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Cooperative and Non-Cooperative Control in IEEE 802.11 WLANs

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Abstract. Numerous techniques for optimal performance of an IEEE 802.11 WLAN have been investigated. These techniques make use of either power control or PHY (physical layer) rate control or both to achieve maximum throughput levels for the network at minimum power consumption. However most of these techniques are non-cooperative by definition. Here, we analyse cooperative and non-cooperative rate and power control in an 802.11 WLAN that uses the Distributed Coordination Function (DCF). We formulate a payoff function comprising of the throughput and costs related to power consumption. The payoff function is optimized and closed form expressions for the optimal PHY rate are obtained. In the cooperative approach we seek to obtain the optimal rates under two different scenarios – *max-min fair* rate and *global multirate* allocation. In the non-cooperative approach we consider only *multirate* allocation. We consider optimization problems for both finite number of nodes n and for the limit $n \to \infty$ and obtain explicit expressions for the optimal PHY rate. Single node throughputs corresponding to the optimal PHY rates are numerically studied and it is observed that network performance in the cooperative scenario is superior to that in the non-cooperative scenario.
Keywords: IEEE 802.11, PHY rate, power control, WLAN, bandwidth sharing, GPS queue

1 Introduction

We analyse *cooperative* and *non-cooperative* power and rate control in an IEEE 802.11 WLAN environment, based on an explicit throughput expression [1] validated in [2] using ns2 simulations. We consider optimizing either the achieved aggregate network throughput (cooperative approach) or an individual node's achieved throughput (in a non-cooperative setup) by adaptively selecting one of the available PHY data rates. In the formulation of the optimization problems we further take into account a cost for power consumption. We formulate a payoff function W_n for n users which comprises a utility part representing the throughput and a cost part related to power consumption. In the cooperative case the global payoff comprising the total network throughput and total transmission power costs of all mobile nodes is maximized. In the non-cooperative game case, each player seeks to maximize its own payoff. The corresponding solution concept is then the Nash equilibrium. In the cooperative control analysis, we seek to maximize the payoff with two different approaches: (i) obtaining an optimal fair assignment of PHY rates, with a *max-min* flavor, to all nodes irrespective of their channel conditions (of course, this means that a channel with bad conditions will have to use larger power); (ii) *global multirate* approach, we allow each node to use a different PHY rate and seek to obtain the optimal rate for each node. In this case, the optimal PHY rate used by each node will depend on its channel conditions. We also present a queueing model that allows us to study the dynamic behavior aspects and expected transfer time and steady state probabilities for data transfers. Our main contribution is in obtaining explicit expressions (or

RTS/CTS frame exchange before the data-ack frame transmission, we assume throughout our discussion that $T_o \geq T_c$. In our analysis in the following sections, we will consider optimization problems for both finite n and for the limit $n \to \infty$. For handling the latter case, we identify here the asymptotic aggregate throughput as $n \to \infty$ (this derivation can be found in [1] for the special symmetric case where all L_i's and C_i's are equal). An appealing feature of the asymptotic case is the *explicit* expression for β.

Asymptotic throughput: In our discussion we use asymptotic throughput in the following two contexts:
(i) In the max-min fair (MMF) case where we assign the same PHY rate to all mobile nodes, we consider all nodes to be symmetric, i.e., they all use the same PHY rate C (they still may have different channel conditions). In this case, if first $K \to \infty$ [1] and then $n \to \infty$, the global throughput is given by Sec. VII.C in [1] as:

$$\tau(C) = \frac{L\left(1-\frac{1}{p}\right)}{\frac{1}{\ln\left(\frac{p}{p-1}\right)} + \left(1-\frac{1}{p}\right)\left(T_o - T_c + \left(\frac{L}{C}\right)\right) + \frac{T_c}{p\ln\left(\frac{p}{p-1}\right)}} \tag{2}$$

