Random Mobility Models for Wireless Ad Hoc Networks and their One-Dimensional Analysis

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Abstract. In the analysis of wireless ad hoc networks, mobility models that are general enough to capture the major characteristics of a realistic movement profile, and yet are simple enough to mathematically formulate its long-run behavior, are highly desirable. In this paper, we propose a generalized random mobility model capable of capturing several mobility scenarios and give a mathematical framework for its exact analysis over one-dimensional mobility terrains. The model provides the flexibility to capture hotspots where mobiles accumulate with higher probability and spend more time. The selection process of hotspots is random and correlations between the consecutive hotspot decisions can be successfully modeled. Additionally, the speed of movement can be a function of distance that is being traveled. Our solution framework formulates the model as a semi-Markov process using a special discretization technique. We provided long-run location and speed distributions by closed-form expressions for one-dimensional regions. Keywords: Mobility Modeling, Long-Run Analysis, semi-Markov Processes, Ad Hoc Networks

1. INTRODUCTION

Wireless ad hoc networks are comprised of wireless mobile nodes that can dynamically form a network in a self-organizing manner without the need for a pre-existing fixed infrastructure. Nodes in an ad hoc network can move according to many different mobility profiles. Therefore, mobility models that dictate the movement behavior of a mobile terminal play a key role in analyzing the impact of dynamically changing topology on the performance of these networks, which can be done through analytical or simulation based studies. In this paper, we consider a generalized random mobility model that is flexible enough to capture different mobility scenarios, and provide its long-run location and speed distributions by closed form expressions for one-dimensional mobility terrains.

In general, a mobility model governs the changes in the moving direction and speed of terminals according to a deterministic approach or a random process. In the former case, movement path of terminals can be restricted to predetermined paths. For ad hoc environments, such mobility models are impractical since wireless ad hoc networks are created “on the fly”, and collecting data to generate the paths for all situations can be very complicated. Thus, a mobility model that dictates the movement of hosts due to a random process, that is, random mobility model, is more appropriate for the performance evaluation of these networks. Surveys for both models are presented in [2,4].

A shortcoming of the random mobility models is that the movement profiles that are generated with respect to them may not be consistent with the major characteristics of a realistic scenario. For instance, as it also mentioned in [2], random walk and random waypoint mobility [11] like models may generate unrealistic movement patterns such as “sudden stops” and “sharp turns”. Furthermore, as it is also criticized in [19,20], selecting speed independently from the distance that is going to be traveled may end up in
unrealistic mobility profiles where mobiles travel long distances with low speeds.

In the analytical studies for the performance analysis of wireless ad hoc networks, closed form expressions for the spatial node distribution are very desirable to understand long-run behavior of the network spatial behavior. For instance, the analysis that are presented in [9,10,13] to estimate the capacity per source to destination pair of these networks are significantly dependent on the spatial distribution of mobile nodes. Additionally, for some scenarios in which terminals can be highly mobile on a wide region, the spatial distribution of offered traffic may not be ignored in determining the capacity of asynchronous MAC layer protocols. Observe that the analysis of this case requires an accurate knowledge of the spatial distribution of nodes. The analytical work presented in [6] also considers the station locations for the MAC layer throughput analysis but the terminals are assumed to be uniformly distributed in the region, which may not be valid for different mobility scenarios. Moreover, this knowledge can be also used in evaluating the connectivity properties of ad hoc networks, which have been extensively studied in [7,17]. In addition to these, the distribution of link distance between mobile terminals, which is an important characteristic of wireless ad hoc networks [15,14], can be obtained from the spatial distribution of terminals.

Hence in this paper we propose a generalized random mobility model that is general enough to capture the major characteristics of a realistic movement profile, and yet is simple enough to mathematically formulate its long-run behavior with analytical expressions. The mobility pattern of a terminal that moves according to this generalized model is composed consecutive movement epochs in a closed region and it is uncorrelated with the movement behavior of other terminals. During each movement epoch, mobile terminal at first moves on the finite line segment joining the starting and destinations points of the epoch at a random speed and then it pauses at the destination for a random amount of time. The generic approach that specifies the characteristics of this model can be explained as follows:

- The distribution of the destination points are assumed to be general and can be conditionally dependent on the starting point of the movement epoch.
- The random speed for each epoch is drawn from a general distribution function that can be conditionally dependent on the starting and destination locations of the movement epoch.
- The pause time at each destination is selected randomly from a distribution that is dependent on the location of the destination point.

These generalizations have number of advantages. First, since destinations are selected from a general distribution, a movement scenario in which terminals select some specific locations, for example, hotspots, as destination with higher probability, can be easily captured. Furthermore, some mobility scenarios may require a Markovian dependency between the destination points of consecutive movement epochs. For instance, the probability of selecting a hotspot as destination can be different from different starting points. This case can be naturally incorporated into our model by employing a distribution function for destinations that is conditionally dependent on the starting points.

