point-to-multipoint optical channel. Performing multicasting at optical layer using light-trees eliminates the need for optical-electronic-optical conversion, but in the absence of wavelength converters such a WDM multicast network is still subject to optical layer constraints: (i) wavelength continuity constraint – a lightpath must use the same wavelength on all the links along its path from source to destination node; (ii) distinct wavelength constraint – all lightpaths using the same link must be allocated distinct wavelengths. Thus, the need for adequate mathematical models that would take into consideration the particularities of wavelength routed networks with multicast calls arises.

The problem of performance analysis of wavelength routed networks with multicast calls has been considered in several studies. In [3] an approximate analytical method to compute efficiently the call-blocking probabilities in wavelength routed networks with multiple classes of both unicast and multicast calls is presented. In this paper the behavior of multicast calls is modeled by a state-dependent Poisson arrival process and in the end approximated by unicast calls. The paper [4] addresses the placement of multicast nodes in wavelength routed all-optical networks. An analytical model for the approximate blocking probability in multicast networks is developed.

In this paper we develop a mathematical model for a linear fragment of a wavelength routed network with unicast and multicast calls and extend the approximate method presented in [5] to the considered model.

In Section 2 we develop mathematical model for a multi-hop linear fragment of a wavelength routed network with unicast and multicast calls. In Sections 3 and 4 we extend the flow modification and decomposition approaches [5, 7] to the considered model in order to approximately calculate blocking probabilities. An efficient recursive algorithm for calculating normalized constant of the equilibrium distribution is presented. In Section 5 we validate our results through numerical solution and show some effects of multicast calls. We conclude the paper in Section 6.

2. MATHEMATICAL MODEL OF A LINEAR FRAGMENT

We consider a multi-hop linear fragment of a wavelength routed network with unicast and multicast calls that consists of several hops representing optical fiber links between nodes. The network fragment provides services for end users by accepting calls in form of setting up unicast or multicast transmission on a circuit-switched basis through a sequence of hops called a route.

We make the following assumptions: (1) network topology is a linear path, $\mathsf{J} := \{ 1,...,J \}$ - the set of all hops; (2) no wavelength converters are installed at the network nodes; (3) each hop supports W wavelength channels (wavelengths), $\mathsf{W} := \{ 1,...,W \}$ - the set of all wavelengths in a hop; (4) there are K classes of calls; each class $k \in \mathsf{K} := \{ 1,...,K \}$ call represents a unicast point-to-point connection or a point-to-multipoint multicast connection; let K_u, K_m - sets of unicast and multicast classes of calls respectively, $\mathsf{K} = \mathsf{K}_u \cup \mathsf{K}_m, \mathsf{K}_u \cap \mathsf{K}_m = \varnothing$; (5) associated with every class k call is

Let $\mathbf{E}_{wk}$ be the state in which only one class k connection on wavelength w is set up,
$\mathsf{F}_k(\mathbf{X}) := \{ w \in \mathsf{W} : \mathbf{X} + \mathbf{E}_{wk} \in \Omega \}$ and $f_k(\mathbf{X}) := |\mathsf{F}_k(\mathbf{X})| \in \{0,1,...,W\}$ for the set and the number of free wavelengths for class k calls at state $\mathbf{X}$ respectively. Let $x_{wk}(t) \in \{0,1\}$ be the number of class k connections set up on wavelength w at some instant of time $t \geq 0$. The behavior of the system in equilibrium is captured by the multidimensional Markov process $\mathbf{X}(t) = \left(x_{wk}(t) \right)_{w \in \mathsf{W}, k \in \mathsf{K},}$, $t \geq 0$ with equilibrium distribution $p(\mathbf{X})$, $\mathbf{X} \in \Omega$ and matrix of transition rates $\mathbf{A} := \left(a(\mathbf{X}, \mathbf{Y}) \right)_{\mathbf{X}, \mathbf{Y} \in \Omega}$. It can be seen that if $W > 1$ and $J > 1$ then the Kolmogorov criterion [6] is not satisfied. Thus, the Markov process $\mathbf{X}(t)$ is not time reversible. Let

$$\mathsf{B}_k := \begin{cases} \{ \mathbf{X} \in \Omega : \mathbf{X} + \mathbf{E}_{wk} \notin \Omega, \forall w \in \mathsf{W} \}, & k \in \mathsf{K}_u \\ \{ \mathbf{X} \in \Omega : \mathbf{X} + \mathbf{E}_{wk} \notin \Omega, \forall w \in \mathsf{W}; x_{\bullet k} = 0 \}, & k \in \mathsf{K}_m \end{cases} \tag{2}$$

be the set of states in which it is not possible to set up a new class k call. Notice that multicast class k calls may be blocked only if there is no corresponding multicast connection set up. Otherwise, a call is “attached” to the existing connection and do not experience blocking. Thus, it is important to distinguish the connection set-up and time blocking probabilities of multicast calls[3]. The probability $B_k := P\{ \mathbf{X}(t) \in \mathsf{B}_k \}$, $k \in \mathsf{K}$ of the process being in set B_k is the time blocking probability of class k calls.