where p is the exponential back-off multiplier, i.e., if b_k is the mean back-off duration (in slots) at the kth attempt for a frame then $b_k = p^k b_0$. According to the IEEE 802.11 specifications $p = 2$.
(ii) In the case where we consider global multirate PHY rate assignment to all nodes, i.e., each node uses one of the c distinct available values of the parameters (C_i, L_i) with $(C_i, L_i) \in \{(C_1, L_1), ..., (C_c, L_c)\}$, we derive here the corresponding asymptotic throughput. Assume that there are m_i nodes using parameters (C_i, L_i). Denote by $\alpha_i(n) = m_i/n$ the fraction of the nodes using (C_i, L_i) among all nodes in the cell. Then the throughput of all nodes using (C_i, L_i) is given by

$$\theta(\alpha_i(n)) = \frac{m_i \beta e^{-n\beta} L_i}{1 + n\beta e^{-n\beta}\left(T_o - T_c + \sum_{i=1}^{c} \frac{\alpha_i(n) L_i}{C_i}\right) + \left(1 - e^{-n\beta}\right) T_c} \tag{3}$$

where we use the Binomial to Poisson approximated version of the throughput expression for the asymptotic case mentioned in Section VII.C of [1]. It is assumed that $\alpha_i(n)$ converges to a limit α_i which is a probability measure. Note that the attempt rate $\beta = \beta(n)$ and the collision probability γ as defined in [1] are not functions of L_i nor C_i. Now, first taking $K \to \infty$ [1] and then taking the limit $n \to \infty$, it can be observed that $\lim_{n\to\infty} n\beta(n) \uparrow \ln\left(\frac{p}{p-1}\right)$ (see Theorem VII.2 in [1]). Combining this result with Equation (3) we get as $n \to \infty$ the following expression for the aggregate throughput of all nodes using (C_i, L_i):

$$\tau(\alpha_i) = \frac{\alpha_i L_i \left(1-\frac{1}{p}\right)}{\frac{1}{\ln\left(\frac{p}{p-1}\right)} + \left(1-\frac{1}{p}\right)\left(T_o - T_c + \sum_{i=1}^{c}\left(\frac{\alpha_i L_i}{C_i}\right)\right) + \frac{T_c}{p\ln\left(\frac{p}{p-1}\right)}} \tag{4}$$

Denote $E_\alpha[L/C] = \sum_{i=1}^{c} \frac{\alpha_i L_i}{C_i}$ and $E_\alpha[L] = \sum_{i=1}^{c} \alpha_i L_i$. Then it follows from Equation (4) that the asymptotic global throughput is given by

$$\tau(\alpha) = \frac{E_\alpha[L]\left(1-\frac{1}{p}\right)}{\frac{1}{\ln\left(\frac{p}{p-1}\right)} + \left(1-\frac{1}{p}\right)\left(T_o - T_c + E_\alpha[L/C]\right) + \frac{T_c}{p\ln\left(\frac{p}{p-1}\right)}} \tag{5}$$

3 Defining the payoff function

In an efficiently working WLAN, the goal of the mobile nodes is to achieve maximum throughput levels with minimized power consumption costs. In a cooperative scenario, the nodes should cooperate to achieve maximum overall network throughput at minimum combined power consumption. If each node uses the highest available PHY rate, which is say common for all nodes, it may not be the best strategy to achieve the most efficient overall network performance. The reason being that under the given channel conditions, a node may be unnecessarily consuming more power by transmitting at the highest available rate if transmitting at a lower PHY rate does not degrade the combined network throughput. Based on

W_n is concave with respect to C (see [6] for proof) and thus has a unique maximizer C^*. In particular, we have the linear and the exponential costs as: $Q_i^{lin}(C) = E[a_i]C, \quad Q_i^{exp}(C) = E[z_i]\Big(e^{\psi C} - 1\Big)$. Denote $u^{lin} = \sum_{i=1}^n \zeta_i a_i$ and $u^{exp} = \sum_{i=1}^n \zeta_i z_i$ and set

$$q_1 = n\beta(1-\beta)^{n-1}L, \quad q_2 = 1 + n\beta(1-\beta)^{n-1}(T_o - T_c) + (1-(1-\beta)^n)\,T_c \tag{8}$$

$$\text{Then} \quad W_n^{lin}(C) = \frac{q_1}{q_2 + q_1/C} - E[u^{lin}]C \qquad W_n^{exp}(C) = \frac{q_1}{q_2 + q_1/C} - E[u^{exp}]\Big(e^{\psi C} - 1\Big) \tag{9}$$

