



asymmetric backbones<sup>3</sup>. They assume that the ISPs connected to the same backbone provide the same quality of Internet access. The ISPs firstly make peering agreements with other ISPs, and then compete in price with the ISPs that attached to the other backbone. The authors conclude that the ISPs connected to the high quality backbone will always peer with each other. But they may or may not peer with the ISPs connected to the low quality backbone. Also, in the latter case, the ISPs of the low quality backbone will peer with each other.

The foresaid issues are analyzed from purely non-cooperative aspect by Narine Badasyan, Subhadip Chakrabarti (2004) and Narine Badasyan (2004). The authors formulate a model with two Internet backbones and six ISPs. They consider both symmetric effect and asymmetric effect in their model. The notion of pairwise stability<sup>4</sup> which was introduced by Jackson and Wolinsky (1996) is used to solve for Subgame Perfect Nash Equilibrium. The main result is that with asymmetries there is inter-backbone peering only if there is intra-backbone peering. However, it is possible to have intra-backbone peering without inter-backbone peering and this is in fact, frequently the case.

In this paper, we propose a sequential game-theoretic model where the first stage deals with the private peering decisions and the second stage with a vertically differentiated competition on the Internet access services. In contrast to aforesaid literature, we take the end users' income into account and analyze the relationship among the users' income, asymmetric quality of networks and the interconnection choice of ISPs. We find that in most cases the ISPs connected to the high quality backbone will make intra-backbone<sup>5</sup> peering with the ISPs attached to the same backbone. However, the low quality ISPs will make intra-backbone and inter-backbone peering only if they have to serve low income users. The rest of the paper is organized as follows. In section 2, we describe the assumptions underlying the model in detail. In section 3, we analyze the second stage and we determine equilibrium prices, demands and network profits. In section 4, we analyze the first stage of the game. Finally, we conclude.

## 2. THE MODEL

We begin with the assumptions underlying the model. We consider two competing Internet backbones A and B and  $n$  symmetric retail ISPs denoted by  $\eta = \{1, 2, \dots, n\}$  in an Internet market. The two backbones interconnect with each other via NAP (Network Access Point) peering or private peering. We assume that each ISP who provides Internet access for end users is connected to one of the backbones, but not both. Without loss of generality, we assume that ISPs  $1, \dots, m$  are connected to backbone A and ISPs  $m + 1, \dots, n$  are connected to backbone B. Denote that club A consists of the first  $m$  ISPs connected to backbone A and the other  $n - m$  ISPs form club B.  $n$  and  $m$  are given exogenously. We illustrate this in Figure 1.

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<sup>3</sup> Every ISP connects to one of the backbones, but not both.

<sup>4</sup> A network is pairwise stable if given the network, there is no incentive to either form links or destroy links.

<sup>5</sup> Intra-backbone peering refers to peering between ISPs connected to the same backbone, whereas inter-backbone peering refers to peering between ISPs connected to different backbones.

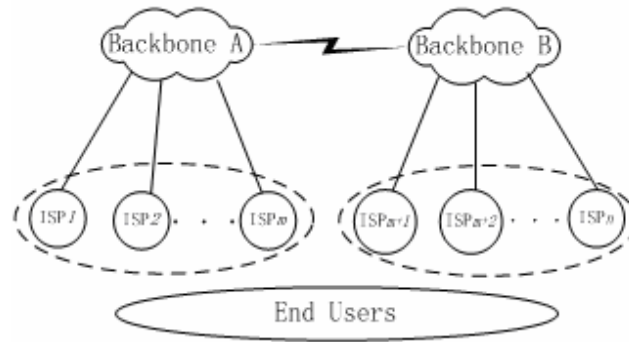


Figure 1 A stylized Internet market

Let  $\lambda_l$  be the quality of Internet access services which the club  $l(l \in \{A, B\})$  provides for the end users. We assume that the two backbones have capacity constraints. Hence the quality of a backbone can suffer from congestion. Every ISP can choose to make private peering agreements with others. Once a peering link is built between two ISPs, they use the link equally with free settlements. Intra-backbone peering reduces congestion in that backbone only and raises the quality of all ISPs connected to that backbone. Inter-backbone peering reduces congestion in both backbones and improves quality of both clubs. Let define  $n_{AA}$  (respectively  $n_{BB}$ ) denote the number of intra-backbone private peering links for backbone A (respectively for backbone B), and  $n_{AB}$  be the number of inter-backbone links.

