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Analytical Model of Switching Networks with Bandwidth Reservation and Several Attempts of Setting up a Connection*

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Abstract. The paper presents a new method of blocking probability calculation in multi-service multi-stage switching networks with bandwidth reservation, point-to-group selection and several attempts of setting up a connection. The basis for the proposed method is the availability distribution. An approximate function of this distribution is chosen in the paper. The availability distribution and the occupancy distribution within the outgoing groups are the direct data for the probability calculations of internal and external blocking in the switching networks considered. Four reservation algorithms for multi-service switching networks are proposed. These algorithms lead to changes in blocking probabilities for different traffic streams in the switching networks. In particular cases, reservation algorithms can be used for blocking probability equalisation of a certain number or of all traffic streams offered. The results of the analytical calculations of blocking probability are compared with the simulation results of three-stage switching networks.

Keywords: bandwidth reservation, multi-service switching networks

1 Introduction

Reservation algorithms are one of possible strategies of the network functions which are responsible for admitting new calls to service in networks with integrated services. They can ensure equalised access to network resources for calls with different bandwidth requirements. Bandwidth reservation has been considered in numerous works [1–4]. In [1,5] an approximate method of the occupancy distribution calculation in a *full-availability group* with reservation was proposed, whereas in [6] the reservation problem was taken up in a *limited-availability group*.

However, determining traffic characteristics of multi-service multi-stage switching networks with reservation is a much more complex problem. Even though much research has been and is still being done on the subject, no method of blocking probability calculation in multi-stage multi-service switching networks with bandwidth reservation and several attempts of setting up a connection has been worked out so far. The methods which have

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appeared in the literature are limited to switching networks with single-rate traffic [7, 8] or to switching networks with multi-rate traffic but without bandwidth reservation [9]. In this paper, a new method PGBNBR (Point-to-Group Blocking probability calculation in multi-stage multi-service switching networks with N attempts of setting up a connection and Bandwidth Reservation) has been proposed. This method is based on the concept of the effective availability [7, 8, 10–12].

The remaining part of this paper is organised as follows. Section 2 describes the reservation algorithms in switching networks. The next section is devoted to the elaboration of the PGBNBR method. In Section 4, the calculation results have been compared with the simulation results. Section 5 concludes the paper.

2 Reservation algorithms in switching systems

Let us consider a switching system with multi-rate traffic. Let us assume that the system services call demands having an integer number of the so-called BBUs (Basic Bandwidth Units) [3]. The system is offered M independent classes of Poisson traffic streams with the intensities: $\lambda_1, \lambda_2, \ldots, \lambda_M$. A class i call requires t_i BBUs. The holding time for calls of particular classes has an exponential distribution with the parameters: $\mu_1, \mu_2, \ldots, \mu_M$. The mean traffic offered to the system by the class i traffic stream is equal to $a_i = \lambda_i/\mu_i$.

2.1 Basic recurrence relations

The multi-rate systems can be determined by the multi-dimensional Markov process. This process cannot be used for practical calculations because of an excessive number of states in which the process can be found. However, the multi-dimensional process can be approximated by the one-dimensional Markov chain, which can be described by the so-called *generalised Kaufman-Roberts recursion* [1, 10]:

$$nP(n) = \sum\nolimits_{i=1}^{M} a_i t_i \sigma_i(n-t_i) P(n-t_i), \tag{1}$$

where $P(n)$ is the probability of n BBUs being busy in the system and $\sigma_i(n)$ is the probability of admission of the class i call to the service when the system is found in the state n. The blocking probability $b(i)$ for the class i stream can be written as follows:

$$b(i) = \sum\nolimits_{n=0}^{V-t_i} P(n)[1-\sigma_i(n)] + \sum\nolimits_{n=V-t_i+1}^{V} P(n), \tag{2}$$

where V is the capacity of the system.

If $\sigma_i(n) = 1$ for all states, (1) is reduced to the recurrent *Fortet-Grandjean* formula [13], which is generally known as the *Kaufman-Roberts recursion* [2]:

$$nP(n) = \sum\nolimits_{i=1}^{M} a_i t_i P(n-t_i). \tag{3}$$

Formula (3) determines the occupancy distribution in the full-availability group (FAG) with different multi-rate traffic streams [3]. The blocking probability in FAG is equal to:

$$b(i) = \sum\nolimits_{n=V-t_i+1}^{V} P(n). \tag{4}$$

According to (2) and (9), the blocking probability for class $i \neq M$ calls:

$$b(i) = \sum_{n=0}^{V-Q} P(n)[1-\sigma_i(n)] + \sum_{n=V-Q+1}^{V} P(n). \tag{11}$$

The blocking probability for class M calls is determined directly by (2). To simplify further discussion, the occupancy distribution in LAGR with Algorithm 1, was designated OD1.