By differentiating the payoff w.r.t. C and equating the derivative to zero, we get the following results:

(i) In the linear case, the unique positive solution of $\frac{dW_n^{lin}(C)}{dC} = 0$ is given by

$$C^* = \frac{q_1}{q_2}\left(\frac{1}{\sqrt{E[u^{lin}]}} - 1\right) \tag{10}$$

provided that $0 < E[u^{lin}] < 1$. If $E[u^{lin}] \geq 1$ then there is no positive solution.

(ii) In the exponential case the unique positive solution of $\frac{dW_n^{exp}(C)}{dC} = 0$ is given by

$$C^* = \frac{2}{\psi} LambertW\left(\frac{1}{2}\frac{q_1}{q_2}\sqrt{\frac{\psi}{E[u^{exp}]}}\exp\left(\frac{1}{2}\frac{q_1}{q_2}\psi\right)\right) - \frac{q_1}{q_2} \tag{11}$$

See [6] for the definition of *LambertW* function. In either the linear or the exponential case, if C^* lies within $\underline{\mathcal{C}}$ then it is the unique globally optimal rate assignment solution for problem (7). If not, then the optimal solution is obtained on one of the two boundary points of $\underline{\mathcal{C}}$. We defer the discussion on the numerical computations of C^* to Section 6.

The asymptotic case: We present here the asymptotic behaviour for large number of nodes. The optimization is based on the expression for the asymptotic throughput given by Equation (2). Here we assume that a_i, z_i and ζ_i have the same distribution for all mobiles. Consider the following problem:

$$\text{Find } C^* \text{ that maximizes } W(C) := \tau(C) - \zeta Q(C) \tag{12}$$

where $Q(C) = E[a]C$ for the linear cost and $Q(C) = E[z]\Big(e^{\psi C} - 1\Big)$ for the exponential one. $W(C)$ turns out to be concave in C (see [6] for proof) and therefore it has a unique maximizer. Writing $W(C)$ for the linear and exponential case as

$$W^{lin}(C) = \frac{q_1}{q_2 + q_1/C} - E[a]C \quad \text{and} \quad W^{exp}(C) = \frac{q_1}{q_2 + q_1/C} - E[z]\Big(e^{\psi C} - 1\Big)$$

$$\text{where} \quad q_1 = L\left(1 - \frac{1}{p}\right), \qquad q_2 = \frac{1 + T_c/p}{\ln\left(\frac{p}{p-1}\right)} + \left(1 - \frac{1}{p}\right)(T_o - T_c). \tag{13}$$

Then the optimal C is obtained by differentiating $W^{lin}(C)$ and $W^{exp}(C)$ and equating them to zero, which gives the following unique positive solution for the linear and exponential cases, respectively:

$$C^*_{lin} = \frac{q_1}{q_2}\left(\frac{1}{\sqrt{\zeta E[a]}} - 1\right), \quad C^*_{exp} = \frac{2}{\psi} LambertW\left(\frac{1}{2}\frac{q_1}{q_2}\sqrt{\frac{\psi}{\zeta E[z]}}\exp\left(\frac{1}{2}\frac{q_1}{q_2}\psi\right)\right) - \frac{q_1}{q_2}.$$

If C^* lies within $\underline{\mathcal{C}}$ then it is the unique globally optimal rate assignment solution for problem (12). If not then the optimal solution is obtained on one of the two boundary points of $\underline{\mathcal{C}}$. Also note that C^* here has the same form as in the finite n case but with different q_1 and q_2.