We adopt the utility function proposed by Shaked and Sutton (1982). The end users are uniformly distributed on the interval of preference  $\theta \in [0, 1]$ . Let  $p$  be the price of per unit Internet traffic charged by ISPs. Also, we call  $U$  the utility derived by an end user characterized by the income  $\theta$  if he sends or receives one unit Internet traffic:  $U(\theta, p) = (\theta - p)\lambda$ . Let  $\lambda_0$  be the marginal utility of the income of end users who do not subscribe to any ISP. Obviously,  $U(\theta, 0) = \theta\lambda_0$ . Finally, we suppose that the ISPs support a unit cost of  $c$  to provide Internet access for end users where  $c$  is given exogenously.

### 3. ANALYSIS OF THE SECOND STAGE

Now we solve the second stage of the game model. We adopt a strong assumption that ISPs connected to the same backbone do not compete in price, but in other dimensions, like the variety of ancillary services or promotion. The price competition occurs only between the two clubs. Without loss of generality, we assume that  $0 \leq \lambda_0 \leq \lambda_B \leq \lambda_A$ . Namely the network quality of club A is higher than that of club B. An end user  $\theta$  is indifferent between subscribing to an ISP of backbone B and not subscribing at all if

$\underline{\theta}\lambda_0 = (\underline{\theta} - p_B)\lambda_B \Rightarrow \underline{\theta} = \frac{p_B\lambda_B}{\lambda_B - \lambda_0}$ . Similarly, an end user  $\bar{\theta}$  is indifferent between an ISP of backbone A

and an ISP of backbone B if  $(\bar{\theta} - p_B)\lambda_B = (\bar{\theta} - p_A)\lambda_A \Rightarrow \bar{\theta} = \frac{p_A\lambda_A - p_B\lambda_B}{\lambda_A - \lambda_B}$ .

The aggregate demands and profits of the ISPs can be written as:

$$D_A = 1 - \bar{\theta} = 1 - \frac{p_A\lambda_A - p_B\lambda_B}{\lambda_A - \lambda_B} \Rightarrow \pi_A = (p_A - c) \left( 1 - \frac{p_A\lambda_A - p_B\lambda_B}{\lambda_A - \lambda_B} \right)$$

$$D_B = \bar{\theta} - \underline{\theta} = \frac{p_A\lambda_A - p_B\lambda_B}{\lambda_A - \lambda_B} - \frac{p_B\lambda_B}{\lambda_B - \lambda_0} \Rightarrow \pi_B = (p_B - c) \left( \frac{p_A\lambda_A - p_B\lambda_B}{\lambda_A - \lambda_B} - \frac{p_B\lambda_B}{\lambda_B - \lambda_0} \right)$$

Hence the equilibrium prices, demands and profits, respectively, of group A and B can be given by:

$$p_A^* = \frac{(\lambda_A - \lambda_0)((2 + 2c)\lambda_A - (2 - c)\lambda_B)}{\lambda_A(4\lambda_A - 3\lambda_0 - \lambda_B)} \quad p_B^* = \frac{\lambda_B((2 + 3c)\lambda_A - \lambda_B) - \lambda_0((1 + c)\lambda_A - (1 - 2c)\lambda_B)}{\lambda_B(4\lambda_A - 3\lambda_0 - \lambda_B)}$$

$$D_A^* = \frac{(2 - 2c)\lambda_A - (2 - c)\lambda_0}{(4\lambda_A - 3\lambda_0 - \lambda_B)} \quad D_B^* = \frac{(\lambda_A - \lambda_0)((1 - c)\lambda_B - (1 + c)\lambda_0)}{(\lambda_B - \lambda_0)(4\lambda_A - 3\lambda_0 - \lambda_B)}$$

$$\pi_A^* = \frac{(\lambda_A - \lambda_B)((2 - c)\lambda_0 + (2c - 2)\lambda_A)^2}{\lambda_A(4\lambda_A - 3\lambda_0 - \lambda_B)^2} \quad \pi_B^* = \frac{(\lambda_A - \lambda_0)(\lambda_A - \lambda_B)((1 + c)\lambda_0 - (1 - c)\lambda_B)^2}{\lambda_B(\lambda_B - \lambda_0)(4\lambda_A - 3\lambda_0 - \lambda_B)^2}$$

For the prices, demands and profits of each group to be positive, we need certain parameter constraints given as follows:  $0 \leq \lambda_0 < \lambda_B \leq \lambda_A$  and  $c \leq 1 - 2\lambda_0 / (\lambda_B + \lambda_0) \leq 1$ .