Second, the generic approach for determining speed provides a unique opportunity to select speed according to the distance that is going to be traveled. Finally, by employing a pause time distribution for each epoch that is a function of destination coordinate, we reached to the flexibility of pausing different times at different locations.

For some sophisticated mobility models, performing its long-run analysis first over one-dimensional regions will be useful in gaining some insight into the methodology that has to be followed for the analysis of higher dimensions. Thus, in this paper, we concentrate our analysis to one-dimensional regions, and develop an analytical framework that provide closed form expressions for the long-run location and speed distributions. We also believe that the analytical results presented can provide a methodology to analytically formulate the fundamental properties of wireless ad hoc networks for number sophisticated mobility scenarios (e.g., capacity, connectivity). Due to lack of space, we refer the reader to [1] for an brief overview of the works presented for the long-run analysis of the mobility models designed for wireless ad hoc networks.
The next section provides the mobility formulation according to our mobility model, basic definitions, and our approach for long-run analysis. In the third section the analytical results are presented. The fourth section concentrates on the example scenarios. Section 5 concludes the paper.

2. MOBILITY FORMULATION

In this section, we provide the formal description of the generalized random mobility model introduced in Section 1 for one-dimensional mobility terrains, and construct an analytical framework for its long-run analysis. Let \( R = [0, a] \) represent the region on which mobile terminals operate, and denote \( X_s \in R \) and \( X_d \in R \) as the random variables corresponding to the starting and destination points of a movement epoch, respectively. Furthermore, let the random variable \( V \) defined on the state space \([v_{\min}, v_{\max}]\), where \( v_{\min} > 0 \), denote the speed of a terminal while moving from \( X_s \) to \( X_d \). In addition, denote the random variable \( T_p \) with state space \([0, \infty)\) as the pause time spent at destination point \( X_d \). With respect to these notations, and the mobility modeling approach we proposed in this paper, we define the following parameters:

- \( f_{X_d|X_s} \): the conditional probability density function (pdf) of \( X_d \) given \( X_s \),
- \( f_{V|X_s,X_d} \): the conditional pdf of \( V \) given \( X_s \) and \( X_d \),
- \( f_{T_p|X_d} \): the conditional pdf of \( T_p \) given \( X_d \).

Hence, the mobility formulation that is performed according to the generalized random mobility model can be characterized by the triplet \( \langle f_{X_d|X_s}, f_{V|X_s,X_d}, f_{T_p|X_d} \rangle \).

Before we proceed further, we note that \( X_s \) and \( X_d \) actually represent the destination points of any two consecutive movement epochs, and the conditional pdf \( f_{X_d|X_s} \) that identifies the distribution of \( X_d \) given \( X_s \) at the embedded points in time where a new epoch starts, is referred as stochastic density kernel by Feller [8].

Now as we have noted in Section 1, each terminals movement is assumed to be independent from others. Thus, it is enough to model a single terminals behavior for the long-run analysis. For this purpose, let \( X(t) \) denote the state of the mobile terminal at time \( t \). According to the specifications of the mobility model we proposed, the stochastic process \( \{X(t), t \geq 0\} \) must be defined on a state space that has separate dimensions for current location, destination, and speed, and more importantly, the ranges of these dimensions must be continuous. However, in the analytical framework we construct, we use a discretization method and describe the mobility behavior of nodes with a stochastic process that is defined on a multidimensional discrete state space. In addition, instead of observing the state of a terminal continuously, we will observe it at embedded times \( T_k \), for \( k \in \mathbb{N} \), such that \( T_0 = 0 \), \( T_{k+1} \geq T_k \), \( \forall k \in \mathbb{Z}^+ \). Also, these embedded times are dependent on the evolution of the system that dictates the movement behavior of the mobile node. The following list formally defines the assumptions that the analytical framework is built on:

- **A1:** The region \( R \) is discretized into \( n \) cells of the same size, that are denoted by \( c_i = [(i-1)\Delta x, i\Delta x) \), \( i = 0 \ldots n-1 \), where \( \Delta x = \frac{a}{n} \) for \( n > 1 \). A mobile terminal is assumed to occupy one of the \( c_i \)'s at any moment in time, and movement epochs start from a cell and ends up at a different destination cell.
- **A2:** The random variable \( V \), which denotes the speed of a mobile during a movement epoch, is approximated by the discrete random variable \( V^* \) taking values in the state space \( S_{V^*} = \{ z_1, z_2, \ldots, z_m \} \) where \( z_r = r \Delta v, r = 1, \ldots, m \), for some discretization parameter \( \Delta v > 0 \), and \( m > 1 \) such that \( \Delta v \leq v_{\min} \) and \( v_{\max} \leq m\Delta v \).
- **A3:** Observation time \( T_k \) points to the time of occurrence of one of the following events:
  - **E1:** The terminal, which is in pause mode, selects a new destination that is different from the current cell occupied, and jumps into moving state at the current cell.
  - **E2:** The terminal, which is traveling in the direction of the target cell, moves out from the current cell and enters the neighbor cell that lies on the path between the current and destination cells,
  - **E3:** The terminal reaches to the destination cell and enters the pause mode at that location.
Notice that as \( \{n,m\} \to \infty \), we converge to model with continuous state space. For the rest of this paper, we will use the term \textit{discretisized mobility formulation} to refer to the version of the mobility modeling approach constructed according to the assumptions \( A_1, A_2, \) and \( A_3 \).