4.2 The dynamic case

So far we have considered a fixed number of nodes in the system. In this section we consider a dynamic setting. Let nodes arrive in a WLAN system according to an independent Poisson process with rate λ. The nth node is assumed to have a service requirement σ_n where σ_n are i.i.d. generally distributed.

With these definitions and by differentiating W_n^{lin} we get

$$\frac{\partial W_n^{lin}}{\partial C_i} = \frac{nq_1^2}{(nq_2^i C_i + q_1)^2} - \zeta_i E[a_i] = \frac{q_1^2}{\left(q_2 + \frac{q_1}{n\hat{C}}\right)^2 nC_i^2} - \zeta_i E[a_i] \tag{18}$$

and similarly by differentiating W_n^{exp} we get

$$\frac{\partial W_n^{exp}}{\partial C_i} = \frac{nq_1^2}{(nq_2^i C_i + q_1)^2} - \psi\zeta_i E[z_i]e^{\psi C_i} = \frac{q_1^2}{\left(q_2 + \frac{q_1}{n\hat{C}}\right)^2 nC_i^2} - \psi\zeta_i E[z_i]e^{\psi C_i} \tag{19}$$

Now by equating the derivatives in Equations (18) and (19) to zero, we obtain:

(i) In the linear case, we get from Equation (18)

$$C_i = \sqrt{\frac{H(\hat{C})}{\zeta_i E[a_i]}} \text{ and also, } \hat{C} = \left(\sum_{i=1}^{n} \sqrt{\frac{\zeta_i E[a_i]}{H(\hat{C})}}\right)^{-1} = \frac{\sqrt{H(\hat{C})}}{Y}, \quad \text{where } Y = \sum_{i=1}^{n} \sqrt{\zeta_i E[a_i]} \tag{20}$$

which implies that the solution $\hat{C}^*$ is given by $\hat{C}^* = \frac{1}{n}\frac{q_1}{q_2}\left(\frac{\sqrt{n}}{Y} - 1\right)$. Substituting the solution of this equation in (20) gives the C_i^*'s.

(ii) In the exponential case, we get from Equation (19)

$$C_i = \frac{2}{\psi} LambertW\left(\frac{1}{2}\sqrt{\frac{\psi H(\hat{C})}{\zeta_i E[z_i]}}\right) \tag{21}$$

Therefore, using the definition of $\hat{C}$, $\hat{C}^*$ is the solution of $\hat{C} = \frac{2}{\psi \sum_{i=1}^{n} \left[LambertW\left(\frac{1}{2}\sqrt{\frac{\psi H(\hat{C})}{\zeta_i E[z_i]}}\right)\right]^{-1}}$

which yields the C_i^*'s through (21).

The above solutions are globally optimal provided they are within the range $\underline{\mathcal{C}}$. We defer the discussion on the numerical computations of C_i^*'s to Section 6.

Large number of nodes: To model the case of a large number of users we shall use a fluid approximation in which there are (non-countably) infinite number of users. We introduce R population classes of mobiles. The parameter z in the exponential cost function will be the same for all mobiles of the same type $r, r = 1, 2, ..., R$ so that mobiles belonging to a given class r have the same channel conditions. We shall thus use the notation $z^{(r)}$ to indicate this dependence. We shall use similarly the notation $a^{(r)}$ for the coefficient appearing in the linear cost. In short, mobiles with the same value of $(a^{(r)}, \zeta^{(r)})$ (in the linear case) or $(z^{(r)}, \zeta^{(r)})$ (in the exponential case) are said to belong to the same class of mobiles having identical channel conditions. We define for each r the vector $\mathbf{x}^{(r)} = (x_1^{(r)}, ..., x_c^{(r)})$ to be the amount of r-type mobiles that use each of the rates $C_1, ..., C_c$. Define $\mathbf{x} = (\mathbf{x}^{(1)}, ..., \mathbf{x}^{(R)})$ to be a multistrategy for all mobiles. With some abuse of notation, let $x_i := \sum_{r=1}^{R} x_i^{(r)}$ denote the global amount of mobiles that use the rate C_i under $\mathbf{x}$. Denote $\overline{\nu}$ to be the total amount of users. Then $\overline{\nu} = \sum_{i=1}^{c} x_i$. Define $\alpha_i(\mathbf{x}) = x_i/\overline{\nu}$. It follows from Equation (5) that