## 4. ANALYSIS OF THE FIRST STAGE

### 4.1. The Effects of Network Differentiation and Access Quality on the Profits

We call  $\lambda_1 = \frac{\lambda_B}{\lambda_A}$  ( $0 < \lambda_1 \leq 1$ ) the ‘‘factor of network differentiation in quality’’ which denotes the

quality differentiation of network A and B. Also we call  $\lambda_2 = \frac{\lambda_A + \lambda_B}{2}$  ( $\lambda_2 > 0$ ) the ‘‘average quality of

Internet access provided by group A and B’’. Moreover, given the complexity of computations, we assume  $c = 0$ . It is reasonable that the cost of transmitting one unit Internet traffic goes near to zero for any Internet backbone. Therefore, the equilibrium profits, respectively, of group A and B can also be written as:

$$\pi_A^* = \frac{(1 - \lambda_1)(4\lambda_2 - 2\lambda_0(1 + \lambda_1))^2}{(3\lambda_0(1 + \lambda_1) + 2(\lambda_1 - 4)\lambda_2)^2} \quad \pi_B^* = \frac{(1 - \lambda_1)(\lambda_0(1 + \lambda_1) - 2\lambda_2)(\lambda_0(1 + \lambda_1) - 2\lambda_1\lambda_2)}{\lambda_1(3\lambda_0(1 + \lambda_1) + 2(\lambda_1 - 4)\lambda_2)^2}$$

We firstly consider the effect of network differentiation on the profit of high quality group A:

$$\frac{\partial \pi_A^*}{\partial \lambda_1} = -\frac{4(\lambda_0 + \lambda_0 \lambda_1 - 2\lambda_2)(3\lambda_0^2 + 6\lambda_0^2 \lambda_1 + 3\lambda_0^2 \lambda_1^2 - 6\lambda_0 \lambda_2 - 20\lambda_0 \lambda_1 \lambda_2 + 2\lambda_0 \lambda_1^2 \lambda_2 + 8\lambda_2^2 + 4\lambda_1 \lambda_2^2)}{(3\lambda_0 + 3\lambda_0 \lambda_1 - 8\lambda_2 + 2\lambda_1 \lambda_2)^3}$$

$$\frac{\partial \pi_A^*}{\partial \lambda_1} < 0 \text{ if } 0 \leq \lambda_0 < \underline{\lambda}_0 \text{ or } \bar{\lambda}_0 < \lambda_0 < \frac{2\lambda_1 \lambda_2}{1 + \lambda_1}; \text{ and } \frac{\partial \pi_A^*}{\partial \lambda_1} > 0 \text{ if } \underline{\lambda}_0 < \lambda_0 < \bar{\lambda}_0^6.$$

If the utility one can derive by not subscribing to any ISP is a bit low ( $0 \leq \lambda_0 < \underline{\lambda}_0$ ) or high ( $\bar{\lambda}_0 < \lambda_0$ ), the effect of network differentiation in quality on profit of group A is negative. The lower the differentiation is, the less profits group A obtains. In these two cases, we have  $\frac{\partial P_A^*}{\partial \lambda_1} < 0$  and  $\frac{\partial D_A^*}{\partial \lambda_1} > 0$ . So the lower the differentiation in quality is, the more intense the price competition is. Conversely, the demand for group A always rises when the network differentiation lowers. The price declining has a greater impact on the profit of group A than the demand rising does when differentiation in quality lowers. So the profit of group A lessens when the network differentiation lowers in the aforesaid cases.