Now, let \( S_k, k \in \mathbb{N} \), denote the state of the mobile terminal at time \( T_k \). Given the assumptions \( A_1, A_2 \) and \( A_3 \), the finite-state space of \( S_k \) will be defined as follows:

\[
S = \{(c_i, c_j, z_r, q) \mid i, j = 0, \ldots, n-1, i \neq j, r = 1, \ldots, m, q = 1\}
\]

\[
\cup \{(c_i, q) \mid i = 0, \ldots, n-1, q = 0\}
\]

(1)

where \( c_i \) is the current cell occupied, \( c_j \) is the destination cell, \( z_r \) is the discretisized speed, and \( q \) is the indicator of moving towards the target cell, or pausing at the destination.

Hence, the stochastic process \( \{X(t), t \geq 0\} \) that represents the state of the mobile terminal at time \( t \), can be redefined on the finite-state space \( S \) by the following expression:

\[
X(t) = S_k, \text{ if } T_k \leq t < T_{k+1}
\]

where the times \( T_1, T_2, \ldots \) are the successive times of transitions of \( X(t) \), and \( S_0, S_1, S_2, \ldots \) represent the successive states occupied by \( X(t) \).

Observe that by constructing a state space that has a separate dimension for the destination cell of moving terminals, the future evolution of the stochastic process \( \{S_k, k \in \mathbb{N}\} \) becomes dependent only on the current state of the mobile terminal, not on its history at previous observation points. Furthermore, for all \( s \in S \) in (1), distribution of sojourn time in state \( s \) would be independent from the previous states occupied can be determined from the components of \( s \).

Therefore, the stochastic process \( \{S_k, T_k; k \in \mathbb{N}\} \) with finite-state space \( S \) in (1) satisfies the conditions for being \textit{Markov Renewal Process}, and the process \( \{X(t), t \geq 0\} \) can be called as the \textit{semi-Markov process} (SMP) associated with \( \{S_k, T_k; k \in \mathbb{N}\} \) [5]. Moreover, since the general distributions for destination, speed, and pause time parameters are assumed to be \textit{time-homogeneous} in the model proposed, the transitions of the process \( X(t) \) from state \( s \) to state \( s' \) at the time instants \( T_k \) can be governed by the \textit{discrete-time} Markov chain (DTMC) \( \{S_k, k \in \mathbb{N}\} \) with finite-state space \( S \) and transition probability matrix \( P = [p_{ss'}] \), where \( p_{ss'} = \Pr \{S_{k+1} = s' \mid S_k = s\} \), such that \( \sum_{s' \in S} p_{ss'} = 1 \) for all \( s \in S \). The process \( \{S_k, k \in \mathbb{N}\} \) is also called \textit{embedded} DTMC of SMP.

Consequently, if the DTMC \( \{S_k, k \in \mathbb{N}\} \) satisfies the \textit{ergodicity} conditions, and if the mean state holding times are finite, then the SMP \( \{X(t), t \geq 0\} \) can be characterized at the long-run. Clearly, if long-run proportion of times spent at the states of the discrete state space \( S \) in (1) are known, then by aggregating the states that has the same current cell and speed components, the long-run location and speed distributions for the discretisized formulation can be easily obtained. After this, by observing the limiting behavior of that discrete result as \( n \to \infty \) and \( m \to \infty \), the continuous result can be derived.

3. **ANALYTICAL RESULTS FOR DISCRETISIZED AND CONTINUOUS MOBILITY FORMULATIONS**

In this section, we apply our framework with the ultimate aim of finding closed form expressions for the long-run location and speed distributions over the mobility terrain \( R = [0, a] \).