Algorithm 2. In this algorithm, the threshold Q is introduced for all traffic classes (also for the oldest class calls). Thus, $\sigma_i(n)$ is equal to:

$$\sigma_i(n) = \begin{cases} (7) & \text{for } n \leq Q, \\ 0 & \text{for } n > Q. \end{cases} \tag{12}$$

Blocking probabilities for all streams are calculated by (11). The occupancy distribution in LAGR with Algorithm 2, determined by (1) and (12), was designated OD2.

Algorithm 3. Algorithms 1 and 2 assume independent strategies of links occupation. They require one reservation mechanism for the group. It is possible to determine other reservation algorithms. In Algorithm 3 the threshold Q is introduced into all ν links of the group. The value of Q is the same in each link and it is equal to $f - t_{\max}$. The occupancy distribution in the case of Algorithm 3 can be approximated by the distribution OD2 [6].

2.4 Distribution of available links in LAG and LAGR

The *distribution of available links* (DAL) determines the probability $P(i,w)$ of an event in which each of arbitrarily chosen w links can carry the class i call [12]. In order to calculate $P(i,w)$, it is indispensable to know the *conditional distribution of available links* $P(i,w|x)$. This distribution determines the probability of an arrangement of x ($x = V-n$) free BBUs, in which each of w arbitrarily chosen links has at least t_i free BBUs, while in each of the remaining $(\nu - w)$ links the number of free BBUs is lower than t_i:

$$P(i,w|x) = \frac{\binom{\nu}{w} \sum_{u=wt_i}^{\Psi} F(u,w,f,t_i) F(x-u, \nu-w, t_i-1, 0)}{F(\nu,x,f,0)}, \tag{13}$$

where: $\Psi = wf,$ if $x \geq wf$, $\quad \Psi = x,$ if $x < wf$. On the basis of $P(i,w|x)$:

$$P(i,w) = \sum_{n=0}^{V} P(n) P(i,w|V-n), \tag{14}$$

where $P(n)$ is the distribution OD0. The distribution of available links in LAG, determined by (13), (14) and the distribution OD0, was designated DAL0.

Subsequently, let us derive DAL for LAGR in which Algorithm 3 has been applied:

$$P^*(i,w) = \sum_{n=0}^{V} P^*(n) P^*(i,w|V-n). \tag{15}$$

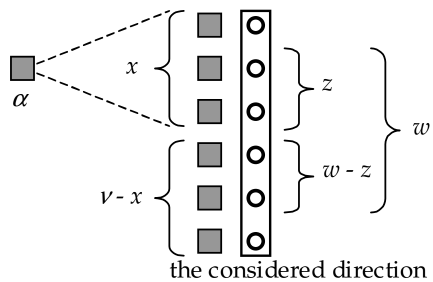
In (15), $P^*(n)$ is the occupancy distribution OD2 and $P^*(i,w|x)$ is equal to:

$$P^*(i,w|x) = P(M,w|x) \quad \text{where } t_M = t_{\max}. \tag{16}$$

The distribution of available links in LAGR, determined by (15), (16) and the distribution OD2, was designated as DAL1.

3.1 Blocking probability in an equivalent switching network

Let us consider a multi-stage multi-service switching network with point-to-group selection and several attempts of setting up a connection. Now, for the class i calls, we can determine an equivalent switching network (Sec. 3.2) carrying a single-rate traffic [12]. Providing that the outgoing direction of the considered switching network has the capacity equal to ν links, let us consider one of possible states of the equivalent switching network (Fig. 2):



- x links available for the first-stage switch α on the incoming link of which a call appears
- $\nu - x$ links non-available for the switch α
- w free links in a given direction
- z free links in the set of x available links for the switch α
- $w - z$ free links in the set of $\nu - x$ non-available links for the switch α

Fig. 2. One of possible states of the outgoing direction of the switching network

The internal blocking probability $P_b(k)$ for k unsuccessful attempts of setting up a connection, in the equivalent switching network, is given by the following equation [9]:

$$P_b(k) = \sum\nolimits_{x=0}^{\nu} P_b(k,x)P(x), \tag{17}$$

where $P(x)$ is the probability of the availability of x outgoing links for the switch α and $P_b(k,x)$ is the probability of the internal blocking for k unsuccessful attempts of setting up a connection, providing the outgoing group has x available links for the switch α:

$$P_b(k,x) = \sum_{z=0}^{w} \frac{\binom{w-z}{k}}{\binom{w}{k}} \frac{\binom{x}{z}\binom{\nu-x}{w-z}}{\binom{\nu}{w}} = \prod_{j=0}^{k-1} \left(1 - \frac{x}{\nu - j}\right). \tag{18}$$