If  $\underline{\lambda}_0 < \lambda_0 < \bar{\lambda}_0$  where the interval is very small, the effect of network differentiation in quality on the profit of group A is positive. The lower the differentiation is, the more profits the group A obtains.

Because of the declining marginal utility of income, one has high marginal utility of income when he has low income and vice versa. Hence, if all the consumers' income is high or low, the group A can increase its profit by strengthening the differentiation in network quality. Now consider the effect of "average quality of Internet access provided by group A and B" on the profit of high quality group A. By deriving the equilibrium profit of group A with respect to the average quality of Internet access, we obtain:

$$\frac{\partial \pi_A^*}{\partial \lambda_2} = \frac{16\lambda_0(1-\lambda_1)^2(1+\lambda_1)(\lambda_0(1+\lambda_1)-2\lambda_2)}{(3\lambda_0(1+\lambda_1)-2(4-\lambda_1)\lambda_2)^3} > 0$$

Because of  $\frac{\partial P_A^*}{\partial \lambda_2} > 0$  and  $\frac{\partial D_A^*}{\partial \lambda_2} > 0$ , we have  $\frac{\partial \pi_A^*}{\partial \lambda_2} > 0$ . The group A can both increase its market

share and raise price to increase its profit by increasing the average quality of Internet access. However, when the average quality of Internet access is very high, the quality increasing has little impact on profit of group A. We now consider the effect of network differentiation on the profit of low quality group B:

$$\frac{\partial \pi_B^*}{\partial \lambda_1} = \frac{-3\lambda_0^3(1+\lambda_1)^3 + 2\lambda_0^2(7+6\lambda_1+11\lambda_1^2+12\lambda_1^3)\lambda_2 - 4\lambda_0(4-3\lambda_1+11\lambda_1^2+6\lambda_1^3)\lambda_2^2 + 8\lambda_1^2(2+\lambda_1)\lambda_2^3}{\lambda_1^2(3\lambda_0+3\lambda_0\lambda_1-8\lambda_2+2\lambda_1\lambda_2)^3}$$

$$^6 \underline{\lambda}_0 = \frac{\lambda_2(3+10\lambda_1-\lambda_1^2) - \lambda_2\sqrt{\lambda_1^4 - 32\lambda_1^3 + 46\lambda_1^2 - 15}}{3(1+\lambda_1)^2} > 0;$$

$$\bar{\lambda}_0 = \frac{\lambda_2(3+10\lambda_1-\lambda_1^2) + \lambda_2\sqrt{\lambda_1^4 - 32\lambda_1^3 + 46\lambda_1^2 - 15}}{3(1+\lambda_1)^2} < \frac{2\lambda_1\lambda_2}{1+\lambda_1}.$$

$$\begin{cases} \frac{\partial \pi_B^*}{\partial \lambda_1} < 0 & \lambda_1^* < \lambda_1 < 1 \text{ or } (0 < \lambda_1 < \lambda_1^* \text{ and } 0 \leq \lambda_0 < \lambda_0^*) \\ \frac{\partial \pi_B^*}{\partial \lambda_1} > 0 & 0 < \lambda_1 < \lambda_1^* \text{ and } \lambda_0^* < \lambda_0 < \frac{2\lambda_1\lambda_2}{1+\lambda_1} \end{cases} \quad 7$$

Denote  $A = 3\sqrt{30045} - 520$ , we have  $\lambda_1^* = \frac{5}{3} + \frac{A^{1/3}}{3 \times 5^{2/3}} - \frac{1}{3(5A)^{1/3}}$  ( $0.50 < \lambda_1^* < 0.55$ ).

When the utility one can derive by not subscribing to any ISP is a bit low ( $0 \leq \lambda_0 < \lambda_0^*$ ) and the network differentiation is a bit high ( $0 < \lambda_1 < \lambda_1^*$ ), or the network differentiation is a bit low ( $\lambda_1^* < \lambda_1 < 1$ ), the effect of network differentiation in quality on the profit of group B is

negative. In these two cases, we have  $\frac{\partial P_B^*}{\partial \lambda_1} < 0$  and  $\frac{\partial D_B^*}{\partial \lambda_1} > 0$ . Hence, the lower the network

differentiation in quality is, the more severe the price competition is. However, group B can increase its market share if the differentiation lowers. The price declining has a greater impact on its profit than the demand rising does when network differentiation lowers. So the profit of group B decreases when the differentiation in quality lowers in the foresaid two cases.