Now to describe the transition probabilities of the embedded DTMC \( \{S_k, k \in \mathbb{N}\} \), we first define:

\[
\tau_{ij} = \Pr \{X_d \in c_j | X_s \in c_i\} = \int_{\Delta x} dx_d f_{X_d | X_s}(x_d | X_s \in c_i), \quad i, j = 0, \ldots, n-1
\]

(2)

Next, since \( V \) is allowed to be dependent on \( X_s \) and \( X_d \), we define the probability mass function of \( V^* \) given \( X_s \in c_i \) and \( X_d \in c_j \)

\[
\nu_{rij} = \Pr \{V^* = z_r | X_s \in c_i, X_d \in c_j\} = \int_{(r-1)\Delta v}^{r\Delta v} f_v | X_s, X_d| \mu(\{v | X_s \in c_i, X_d \in c_j\}) \, dv, \quad r = 1, \ldots, m
\]

(3)
Table 1
Transition probabilities of the process \{S_k, k \in \mathbb{N}\}

<table>
<thead>
<tr>
<th>Event</th>
<th>Transition</th>
<th>Probability</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_1</td>
<td>((c_i, 0) \rightarrow (c_i, c_j, z_r, 1))</td>
<td>(\frac{\nu_{ij}}{1-\tau_{ij}})</td>
<td>(i \neq j)</td>
</tr>
<tr>
<td>E_2</td>
<td>((c_i, c_j, z_r, 1) \rightarrow (c_{i+1}, c_j, z_r, 1))</td>
<td>1</td>
<td>(j &gt; i + 1)</td>
</tr>
<tr>
<td></td>
<td>((c_i, c_j, z_r, 1) \rightarrow (c_{i-1}, c_j, z_r, 1))</td>
<td>1</td>
<td>(j &lt; i - 1)</td>
</tr>
<tr>
<td>E_3</td>
<td>((c_i, c_j, z_r, 1) \rightarrow (c_j, 0))</td>
<td>1</td>
<td>(</td>
</tr>
</tbody>
</table>

Based on the events \(E_1, E_2,\) and \(E_3\) that cause state changes, and \(\tau_{ji}\) in (2) and \(\nu_{ij}\) in (3), the possible transitions and the corresponding transition probabilities of the embedded DTMC can be grouped as in Table 1.

Having identified the transition probabilities of the process \(\{S_k, k \in \mathbb{N}\}\), we define the ordering we impose on the states in \(S\) as follows:

\[ S = \{S_0, S_1, \ldots, S_{n-1}\}, \quad (4) \]
\[ S_i = \{(c_i, 0, z_1, 1), \ldots, (c_i, c_0, z_m, 1), \ldots, (c_i, c_{i-1}, z_1, 1), \ldots, (c_i, c_{n-1}, z_1, 1), \ldots, (c_i, c_{n-1}, z_m, 1)\}, \quad (5) \]

Before we can proceed with the long-run analysis of the SMP \(\{X(t), t \geq 0\}\), we must first find the steady-state distribution of the embedded DTMC \(\{S_k, k \in \mathbb{N}\}\) with the transition probability matrix \(P\) defined on the state space \(S\) in (4) according to the transition probabilities given in Table 1. Clearly this distribution exists if and only if a steady-state distribution exists for \(X_s\) and \(\{S_k, k \in \mathbb{N}\}\) satisfies the ergodicity conditions. Hence, we focus on these issues now.

Under the “mild” regularity conditions defined by Feller [8] on \(f_{X_{d|s}}(x_d|x_s)\), there exists a steady-state distribution for \(X_s\) with pdf \(f_{X_s}(x_d)\), which can be uniquely determined from the solution of the following integral equation

\[ f_{X_s}(x_d) = \int_0^a f_{X_{d|s}}(x_d|x_s) f_{X_s}(x_s) dx_s \quad (6) \]

Thus, if the pdf \(f_{X_s}(x_d)\) can be uniquely determined from the solution of (6), then the probability of starting a movement epoch from cell \(c_i\) at the steady-state, denoted by \(\varphi_i\), \(i = 0, \ldots, n - 1\), is

\[ \varphi_i = \int_{x_i}^{x_{i+1}} dx_d f_{X_s}(x_d) \quad (7) \]

Having clarified this issue, we now we examine the ergodicity of \(\{S_k, k \in \mathbb{N}\}\) and its steady-state distribution.

**Lemma 1** If the pdf \(f_{X_{s}}(x_d)\) can be uniquely determined from the integral equation in (6), and if \(\nu_{r|i,j} > 0\), \(i, j = 0, \ldots, n - 1\) and \(r = 1, \ldots, m\), then the embedded DTMC \(\{S_k, k \in \mathbb{N}\}\) will be irreducible and aperiodic, and the steady-state distribution of the states in \(S_i\) in (5), denoted by the vector \(\pi_i\), is given by

\[ \pi_i = \sum_{\ell=1}^{n-1} \varphi_{i, \ell} \tau_{0|\ell} \nu_{m|\ell, 0} + \sum_{\ell=i}^{n-1} \varphi_{i, \ell} \tau_{i-1|\ell} \nu_{m|\ell, i-1} \varphi_i \left(1 - \tau_{i|i}\right), \]

\[ \sum_{\ell=i}^{n-1} \varphi_{i, \ell} \tau_{i+1|\ell} \nu_{m|\ell, i+1} + \cdots + \sum_{\ell=0}^{i} \varphi_{i, \ell} \tau_{n-1|\ell} \nu_{m|\ell, n-1} / \sum_{i=0}^{n-1} |\pi_i| \quad (8) \]

where \(\nu_{m|i,j} = [\nu_{1|i,j}, \ldots, \nu_{m|i,j}], i, j = 0, \ldots, n - 1\).

