FD (Flow Deviation) algorithm for the solution of MC flow problems [2].

If \( f^k \) is the current feasible solution, the direction of descent can be found by solving the following problem:
\[ \min z = \nabla T(f^k) \cdot f \] (15)
subject to (4) - (11).

where \( \nabla T(f^k) \) is the gradient of the objective function (1) at point \( f^k \). Let \( \Psi^k \) denote the solution of (15), then the feasible direction of descent for the steepest descent algorithm is \( \Psi^k - f^k \). By restricting the search to the straight line segment that joins \( f^k \) and \( \Psi^k \), a minimum is found by using any convenient line search method (such as golden section technique). By repeating the process of finding a feasible direction of descent and performing a line search, solutions with decreasing objective function values are obtained until a minimum is reached.

The problem (15) is a linear programming problem, and, consequently, it can be solved using, e.g., the simplex method. If the TR relation equations (10) - (11) are omitted, the problem (15) becomes an unconstrained linear minimum cost MC flow problem. In the FD algorithm, it is solved using a minimum path algorithm, which is much faster than the simplex method. The length of each arc \( a(i,j) \) in FD is given by
\[ i_{i,j} = \frac{\partial T(f^k)}{\partial f_{i,j}} . \] (16)

As the queuing and processing delay in a node are ignored in [2], the length for each node is zero. The problem (15) is solved by assigning each commodity along its shortest path, since the solution of a linear minimum cost MC flow problem is always an extremal flow [2].

Clearly, the RMC flow problem considered here can be solved efficiently, if the minimum path algorithm could be applied to the solution of the problem (15), subject to the TR relation equations (10) - (11). However, under the metric defined by (16), the shortest path from node \( o \) to node \( d \) is not the same as that from node \( d \) to node \( o \). Obviously, we are faced with the following contradictions: If we would assign the transmit and the receive traffic separately, each along its own shortest path, the TR relation constraint would be violated; if on the other hand, we would assign the two parts of traffic jointly, both along the same path, either the shortest path for transmit or that for the receive direction, then the objective function of (15) would not be minimized.

Fortunately, we can rewrite problem (15) according to Eq. (12) as follows
\[ \min z = \sum_{o=1}^{N} \sum_{d=1}^{N} \nabla T(f^k) \cdot f^{(o,d)} \] (17)
subject to (4) - (11).

Since each related commodity is independent of the other, problem (17) can be separated into \( N(N - 1) \) subproblems — one for each related commodity — where the subproblem for each \( rcm(o,d) \) is
\[ \min z^{(o,d)} = \nabla T(f^k) \cdot f^{(o,d)} \] (18)
subject to (4) - (11).
We shall refer to (18) as a related single commodity (RSC) flow problem. Let $Q^{(o,d)}$ denote the transmit-receive (TR) ratio for each $rcm(o, d)$, then we have

$$Q^{(o,d)} = \frac{r^{(o,d)}_r}{r^e_f},$$

(19)

For the solution of problem (18), we have the following theorem.

**Theorem:** Under the metrics given by

$$I_{i,j} = \frac{\partial T(f^k)}{\partial f_{i,j}} + Q^{(o,d)} \cdot \frac{\partial T(f^k)}{\partial f_{j,i}},$$

(20)

\(\forall a(i,j), a(j,i) \in A\)

and

$$I_n = (1 + Q^{(o,d)}) \cdot \frac{\partial T(f^k)}{\partial \phi_n},$$

(21)

\(\forall \phi_n \in V\)

a minimum path algorithm solves the linear cost RSC flow problem (18).

This theorem is proven in the appendix. We shall now describe our algorithm whose central steps are based on the above theorem.

Once a starting RMC flow has been found, the main steps of our algorithm to compute the iterated flow $f^{k+1}$ from the current flow $f^k$ are:

1. **Step 0:** Set $k = 0$ and compute $T$ according to (1) from the initial feasible flow $f^0$.

2. **Step 1:** Calculate an extremal flow $\Psi^k$ with the following substeps performed for each $rcm(o, d)$:
   1. **Step 1.a:** Compute the metric for each arc and node according to Eq. (20) - (21).
   2. **Step 1.b:** Route the $rcm(o, d)$ along its shortest path under the metrics calculated at Step 1.a.
   3. **Step 1.c:** If all related commodities are considered, go to Step 2; Otherwise, go to Step 1.a.