For  $0 < \lambda_1 < \lambda_1^*$  and  $\lambda_0 > \lambda_0^*$ , the differentiation in quality has a positive impact on the profit of group B. The lower the network differentiation is, the more profits the group B obtains and vice versa. So the low quality group B has strong incentives to improve its quality of Internet access services. Namely the group B can increase its profit by lowering the network differentiation when all the consumers' income is high. Now analyze the effect of "average quality of Internet access provided by group A and B" on the profit of group B. By deriving the equilibrium profit of group B with respect to the average quality of

Internet access, we have  $\frac{\partial \pi_B^*}{\partial \lambda_2} = \frac{2\lambda_0(1-\lambda_1)^2(1+\lambda_1)(5\lambda_0(1+\lambda_1)-2(4+\lambda_1)\lambda_2)}{\lambda_1(3\lambda_0(1+\lambda_1)-2(4-\lambda_1)\lambda_2)^3} > 0$ .

The higher the average quality of Internet access provided by group A and B is, the more profits the group B obtains. When the network differentiation is a bit high, we have  $\frac{\partial P_B^*}{\partial \lambda_2} > 0$  and  $\frac{\partial D_B^*}{\partial \lambda_2} > 0$ . In this

case the group B can both increase its market share and raise the price to increase its profit by improving

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$$^7 \lambda_0^* = \frac{2b\lambda_2}{c} + \frac{2^{1/2} \times \lambda_2(4ac - 4b)}{c\lambda_2(d + (4(4ac - 4b)^3 + d^2)^{1/2})^{1/3}} - \frac{\lambda_2(d + (4(4ac - 4b)^3 + d^2)^{1/2})^{1/3}}{2^{1/3} \times c}, \text{ where}$$

$$a = 4 - 3\lambda_1 + 11\lambda_1^2 + 6\lambda_1^3, b = 7 + 6\lambda_1 + 11\lambda_1^2 + 12\lambda_1^3, c = 9(1 + \lambda_1)^3,$$

$$d = 560 + 4680\lambda_1 + 480\lambda_1^2 - 4800\lambda_1^4 - 28080\lambda_1^5 + 5920\lambda_1^6 + 37440\lambda_1^7 - 2160\lambda_1^8 - 14040\lambda_1^9.$$

the average network quality. When the differentiation is a bit low, we have  $\frac{\partial P_B^*}{\partial \lambda_2} > 0$  and  $\frac{\partial D_B^*}{\partial \lambda_2} < 0$ . By

increasing the average quality of Internet quality, the low quality group B can raise its price while the demand for it declines. Improving the average quality of Internet access contributes little to the profit of group B in this case. Here we present the main results in form of Proposition 1.

**Proposition 1** When all consumers' income is high ( $0 \leq \lambda_0 < \min\{\underline{\lambda}_0, \lambda_0^*\}$ ), All the ISPs connected to the two backbones can increase their profits by strengthening the differentiation in network quality. When all the consumers' income is neither too high nor too low ( $\max\{\underline{\lambda}_0, \lambda_0^*\} < \lambda_0 < \bar{\lambda}_0$ ) and the network differentiation in quality is high, All the ISPs connected to the two backbones can increase their profits by lowering the differentiation in network quality. All the ISPs connected to the two backbones can increase their profits by improving the average quality of Internet access provided by group A and B.

#### 4.2. The Effects of Peering on the Access Quality and Profits

We consider the effects of intra-backbone peering and inter-backbone peering on the network quality and profits in this section. Intra-backbone peering refers to peering between ISPs connected to the same backbone, whereas inter-backbone peering refers to peering between ISPs connected to different backbones. We begin with the intra-peering in group A.