**Proof** Refer to [1].
Finally, to characterize the SMP \(\{X(t), t \geq 0\}\) at the long-run, it remains to formulate the expected state holding times which we denote by \(\bar{t}_s, \forall s \in S\). Recall that in Section 2, we decomposed the state space \(S\) in (1) into two groups that represent moving, and pausing terminals. Thus, the expected time that is going to be spent in a cell \(c_i\) by moving terminals is simply \(\bar{t}_s = \frac{\Delta x}{z_r}\). To formulate the mean time that is spent in a state of the form \(s = (c_i, 0), i = 0, \ldots, n - 1\), we define

\[
\bar{t}_s = E[T_{p_i}] = E[T_p|X_s \in c_i] = \int_0^\infty \Pr\{T_p > t_p|X_s \in c_i\} \, dt_p
\]

(9)

Finally, to characterize the SMP \(\{X(t), t \geq 0\}\) at the long-run, the following must be satisfied [5]:

\[
\sum_{s \in S} \pi_s \bar{t}_s < \infty
\]

(10)

Hence, by applying the theory of semi-Markov processes we obtained the long-run proportion of time that the SMP \(\{X(t), t \geq 0\}\) is in a state \(s \in S\). After aggregating the states in \(S\) that has the same “current location” and “speed” components, including the ones with zero speed (i.e, \(s = (c_i, 0)\)), we reached to the following result.

**Lemma 2** For the mobile terminal, whose mobility pattern is characterized according to the discretized version of the \(< f_{X_d|X_s}, f_{X|X_s, \tilde{V}|X_s}, f_{T_p|X_s} >\) mobility formulation, let \(p_i\) denote the long-run proportion of time that terminal stays in cell \(c_i\). Similarly, denote \(\psi_r\) as the long-run proportion of time that mobile possesses speed \(z_r = \tau \hat{v}_r, r = 0, \ldots, m\). If the conditions given by Lemma 1 holds, and if the equation (10) is satisfied, then

\[
p_i = \frac{\varphi_i (1 - \tau_{i|i}) E[T_{p_i}] + \sum_{j=0}^{n-1} \sum_{\ell=1}^{i-1} \varphi_{\ell} \tau_{j|\ell} \sum_{r=1}^{m} \frac{\Delta x}{z_r} \nu_r|\ell,j} {\sum_{i=0}^{n-1} \varphi_i (1 - \tau_{i|i}) E[T_{p_i}] + \hat{D}_n \Delta x},
\]

(11)

\[
\psi_r = \begin{cases} \left( \sum_{i=0}^{n-1} \varphi_i (1 - \tau_{i|i}) E[T_{p_i}] \right) / \hat{N}, & r = 0 \\ \left( \sum_{i=0}^{n-1} \Delta x \left( \sum_{j=0}^{i-1} \sum_{\ell=1}^{n-1} \varphi_{\ell} \tau_{j|\ell} \frac{1}{z_r} \nu_r|\ell,j + \sum_{j=i+1}^{n-1} \sum_{\ell=0}^{i-1} \varphi_{\ell} \tau_{j|\ell} \frac{1}{z_r} \nu_r|\ell,j \right) \right) / \hat{N}, & r = 1, \ldots, m \end{cases}
\]

(12)

where

\[
\hat{D}_n = \sum_{i=0}^{n-1} \left( \sum_{j=0}^{i-1} \sum_{\ell=1}^{n-1} \varphi_{\ell} \tau_{j|\ell} \sum_{r=1}^{m} \frac{1}{z_r} \nu_r|\ell,j + \sum_{j=i+1}^{n-1} \sum_{\ell=0}^{i-1} \varphi_{\ell} \tau_{j|\ell} \sum_{r=1}^{m} \frac{1}{z_r} \nu_r|\ell,j \right)
\]

(13)

and \(\hat{N} = \sum_{i=0}^{n-1} \varphi_i (1 - \tau_{i|i}) E[T_{p_i}] + \hat{D}_n \Delta x\).