3. **Step 2:** Find the convex combination of $f^k$ and $\Psi^k$:

$$f^{k+1} = (1 - \alpha) \cdot f^k + \alpha \cdot \Psi^k$$

(22)

which minimizes the objective function $T(f)$.

4. **Step 3:** If $|T(f^{k+1}) - T(f^k)|/T(f^k) \leq \epsilon$, where $\epsilon$ is some predetermined level of tolerance, then STOP; Otherwise, let $k = k + 1$ and go to Step 1.

For finding a starting MC flow, several methods have been published. To find an RMC starting flow, these methods can be modified properly. Based on the results presented in this paper, the modification can be made in a straightforward way. Therefore, we shall not describe it here.

**IV RESULTS AND APPLICATIONS**

In [2] it was shown that the optimal routing based on the MC flow model is purely destination-oriented. As shown in Section III, the metrics defined by Eq. (20) - (21) are dependent on the TR ratio of a specific related commodity. Therefore, the resulting routing is also dependent on it. However, if the TR ratios of the related commodities with the same destination all have the same value, then the resulting routing is also purely destination-oriented. In this case, although the network metrics given by (20) - (21) are still dependent on the value of the TR ratio, they are not different for each related commodity with the same destination. Consequently, all extremal flows found by the minimum path algorithm can be described by the shortest route matrix, and hence, every routing scheme which corresponds to a convex combination of such extremal flows is purely destination-oriented.

Specifically, if we have $Q^{(o,d)} = 1$ for all $rcm(o, d)$, then the optimal routing is not only purely destination-oriented but also symmetrical, because the metric of arc $a(i,j)$ has the same value as that of arc $a(j,i)$. The other special case is that $Q^{(o,d)} = 0$ for all $rcm(o, d)$, where the TR relation constraint disappears actually and we obtain the traditional (unrelated) MC flow model.

Obviously, if the routing problem of a VC network is modeled as an MC instead of an RMC flow problem, the results of the optimization such as the routing scheme and the value of packet delay will be incorrect. Furthermore, as the TR relation is indeed a practical constraint in VC networks, the routing scheme obtained from the optimization with inconsistent MC flow model could cause some performance degradations.

**Figure 2:** Topology of the DATEX-P network

To illustrate our results, we shall take the DATEX-P network as an example and solve the routing problem for it. The DATEX-P network is the German public packet net-
work provided by the German Telecom (Telekom) since 1980. A detailed description of the network is given in [3],[9]. The network has currently more than 80 nodes at 18 locations. The system is a product of Northern Telecom Canada Ltd. in SL-10 technology. As the network grows rapidly, the German Telecom has decided to introduce new technology: the high-performance EWSP systems of Siemens AG, with which only one switch per location is needed [4]. One of the proposed topologies for DATEX-P network [3] is given in Fig. 2. The capacity of each trunk is assumed to be 64 (kbits/second) and the average packet length is 1000 (bits/packet). For simplicity, we assume that the nodal queuing and processing time has a constant value of 35 ms, which corresponds to the node packet delay at a system load of 50% [3][4].

The value of the transmit packet arrival rate for each OD node pair is a random number drawn from a uniform distribution over an interval \([a, b]\). While the traffic level of the network varies, the ratio of \(a\) to \(b\) is kept constantly \((b/a = 10)\). The TR ratios in a network are generally very different. As described in [10], the TR ratio for a character interactive VC is about one, and that for a block interactive VC is about four. For our example, the TR ratio for each OD node pair is also randomly generated according to a uniform distribution over an interval \([1,4]\). To compare the results, both the MC and the RMC flow model are applied for solving the optimal routing problem. Since the MC flow model uses only one traffic matrix \(R\) for the description of the external traffic, there is no possibility to distinguish \(r_{o,a}\) from \(r_{t,a}\), and we have \(R = R_t + R_t^T\) where \(R_t^T\) is the transpose of \(R_t\).