$$\frac{\partial \pi_A^*}{\partial n_{AA}} = \frac{\partial \pi_A^*}{\partial \lambda_1} \frac{\partial \lambda_1}{\partial \lambda_A} \frac{\partial \lambda_A}{\partial n_{AA}} + \frac{\partial \pi_A^*}{\partial \lambda_2} \frac{\partial \lambda_2}{\partial \lambda_A} \frac{\partial \lambda_A}{\partial n_{AA}} = \frac{\partial \lambda_A}{\partial n_{AA}} \left( -\frac{\lambda_B}{(\lambda_A)^2} \frac{\partial \pi_A^*}{\partial \lambda_1} + \frac{1}{2} \frac{\partial \pi_A^*}{\partial \lambda_2} \right)$$

$$\left\{ \begin{array}{l} \frac{\partial \pi_A^*}{\partial n_{AA}} < 0 \quad \frac{9-\sqrt{73}}{2} < \lambda_1 \leq \frac{2}{7} \text{ and } \underline{\lambda}'_0 \leq \lambda_0 < \bar{\lambda}'_0 \\ \frac{\partial \pi_A^*}{\partial n_{AA}} > 0 \quad 0 < \lambda_1 \leq \frac{9-\sqrt{73}}{2} \text{ or } \left( \frac{9-\sqrt{73}}{2} < \lambda_1 \leq \frac{2}{7} \text{ and } \lambda_0 \geq \bar{\lambda}'_0 \right) \text{ or } \frac{2}{7} \leq \lambda_1 \leq 1 \end{array} \right. \quad 8$$

The group A improves its own quality by peering with each other, which makes the group A to increase its profit in most cases. Hence, the high quality ISPs have strong incentives to make intra-backbone peering agreements. They should always choose to peer with each other. However, increasing the intra-backbone peering agreements will reduce the

profit of the group A if  $\frac{9-\sqrt{73}}{2} < \lambda_1 \leq \frac{2}{7}$  and  $\underline{\lambda}'_0 \leq \lambda_0 < \bar{\lambda}'_0$ . So the high quality ISPs in group

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$$^8 \underline{\lambda}'_0 = \frac{-2\lambda_A^2 + 9\lambda_A\lambda_B - \lambda_B^2 - (\lambda_A - \lambda_B)\sqrt{4\lambda_A^2 - 28\lambda_A\lambda_B + \lambda_B^2}}{6\lambda_B};$$

$$\bar{\lambda}'_0 = \frac{-2\lambda_A^2 + 9\lambda_A\lambda_B - \lambda_B^2 + (\lambda_A - \lambda_B)\sqrt{4\lambda_A^2 - 28\lambda_A\lambda_B + \lambda_B^2}}{6\lambda_B}.$$

A has negative incentives to make intra-backbone peerings.

The inter-backbone peering increases not only the quality of group A, but also the quality of the rival group B, which makes the effect of inter-backbone peering on the profit of group

B complex. 
$$\frac{\partial \pi_A^*}{\partial n_{AB}} = \frac{\partial \pi_A^*}{\partial \lambda_1} \left( \frac{\partial \lambda_1}{\partial \lambda_A} \frac{\partial \lambda_A}{\partial n_{AB}} + \frac{\partial \lambda_1}{\partial \lambda_B} \frac{\partial \lambda_B}{\partial n_{AB}} \right) + \frac{\partial \pi_A^*}{\partial \lambda_2} \left( \frac{\partial \lambda_2}{\partial \lambda_A} \frac{\partial \lambda_A}{\partial n_{AB}} + \frac{\partial \lambda_2}{\partial \lambda_B} \frac{\partial \lambda_B}{\partial n_{AB}} \right)$$

We assume  $\frac{\partial \lambda_A}{\partial n_{AB}} = \frac{\partial \lambda_B}{\partial n_{AB}}$ , then:

$$\frac{\partial \pi_A^*}{\partial n_{AB}} = \frac{\partial \lambda_A}{\partial n_{AB}} \left( - \frac{4(\lambda_0 - \lambda_A)(\lambda_A - \lambda_B)(3\lambda_0^2 + \lambda_0(-7\lambda_A + \lambda_B) + \lambda_A(2\lambda_A + \lambda_B))}{\lambda_A^2(3\lambda_0 - 4\lambda_A + \lambda_B)^3} \right)$$

$$\left\{ \begin{array}{l} \frac{\partial \pi_A^*}{\partial n_{AB}} < 0 \quad 0 < \lambda_1 < \frac{2}{3} \text{ or } \left( \frac{2}{3} < \lambda_1 < \frac{3}{4} \text{ and } 0 < \lambda_0 < \lambda_0^{**} \right) \text{ or } \frac{3}{4} < \lambda_1 < 1 \\ \frac{\partial \pi_B^*}{\partial n_{AB}} > 0 \quad \frac{2}{3} < \lambda_1 < \frac{3}{4} \text{ and } \lambda_0^{**} < \lambda_0 < \lambda_B \end{array} \right. \quad 9$$