**Proof** Refer to [1].

Now let the random variables \(X(t)\) and \(\tilde{V}(t)\) denote the location and the speed of a mobile terminal at time \(t\), respectively. Note that since the mobile can be in moving or pausing modes at any point in time, \(\tilde{V}(t)\) is either equal to 0, or \(\in [\tilde{v}_{\min}, \tilde{v}_{\max}]\). Because we are interested in the long-run distributions, let \(X\) and \(\tilde{V}\) respectively denote random variables having the long-run distribution of \(X(t)\) and \(\tilde{V}(t)\). Hence, based on the distributions given in Lemma 2 and these notations we present our fundamental result as follows.
Theorem 1 For the mobile terminal, whose mobility pattern is characterized by the triplet $< f_{X_d|x_s}, f_{V|X_s,X_d}, f_{T_p|X_d} >$, if the pdf $f_{X_s}(x_d)$ can be uniquely determined from the integral equation (6), and $E[T_p|x_s = x_s] < \infty$, $\forall x_s \in [0, a]$, and $f_{V|X_s,X_d} > 0$, $\forall v \in [v_{min}, v_{max}]$, and $\forall x_s, x_d \in [0, a]$, then pdf of $X$ and $\tilde{V}$, denoted by $f_X$ and $f_{\tilde{V}}$, respectively, and $E[\tilde{V}]$ are given by

$$f_X(x) = \frac{f_{X_s}(x)E[T_p|x_s = x] + \int_{v_{min}}^{v_{max}} dv k(x, v)}{E[T_p|0 \leq X_s \leq a] + \tilde{D}}$$  \hspace{1cm} (14)$$

$$f_{\tilde{V}}(\tilde{v}) = \begin{cases} \frac{E[T_p|0 \leq X_s \leq a] \delta(\tilde{v})}{E[T_p|0 \leq X_s \leq a] + \tilde{D}}, & \tilde{v} = 0 \\ \frac{\int_0^a dx k(x, \tilde{v})}{E[T_p|0 \leq X_s \leq a] + \tilde{D}}, & \tilde{v} \in [v_{min}, v_{max}] \end{cases}$$ and $E[\tilde{V}] = \frac{\tilde{D}}{E[T_p|0 \leq X_s \leq a] + \tilde{D}}$  \hspace{1cm} (15)$$

where

$$k(x, v) = \int_0^x dx_d \int_0^x dx_s f_{X_s}(x_s) f_{X_d|x_s}(x_d|x_s) f_{V|X_s,X_d}(v|x_s, x_d)$$

$$+ \int_x^a dx_d \int_0^x dx_s f_{X_s}(x_s) f_{X_d|x_s}(x_d|x_s) f_{V|X_s,X_d}(v|x_s, x_d),$$  \hspace{1cm} (16)$$

$$\tilde{D} = \int_0^a dx \int_{v_{min}}^{v_{max}} dv k(x, v),$$  \hspace{1cm} (17)$$

and

$$\tilde{D} = \int_0^a dx_d \int_0^{x_d} dx_s (x_d - x_s) f_{X_s}(x_s) f_{X_d|x_s}(x_d|x_s) + \int_0^a dx_d \int_0^{x_d} dx_s (x_d - x_s) f_{X_s}(x_s) f_{X_d|x_s}(x_d|x_s)$$  \hspace{1cm} (18)$$

Proof Refer to [1].

It should be noted that, if the distributions of $X_d$ and $T_p$ are independent from $X_s$, and if distribution of $V$ is also independent of $X_s$ and $X_d$, then the formulation of $f_{\tilde{V}}(v)$ and $E[\tilde{V}]$ in (15) for this simplified case will match to the results that are derived in [20] for a class of mobility models where speed is selected independently from the distance that is going to be traveled.

4. VARIANTS OF MOBILITY CHARACTERIZATIONS

Example 1 The random waypoint model [11] can be considered as the simplest nontrivial case for the mobility characterizations that can be done according to the triplet $< f_{X_d|x_s}, f_{V|X_s,X_d}, f_{T_p|X_d} >$. For this model, the distributions of $X_d$ and $V$ are assumed to be uniform in the regions $R$ and $[v_{min}, v_{max}]$, respectively. Moreover, the distribution of $T_p$ is considered to be the same at all destinations. Therefore, for $R = [0, a]$ we have $f_{X_s}(x_d) = \frac{1}{a}$, $x_d \in R$. Hence, it follows from Theorem 1 that

$$f_X(x) = \frac{\frac{1}{a} E[T_p] + \frac{2x(a-x)}{a^2} E[\frac{1}{v}]}{E[T_p] + \frac{\partial}{\partial x} E[\frac{1}{v}]} \text{ and } E[\tilde{V}] = \frac{a}{E[T_p] + \frac{\partial}{\partial x} E[\frac{1}{v}]}$$  \hspace{1cm} (19)$$

where $E[\frac{1}{v}] = \ln(\frac{v_{max}}{v_{min}})/(v_{max} - v_{min})$.

We also note that the work presented in [3], considers a variant of this result where speed is deterministic with a parameter $v$. If we substitute $E[\frac{1}{v}]$ with $\frac{1}{v}$, the two results will match with each other.