In Equation (18), the following expression:

$$C_k(x) = \prod\nolimits_{j=0}^{k-1} (\nu - x - j) = a_k x^k + a_{k-1}x^{k-1} + \ldots + a_1 x + a_0 \tag{19}$$

is a polynomial of degree k with real coefficients a_j, where $0 \leqslant j \leqslant k$. As the j-order moment of the random variable x is equal to:

$$M_j(x) = \sum\nolimits_{x=0}^{\nu} x^j P(x), \tag{20}$$

and taking into account (19) and (20), (17) can be expressed as follows:

$$P_b(k) = \frac{1}{\prod_{j=0}^{k-1}(\nu - j)} \left[a_k M_k(x) + \ldots + a_0\right] = \frac{1}{\prod_{j=0}^{k-1}(\nu - j)} A_k(x), \tag{21}$$

where $A_k(x) = C_k(x)P(x)$ is a polynomial of the random variable of availability equal to x with the same real coefficients as in $C_k(x)$ polynomial.

Table 1. External blocking probability calculation in the PGBNBR method

Reservation algorithm in an outgoing direction	Occupancy distribution	External blocking probability $B_{ex}(i)$
without reservation	OD0	(2) for each i
Algorithm 1	OD1	(2) for $i = M$ and (11) for $i \neq M$
Algorithm 2	OD2	(11) for each i
Algorithm 3	OD2	(11) for each i

– $Y_j(i)$ – an average value of the fictitious traffic served by the stage j switch: $Y_j(i) = lb_j(i)$;
– l – the number of outgoing links from the last stage switch;
– η – a portion of the average fictitious traffic from a switch of the first stage, which is carried by the direction in question;
– $\sigma_z(i)$ – the so-called *secondary availability coefficient* [12] which is the probability of an event in which a connection path of the class i connection passes through directly available switches of intermediate stages:

$$\sigma_z(i) = 1 - \prod_{r=2}^{z-1} [\pi_r(i)]. \tag{26}$$

3.3 Blocking probability in a multi-rate switching network

Having determined the internal blocking probability $P_b(k)$ for k ($k \leq N$) unsuccessful attempts of setting up a connection in an equivalent switching network (determined for class i calls), the internal blocking probability for class i calls when N attempts of setting up a connection are allowed can be calculated:

$$B_{in}(i, N) = \frac{1}{1 - P(w = 0)} \sum_{w=1}^{\nu} P_b(k) P(w), \quad \text{where } k = \begin{cases} w & \text{for } w < N \\ N & \text{for } w \geqslant N \end{cases} \tag{27}$$

and $P(w)$ is the distribution of free outgoing links of a given direction in an equivalent switching network. In a real switching network this is the distribution of free links for class i calls. In the proposed method, this distribution is approximated by DAL0 distribution – if the reservation mechanism is not introduced into outgoing links (or introduced according to Algorithm 1 or 2) – and by DAL1 distribution – if the reservation mechanism is introduced into outgoing links, according to Algorithm 3.

External blocking probability $B_{ex}(i)$ depends on the reservation algorithm adopted. A list of the occupancy distributions in relation to the reservation algorithms used in outgoing groups is presented in Table 1.

Total blocking probability $B(i, N)$ for the class i call is a sum of external and internal blocking probabilities. Assuming the independence of these blocking events:

$$B(i, N) = B_{ex}(i) + B_{in}(i, N)[1 - B_{ex}(i)]. \tag{28}$$

4 Calculation and simulation results

In order to confirm the assumptions adopted in the PGBNBR method, the results of calculations were compared with the results of simulation of the switching network (Fig. 1)

5 Conclusions

In the paper the PGBNBR method has been derived for the switching networks with several attempts of setting up a connection and with bandwidth reservation. A simulation study has confirmed fair accuracy of the proposed analytical method. The obtained accuracy is comparable to that in the switching networks without bandwidth reservation [7–9]. The simulation research carried out by the authors proves that the proposed method ensures fair accuracy of calculations for switching networks with any number of attempts of setting up a connection, for various network structures, for various traffic structures and for different reservation algorithms. The proposed PGBNBR method can be used to determine the most effective number of attempts of setting up a connection in multi-service switching networks with bandwidth reservation.

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