The ISPs are not willing to make inter-backbone peerings in most cases. Because the inter-backbone peering increases not only the quality of group A, but also the quality of the rival one, which lowers the network differentiation in quality and greatly promotes stiffer price competition. The net effect of inter-backbone peering on the profit of group A is negative. Conversely if  $\frac{2}{3} < \lambda_1 < \frac{3}{4}$  and  $\lambda_0^{**} < \lambda_0 < \lambda_B$ , the ISPs connected to group A have strong incentives to make inter-backbone peering agreements with ISPs in group B.

Similarly we can evaluate the incentives of ISPs in group B to peer with each other by differentiating their equilibrium profits. The effect of intra-backbone peering on the profit of

group B is given by: 
$$\frac{\partial \pi_B^*}{\partial n_{BB}} = \frac{\partial \pi_B^*}{\partial \lambda_1} \frac{\partial \lambda_1}{\partial \lambda_B} \frac{\partial \lambda_B}{\partial n_{BB}} + \frac{\partial \pi_B^*}{\partial \lambda_2} \frac{\partial \lambda_2}{\partial \lambda_B} \frac{\partial \lambda_B}{\partial n_{BB}} = \frac{\partial \lambda_B}{\partial n_{BB}} \left( \frac{1}{\lambda_A} \frac{\partial \pi_B^*}{\partial \lambda_1} + \frac{1}{2} \frac{\partial \pi_B^*}{\partial \lambda_2} \right)$$

$$\frac{\partial \pi_B^*}{\partial n_{BB}} < 0 \text{ if } 0 < \lambda_0 < \lambda_0^{***} \text{ }^{10}; \text{ and } \frac{\partial \pi_B^*}{\partial n_{BB}} > 0 \text{ if } \lambda_0^{***} < \lambda_0 < \lambda_B.$$

If all the consumers' income is high ( $0 < \lambda_0 < \lambda_0^{***}$ ), increasing the number of intra-backbone peering reduces the aggregate profit of the ISPs in the group B. The ISPs will refuse any intra-backbone peering agreements. But the ISPs connected to backbone B can

<sup>9</sup>  $\lambda_0^{**} = \frac{1}{6} \left( 7\lambda_A - \lambda_B - \sqrt{25\lambda_A^2 - 26\lambda_A\lambda_B + \lambda_B^2} \right).$

<sup>10</sup>  $\lambda_0^{***} = \frac{4\lambda_A^2 - 3\lambda_A\lambda_B + 5\lambda_B^2 - (\lambda_A - \lambda_B)\sqrt{16\lambda_A^2 + 8\lambda_A\lambda_B + 25\lambda_B^2}}{6\lambda_A}.$



increase their aggregate profit by making more intra-backbone peering agreements if all the consumers' income is low ( $\lambda_0^{***} < \lambda_0 < \lambda_B$ ). They should make intra-peering agreements with each other in the latter case.

Finally we consider the effect of inter-backbone peering on the profit of low quality group B:

$$\frac{\partial \pi_B^*}{\partial n_{AB}} = \frac{\partial \pi_B^*}{\partial \lambda_1} \left( \frac{\partial \lambda_1}{\partial \lambda_A} \frac{\partial \lambda_A}{\partial n_{AB}} + \frac{\partial \lambda_1}{\partial \lambda_B} \frac{\partial \lambda_B}{\partial n_{AB}} \right) + \frac{\partial \pi_B^*}{\partial \lambda_2} \left( \frac{\partial \lambda_2}{\partial \lambda_A} \frac{\partial \lambda_A}{\partial n_{AB}} + \frac{\partial \lambda_2}{\partial \lambda_B} \frac{\partial \lambda_B}{\partial n_{AB}} \right)$$