Example 2 In the random waypoint mobility model we analyzed by Example 1, $V$ is assumed to be independent from $|X_s - X_d|$, that is, the distance traveled during a movement epoch. However, in most of the realistic scenarios, $V$ tends to increase as $|X_s - X_d|$ does. Thus, for this example, we make an improvement on the random waypoint mobility model by proposing a $f_{V|X_s,X_d}$ that provides the opportunity to determine $V$ proportional to the random variable $D = |X_s - X_d|$ with high probability.
Now, considered a truncated normal distribution [12] for $V$ according to the pdf given by

$$f_{V|X_s,X_d}(v|x_s,x_d) = \frac{Z \left( \frac{v - \mu(x_s,x_d)}{\sigma} \right)}{\sigma \left( \Phi \left( \frac{v_{\text{max}} - \mu(x_s,x_d)}{\sigma} \right) - \Phi \left( \frac{v_{\text{min}} - \mu(x_s,x_d)}{\sigma} \right) \right)}, \quad (20)$$

where $\sigma > 0$, $v_{\text{min}} \leq v \leq v_{\text{max}}$, and $\mu(x_s,x_d) = v_{\text{min}} + \frac{(v_{\text{max}} - v_{\text{min}})}{a} |x_s - x_d|$, and $Z$ and $\Phi$ are the probability density and cumulative distribution functions for the normal distribution, respectively [12]. Hence, we reached to the following results for this improved case:

$$f_X(x) = \frac{1}{a} \frac{E[T_p]}{E[T_p] + D} + k_X(x) \quad \text{and} \quad E[\hat{V}] = \frac{a/3}{E[T_p] + D} \quad (21)$$

where

$$k_X(x) = \frac{2}{a^2} \int_0^x dx_d \int_x^a dx_s \int_{v_{\text{min}}}^{v_{\text{max}}} dv \frac{1}{v} f_{V|X_s,X_d}(v|x_s,x_d) \quad \text{and} \quad \hat{D} = \int_0^a dx k_X(x) \quad (22)$$

Clearly, because of the complicatedness of $f_{V|X_s,X_d}$, $k_X(x)$ can only be evaluated numerically for a given $x \in [0,a]$, and also $\hat{D}$. However, for the extreme case $\sigma \rightarrow 0$, we have $f_{V|X_s,X_d}(v|x_s,x_d) = \delta(v - \mu(x_s,x_d))$, where $\delta(v)$ corresponds to direc delta function, and $k_X(x)$ simplifies to

$$k_X(x) = 2 \left( \frac{\ln((v_{\text{max}}(a-x)+v_{\text{min}})/a)\ln((x(v_{\text{max}}-v_{\text{min}})-v_{\text{max}}a)\ln((x(v_{\text{max}}-v_{\text{min}})+v_{\text{min}}a)/a)(x(v_{\text{max}}-v_{\text{min}})-av_{\text{min}})}{a(v_{\text{max}}+v_{\text{min}})^2} \right) \quad (23)$$

and $\hat{D}$ will be given by

$$\hat{D} = \left( a(v_{\text{max}}^2-v_{\text{min}}^2-2v_{\text{min}}v_{\text{max}} \ln(v_{\text{max}}/v_{\text{min}})) \right)/(v_{\text{max}}-v_{\text{min}})^3 \quad (24)$$

Obviously, the other extreme case of interest, that is, $\sigma \rightarrow \infty$, simplifies to the scenario where $V$ is uniformly distributed in $[v_{\text{min}}, v_{\text{max}}]$.

Now, after substituting the $\hat{D}$ given above by (24) to the equation for $E[\hat{V}]$, a comparison of that $E[\hat{V}]$ with the one defined in Example 1 for the original random waypoint mobility model reveals out that since $\hat{D}$ in (24) is less than $(a/3)(\ln(v_{\text{max}}/v_{\text{min}}))$ (i.e., $\hat{D}$ in Example 1) for all $v_{\text{max}} > v_{\text{min}} > 0$, the $E[\hat{V}]$ obtained for the uniformly distributed $V$ is always smaller than its counterpart for the $V$ that is directly proportional to $|X_s - X_d|$ with probability one. This is consistent with the intuitive expectations because in this case the possibility of moving long distances with low speeds becomes zero. On the other hand, for the other case, lower speeds might be selected for longer distances and as a result, expected value of the long-run speed decreases. Therefore, we conclude that, if $f_{V|X_s,X_d}$ is defined according to (20) for the original random waypoint mobility model, then the lower bound for $E[\hat{V}]$ is $(a/3)/(E[T_p] + E[1/V] \hat{D})$, and the upper bound for it is given by $(a/3)(E[T_p] + \hat{D})$ with the $\hat{D}$ defined as in (24). Obviously, the difference between these bounds decreases as $E[T_p] \rightarrow \infty$, or $v_{\text{min}} \rightarrow v_{\text{max}}$. Additionally, in Fig. 1 we plotted $f_X$ after evaluating it numerically. Observe that as $\sigma$ decreases the probability of the mobile terminal to be located at the center becomes smaller, but the deviation is not substantial.
Example 3 As an example of a scheme where distribution of $X_d$ is dependent on $X_s$, consider a partitioning of the region $R = [0,a]$ into $M$ subregions $R_i = [a_i,a_{i+1})$, $i = 1,\ldots,M$ such that $a_{i+1} > a_i$ with $a_1 = 0$, $a_{M+1} = a$, and let the stochastic density kernel be defined by