![Figure 3: Results from the MC and RMC flow model](image)

The minimum average packet delay is evaluated for both the RMC model (RMC-VC) and the MC model (MC-DG) at different traffic levels. As shown in Fig. 3, no visible difference between their resulting delay values can be observed. This is because each trunk in the network is assumed to be duplex, the network is actually able to operate with the same average packet delay for VC mode as for the DG mode. However, as discussed earlier, the flow pattern resulting from the optimization based on the MC model can never be implemented in a VC network, since the TR relation constraint, which is omitted in the MC model, does exist in the VC network.

To evaluate the practical effect of using the MC model for optimization of a VC network, we assign the related traffic flows according to the routing table obtained with the MC flow model, whereas the TR-relation is applied. The resulting delay values (MC-VC) are also shown in Fig. 3. As can be expected, the network performance is evidently degraded, because no bandwidth reservation for the receive traffic of the VCs is guaranteed. At the higher traffic levels, the network would be saturated if the routing scheme resulting from the MC flow model would be used.

![Figure 4: Sample case of connectivity equal to 4 for a symmetrical network of 18 nodes](image)

![Figure 5: Dependence of performance degradation on the network connectivity](image)

To explore dependence of the performance degradation on network topology, we have investigated a set of symmetrical networks with 18 nodes. The connectivity (node degree)
varies from 2 (a ring network) to 17 (a fully connected network). Fig. 4 shows a symmetrical network with the connectivity equal to 4. Generally, there are several such networks for each connectivity. All networks we investigated have the property that the average number of hops (the average is over all OD node pairs), using minimum number of hops routing, is minimal among all possible topologies. Again the value of the transmit packet arrival rate for each OD node pair is randomly drawn from a uniform distribution over an interval [a, b] with a/b = 10. For simplicity, all TR ratios are assumed to be 4. To reduce dependence on specific traffic patterns, several traffic matrices were generated, and the results were averaged. The maximum throughput of these networks obtained by using the RMC model and MC model are shown in Fig. 5. The throughput degradation increases monotonically with the increase of the network connectivity because the number of alternative paths between each OD node pair is growing.

V Conclusions

For solving the optimal routing problem in VC networks, we have presented a new model—the RMC flow model—which accounts for an important practical constraint, i.e., the TR relation constraint in VC networks. The solution technique based on the new metric presented here is very efficient due to small computational overhead. We have shown the impact of the TR relation constraint on optimal routing, especially the performance degradation caused by using the MC flow model for the optimization of the routing in a VC network and its dependence on network topology. The impact of the TR relation on other related network design problems such as the simultaneous routing and capacity assignment problem is being studied currently.

The model and solution methods presented in this paper are by no means restricted to traditional packet-switched VC networks. They can also be applied for solving design problems of other VC networks, such as the design of the frame-relay networks or the ATM networks.

Appendix

Proof: First we drop the argument o, d and f^k, from (18) for convenience and rewrite it as follows:

\[
\text{min } z_1 = \ldots + f(i, j) \frac{\partial T}{\partial f_{i,j}} + \\
+f(j, i) \frac{\partial T}{\partial f_{j,i}} + \ldots + \\
+\varphi(n) \frac{\partial T}{\partial \varphi_n} + \ldots
\]  

(A.1)

subject to (4) – (11).

Using the definition (19), the TR-relation constraints (10) – (11) can be written as

\[ f_r(i, j) = Q \cdot f_t(j, i), \quad \forall a(i, j), a(j, i) \in A \]  

(A.2)

and

\[ \varphi_r(n) = Q \cdot \varphi_t(n), \quad \forall \varphi_n \in V. \]  

(A.3)

Substitute (2) – (3) and (A.2)-(A.3) in (A.1) and consider that constraints (10) – (11) are now included therein, we have

\[
\text{min } z_1 = \ldots + f_t(i, j) \left( \frac{\partial T}{\partial f_{t,i,j}} + Q \frac{\partial T}{\partial f_{t,j,i}} \right) + \\
+f_t(j, i) \left( \frac{\partial T}{\partial f_{t,j,i}} + Q \frac{\partial T}{\partial f_{t,i,j}} \right) + \ldots + \\
+\varphi_t(n) (1 + Q) \frac{\partial T}{\partial \varphi_t(n)} + \ldots
\]  

(A.4)

subject to (4) – (9).

Problem (A.4) becomes now an unconstrained linear minimum cost (unrelated) MC flow problem, and it can be solved by using a minimum path algorithm under metrics given by (20) – (21).

References