Similarly we assume  $\frac{\partial \lambda_A}{\partial n_{AB}} = \frac{\partial \lambda_B}{\partial n_{AB}}$ , then:

$$\frac{\partial \pi_B^*}{\partial n_{AB}} = \frac{\partial \lambda_B}{\partial n_{AB}} \left( - \frac{(\lambda_A - \lambda_B) (3\lambda_0^3 - \lambda_B^2 (2\lambda_A + \lambda_B) - \lambda_0^2 (7\lambda_A + 2\lambda_B) + \lambda_0 (4\lambda_A^2 + \lambda_A \lambda_B + 4\lambda_B^2))}{\lambda_B^2 (3\lambda_0 - 4\lambda_A + \lambda_B)^3} \right)$$

$$\frac{\partial \pi_B^*}{\partial n_{AB}} < 0 \text{ if } 0 < \lambda_0 < \lambda_0^{****} \text{ }^{11}; \text{ and } \frac{\partial \pi_B^*}{\partial n_{AB}} > 0 \text{ if } \lambda_0^{****} < \lambda_0 < \lambda_B.$$

The effect on the profit of group B depends on the consumers' income. If all consumers' income is high ( $0 < \lambda_0 < \lambda_0^{****}$ ), increasing the number of inter-backbone peering reduces the profit of group B. ISPs in group B will not make any inter-backbone peerings with the ISPs in the group A. If all consumers' income is low ( $\lambda_0^{****} < \lambda_0 < \lambda_B$ ), ISPs in group B can increase the aggregate profit by increasing the number of inter-backbone peering. So ISPs in group B have strong incentives to make inter-peerings with the ISPs in group A. The different results can be summarized in the following propositions.

**Proposition 2** The ISPs connected to the high quality backbone have strong incentives to improve their network quality in most cases. The ISPs connected to the low quality one have strong incentives to improve their network quality only if all consumers' income is low.

**Proposition 3** The ISPs interconnected to the high quality backbone have strong incentives to make intra-backbone peering agreements in most cases. But they refuse any inter-backbone peerings in most cases. The ISPs interconnected to the low quality backbone will try their best to peer with all the other ISPs only if all the consumers' income is low.

**Proposition 4** If all consumers' income is low, the equilibrium result is that ISPs make intra-backbone peerings with all the other ISPs connected to the same backbone and there are no inter-backbone peerings in most cases. If all consumers' income is high, the equilibrium is that ISPs connected to the high quality backbone make intra-backbone peerings with all the other ISPs connected to the same backbone and there are neither any inter-backbone peerings

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<sup>11</sup>  $\lambda_0^{****} = \frac{1}{9} (7\lambda_A + 2\lambda_B) - \frac{2^{1/3} \times a'}{9(b' + \sqrt{4(a')^3 + (b')^2})^{1/3}} + \frac{(b' + \sqrt{4(a')^3 + (b')^2})^{1/3}}{9 \times 2^{1/3}}$ , where

$a' = -13\lambda_A^2 - 19\lambda_A \lambda_B + 32\lambda_B^2$ ,  $b' = -70\lambda_A^3 + 183\lambda_A^2 \lambda_B - 156\lambda_A \lambda_B^2 + 43\lambda_B^3$ .

nor intra-backbone peerings among the ISPs in low quality backbone in most cases.

## 5. CONCLUSIONS

The current analysis is quite preliminary, but we find that both the quality differentiation and the consumers' income utility have impacts on the ISPs' incentives of intra-backbone and inter-backbone peering. The effects are complex. Peering among retail ISPs have both positive and negative effects with regard to the ISPs profits. On the positive side, peering improves quality of Internet access services. On the negative side, it reduces differentiation and intensifies price competition. Reduction of congestion brings overall prices down. Moreover, the income utility of consumers also have a great impact on the demand and the profits of the retail ISPs. Hence, before peering, the retail ISPs will by no means take the market conditions and the quality differentiation into account. In most cases the high quality ISPs will privately peer with one another that interconnected to the same backbone and refuse to privately peer with the low quality ISPs that interconnected to the other backbone. But the low quality ISPs will increase peering agreements only if the consumers' income is low.

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