$$f_{X_d|X_s}(x_d|x_s) = \begin{cases} \frac{A_{i,j}}{a_{j+1} - a_i}, & \text{if } x_s \in R_i, \text{ and } x_d \in R_j, i,j = 1,\ldots,M \\ 0, & \text{otherwise} \end{cases}$$

(25)

where $A_{i,j}$ denote the probability of selecting $X_d$ uniformly in subregion $R_j$ given that $X_s$ is located in subregion $R_i$.

Now it can be shown that if the DTMC $\{X_d,k \in \mathbb{N}\}$ with the state space $\{R_1,\ldots,R_M\}$ and the $M \times M$ transition probability matrix $A = [A_{i,j}]$ is irreducible and aperiodic, then the stationary pdf of the destination points is given by

$$f_{X_s}(x_d) = \begin{cases} \frac{\pi_A}{a_{j+1} - a_i}, & \text{if } x_d \in R_i, i = 1,\ldots,M \\ 0, & \text{otherwise} \end{cases}$$

(26)

where $\pi_A = [\pi_A_1,\ldots,\pi_A_M]$ is the solution of the linear system $\pi_A A = \pi_A$, $|\pi_A|_1 = 1$.

As an application of this scenario, we focused on the one-dimensional version of the random direction model described in [18]. In this model, nodes are restricted to move between the destinations that are located at the $\epsilon$ neighborhood of boundaries. After reaching the destination, mobile pauses for a specified amount of time, and travels to a new destination, which is also located at the $\epsilon$ neighborhood of boundaries. Similar to the random waypoint mobility model, for each movement epoch, $V$ is selected independently from $|X_s - X_d|$.

Now, in order to capture this model with the $f_{X_d|X_s}$ defined by (25) on a one-dimensional topology, we have to set $M = 3$, and divide $R$ into subregions $R_1 = [0,\epsilon)$, $R_2 = [\epsilon,a-\epsilon)$, and $R_3 = [a-\epsilon,a)$. Since, the stochastic matrix $A$ must be irreducible and aperiodic, we define it by

$$A = \begin{bmatrix} R_1 & 0 & \epsilon & 1 - \epsilon \\ R_2 & 1/2 & 0 & 1/2 \\ R_3 & 1 - \epsilon & \epsilon & 0 \end{bmatrix}$$

(27)

where $0 < \epsilon < 1$. Obviously, since $\epsilon$ cannot be equal to zero, mobile terminals may select destination points located at $R_2$. However, as $\epsilon \rightarrow 0$, the possibility of this case diminishes, and we reach to desired scenario.

Hence, after obtaining the $f_{X_s}(x_d)$ from (26) for a nonzero $\epsilon$, applying Theorem 1, and finally, by taking the limit of the result as $\epsilon \rightarrow 0$, we derived the following for the long-run location distribution of this mobility model:

$$f_X(x) = \begin{cases} \frac{E[T_p]/(2\epsilon) + E[1/V]\epsilon}{E[T_p] + D}, & \text{if } x \in [0,\epsilon) \\ \frac{E[1/V]}{E[T_p] + D}, & \text{if } x \in [\epsilon,a-\epsilon) \\ \frac{E[T_p]/(2\epsilon) + E[1/V](a-x)/\epsilon}{E[T_p] + D}, & \text{if } x \in [a-\epsilon,a) \end{cases}$$

(28)

where $D = E[1/V](a-\epsilon)$, and $E[T_p]$ is the expected pause time spent at the destinations. Notice that, $f_X(x)$ converges to $\frac{1}{a}$ as $E[T_p] \rightarrow 0$ and $\epsilon \rightarrow 0$, which is consistent with the results given in [16] for the properties of the random direction models.
5. CONCLUSIONS

For ad hoc wireless networks, we proposed a generalized random mobility model capable of capturing several scenarios, including hotspots and displacement dependent speed distributions. The analytical framework we presented for the long-run analysis of this generic mobility model over one-dimensional mobility terrains provided closed form expressions for the long-run location and speed distributions. Our example scenarios verify the usefulness of our analytical framework for the mobility analysis and yield significant insights into how realistic mobility scenarios can be brought into the capacity analysis of wireless ad hoc networks. Future work will consider the extension of these results to two-dimensional regions.

REFERENCES