3. FLOW MODIFICATION APPROXIMATE ANALYSIS

The nature and dimensions of the matrix of transition rates $\mathbf{A}$ made it impossible for us to solve equilibrium equations. In order to obtain a closed-form solution we will extend the method presented in [5] to the considered model. The point is that the functioning of a linear fragment in equilibrium can be approximated by a multidimensional Markov process. It can be seen that by modification of several transition rates of the considered process $\mathbf{X}(t)$ we

---

[3] In [3] Engset-like traffic model is used to describe multicast flows. Associated with each multicast connection is a source which may be in “on” or “off” state and there are no calls generated when the corresponding multicast connection is on. Therefore, only connection set-up blocking probabilities may be analyzed. Since setting up unicast or multicast connections over the same route in the linear fragment require allocation of same resources then connection set-up blocking probabilities for unicast and multicast connections are also the same. In this paper we use another approach to model multicast calls.

Define matrix $\mathbf{S} := (s_{kl})_{k,l \in \mathrm{K}}$, where s_{kl} is indicator of whether routes of class k and l connections intersect or not. Without loss of generality we may assume that classes are ordered so that $h_{\bullet 1} \le ... \le h_{\bullet K}$. Then, using the definitions

$$\mathrm{N}(k, \vec{w}) := \left\{ (n_1, ..., n_k) : 0 \le n_k \le w_k - \sum_{m=i+1}^{k} n_m s_{im}, i = \overline{1, k} \right\}$$

and

$$g(k, \vec{w}) := \sum_{\vec{n} \in \mathrm{N}(k, \vec{w})} \prod_{i=1}^{k} \frac{\rho_i^{n_i}}{n_i!}$$

we may see that $\mathrm{N} = \mathrm{N}(K, (W, ..., W))$ and $G = g(K, (W, ..., W))$. It can be shown that the function $g(k, \vec{w})$ may be calculated recursively as follows:

$$g(k, \vec{w}) = \sum_{l=0}^{w_k} g(k-1, \vec{w} - l\vec{s}_k) \frac{\rho_k^l}{l!} \quad , \quad k = \overline{1, K} \quad , \quad \vec{w} \in \{1, ..., W\}^k \quad ,$$
(6)

where $g(0, \vec{0}) = 1$.

Formula (6) defines an effective recursive method to calculate normalized constant (5). These results enable us to calculate approximate equilibrium distribution for a linear fragment and thus any other of its stochastic characteristics. We will approximate the blocking probability of the linear fragment as

$$B_k \approx \tilde{B}_k := P\left\{ \tilde{\mathbf{X}}(t) \in \mathrm{B}_k \right\}$$
(7)

Approximate blocking probabilities are given by

$$\tilde{B}_k = \sum_{\vec{n} \in \mathrm{N}} \beta_k(\vec{n}) \tilde{p}(\vec{n}) \quad ,$$
(8)

where $\beta_k(\vec{n}) := P\{ \mathbf{X} \in \mathrm{B}_k \mid \mathbf{X} \in \Omega(\vec{n}) \}$ is the proportion of blocking states $\mathbf{X}$ among all states with $\vec{n}$ connections set up. Expressions for calculating $\beta_k(\vec{n})$ are not shown here for brevity.

4. DECOMPOSITION ANALYSIS OF LONG LINEAR FRAGMENTS

(9) $$\lambda_1^{(2)} = \lambda_3, \lambda_2^{(2)} = \lambda_4, \lambda_3^{(2)} = \lambda_5(1-\tilde{L}_5^{(2)}), \lambda_4^{(2)} = \lambda_6(1-\tilde{L}_6^{(2)})$$

(10)

for the first segment and for the second one respectively. Here, $\tilde{L}_k^{(s)}$ is the estimate of probability that a class k call that span both segments is blocked despite there exist enough resources on the segment s. The rationale of such modification of the arrival rates is that any route of a segment offered not only calls that originate and end on that very segment but also calls that go through the other segment as well. We will approximate the blocking probabilities B_k, $k \in \mathrm{K}$ of the original fragment as follows:

$$B_1 \approx \tilde{B}_1^{(1)}, B_2 \approx \tilde{B}_2^{(1)}, B_3 \approx \tilde{B}_1^{(2)}, B_4 \approx \tilde{B}_2^{(2)}, B_5 \approx \tilde{L}_5, B_6 \approx \tilde{L}_6 .$$

(11)

where $\tilde{L}_k, \tilde{L}_k^{(s)}, k = 5,6$ are given by

$$\tilde{L}_k = \sum_{f_1=0}^{W} \sum_{f_2=0}^{W} \frac{\binom{W-f_1}{f_2}}{\binom{W}{f_2}} R_{k_1}^{(1)}(f_1) R_{k_2}^{(2)}(f_2) \quad , \quad \tilde{L}_k^{(s)} = \sum_{f_1=0}^{W} \sum_{f_2=0}^{W} \frac{\binom{W-f_1}{f_2}}{\binom{W}{f_2}} R_{k_1}^{(1)}(f_1) R_{k_2}^{(2)}(f_2) u(f_s) .$$

(12)

Here, k_s are the classes of calls on segments $s = 1,2$ that correspond to the class k call of the original linear fragment, $u(x) = \begin{cases} 1, & x > 0 \\ 0, & x \le 0 \end{cases}$ and $R_k^{(s)}(f) = P\left\{\mathbf{X} \in \Omega^{(s)} : f(\mathbf{X}) = f\right\}$, $k \in \mathrm{K}^{(s)}$ is the probability that there are f free lightpaths for class k calls, $k \in \mathrm{K}^{(s)}$.