$$\tau(\alpha(\mathbf{x})) = \frac{E_{\alpha(\mathbf{x})}[1]q_1}{q_2 + q_1 E_{\alpha(\mathbf{x})}[1/C]} = \frac{\overline{\nu}q_1}{\overline{\nu}q_2 + q_1 \sum_{i=1}^{c} x_i C_i^{-1}}$$

where q_1 and q_2 are given by Equation (13). To simplify, we shall denote $\tau(\mathbf{x}) = \tau(\alpha(\mathbf{x}))$. Define $Q_i^{(r)}(x_i^{(r)}) = a^{(r)}C_i$ for the linear cost and $Q_i^{(r)}(x_i^{(r)}) = z^{(r)}\left(e^{\psi C_i} - 1\right)$ for the exponential cost. Then our problem of maximizing the payoff function turns out to be a non-linear optimization problem defined by:

$$\max_{\mathbf{x}} W(\mathbf{x}) \quad \text{where } W(\mathbf{x}) := \tau(\mathbf{x}) - \sum_{r=1}^{R} \zeta^{(r)} \sum_{i=1}^{c} x_i^{(r)} Q_i^{(r)}(x_i^{(r)})$$

$$= \frac{\overline{\nu}q_1}{\overline{\nu}q_2 + q_1 \sum_{i=1}^{c}\left(\sum_{r=1}^{R} x_i^{(r)}\right) C_i^{-1}} - \sum_{r=1}^{R} \zeta^{(r)} \sum_{i=1}^{c} x_i^{(r)} Q_i^{(r)}(x_i^{(r)})$$

$$\text{subject to} \qquad \sum_{i=1}^{c} x_i^{(r)} = g_r, \forall r, \quad x_i^{(r)} \geq 0, \forall i, r$$

the single node throughput in the cooperative global multirate case (Figure 5) is around 11% higher than in the non-cooperative multirate case (Figure 6). In fact with increasing n the single node throughput percentage gain in the cooperative global multirate scenario over the non-cooperative multirate scenario goes from 11% for $n = 2$ to up to more than 200% for $n = 10$. When the cost associated with power consumption is *exponential* the single node throughput percentage gain in the cooperative allocation (Figure 11) over the non-cooperative allocation (Figure 12) varies from around 12% for $n = 2$ to up to 100% for $n = 10$. We also observe that the cooperative max-min fair scheme (Figure 4,10) performs almost equally well as the cooperative global multirate scheme (Figure 5,11). These observations clearly illustrate that cooperative rate allocation strategy results in higher single node throughputs and hence higher total network throughput as against non-cooperative strategy. Our analysis thus confirms the results obtained by Tan et al. in [4]. Indeed the DCF protocol under non-cooperative setting is not efficient.

7 Conclusion and future work

In this paper, we have analysed cooperative and non-cooperative rate and power control in an IEEE 802.11 WLAN, by optimizing a payoff function that comprises of the throughput and costs related to power consumption. It is observed through numerical studies that cooperative control is more efficient than non-cooperative control. With a linear cost approximation, the single node throughput in the cooperative approach is observed to be 11% to 200% more than in the non-cooperative game approach. The improvement varies from 12% to 100% in the exponential cost approximation case. Thus a first glimpse of cooperative and non-cooperative control in an 802.11 WLAN by our analysis shows that the currently used mandatory DCF protocol in 802.11 does not perform with the highest efficiency in a non-cooperative setting. Our future work will include designing an efficient cooperative rate and power control algorithm based on the analysis illustrated in this paper.

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