Hence, the algorithm for calculation blocking probabilities of various calls offered to the four-hop linear fragment may be implemented as follows.

Algorithm. A four-hop linear fragment depicted in Figure 2 (a) is decomposed into two two-hop segments as shown in Figure 2 (b). Let ε define the precision of the results.

1. let $i \leftarrow 0$, $\tilde{L}_k \leftarrow 0$, $\tilde{L}_k^{(s)} \leftarrow 0$, $k = 5,6$, $s = 1,2$;
2. let $i \leftarrow i+1$;
3. calculate $\lambda_k^{(s)(i)}$, $k \in \mathrm{K}^{(s)}$, $s = 1,2$ using (9), (10);
4. calculate $\tilde{B}_k^{(s)}$, $k \in \mathrm{K}^{(s)}$, $s = 1,2$ using (7);
5. calculate $\tilde{L}_k^{(s)(i)}$, $\tilde{L}_k$, $k = 5,6$, $s = 1,2$ using (12);

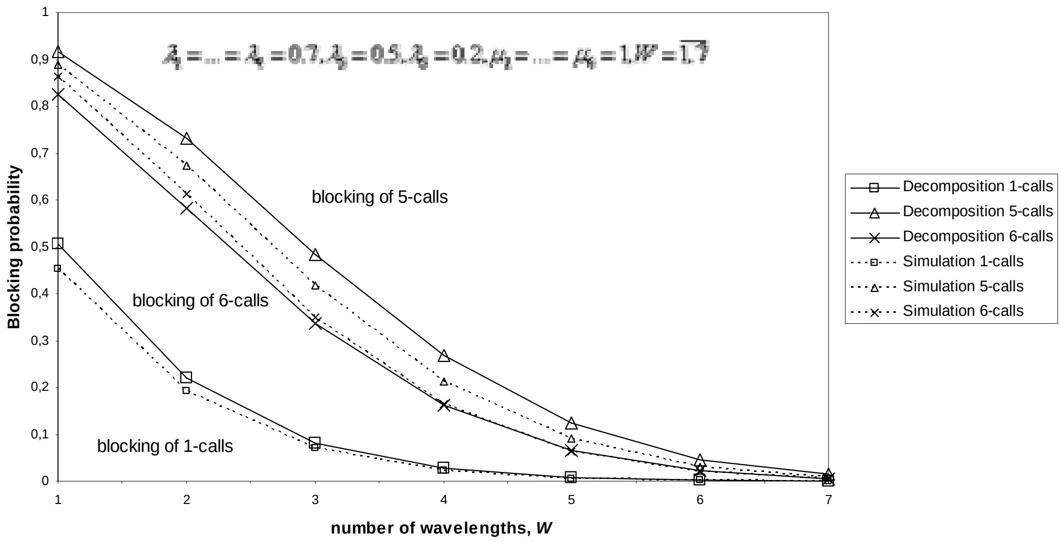

Figure 4. Blockings of the 4-hop linear fragment vs. number of wavelengths, W.

In Figure 4 we plot blocking probability of class 1, class 5 and multicast class 6 calls. As expected, the blocking probabilities decrease as we increase number of wavelengths in links. This type of graph can be used to determine the minimum number of wavelengths needed to guarantee the needed quality of service. The figures show good accordance of the approximate results with the results obtained by simulation or numerical solution of equilibrium equations.

6. CONCLUDING REMARKS

We have presented a mathematical model for a multi-hop linear fragment of a wavelength routed network characterized by the absence of wavelength converters, fixed routing and random wavelength assignment schemes used. The network fragment provides services for end users by accepting calls in form of setting up unicast or multicast transmission on a circuit-switched basis through a sequence of hops called a route. In order to model multicast connections we have used approach proposed in [2]. We presented an effective method to approximately calculate blocking probabilities for not very long linear fragments. We have also showed how the decomposition approach (see [5]) may be applied to wavelength routed networks with multicast calls for longer linear fragments.

The results of the paper may be applied for performance analysis of multicast enabled wavelength routed WDM all-optical networks.

REFERENCES

1. Siva Ram Murthy C., Gurusamy M. WDM Optical Networks: concept, design, and algorithms, *Prentice Hall PTR*, 2001.
2. Rykov V., Samouylov K. Product Form Solution for Stochastic Model of Dynamic Multicast Connections, In Proc. of *The Int. Conf. on Distributed Computer Communication*