Analytical Model of Switching Networks with Bandwidth Reservation and Several Attempts of Setting up a Connection* 

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Abstract. The paper presents a new method of blocking probability calculation in multi-service multi-stage switching networks with bandwidth reservation, point-to-group selection and several attempts of setting up a connection. The basis for the proposed method is the availability distribution. An approximate function of this distribution is chosen in the paper. The availability distribution and the occupancy distribution within the outgoing groups are the direct data for the probability calculations of internal and external blocking in the switching networks considered. Four reservation algorithms for multi-service switching networks are proposed. These algorithms lead to changes in blocking probabilities for different traffic streams in the switching networks. In particular cases, reservation algorithms can be used for blocking probability equalisation of a certain number or of all traffic streams offered. The results of the analytical calculations of blocking probability are compared with the simulation results of three-stage switching networks.

Keywords: bandwidth reservation, multi-service switching networks

1 Introduction

Reservation algorithms are one of possible strategies of the network functions which are responsible for admitting new calls to service in networks with integrated services. They can ensure equalised access to network resources for calls with different bandwidth requirements. Bandwidth reservation has been considered in numerous works [1–4]. In [1,5] an approximate method of the occupancy distribution calculation in a full-availability group with reservation was proposed, whereas in [6] the reservation problem was taken up in a limited-availability group.

However, determining traffic characteristics of multi-service multi-stage switching networks with reservation is a much more complex problem. Even though much research has been and is still being done on the subject, no method of blocking probability calculation in multi-stage multi-service switching networks with bandwidth reservation and several attempts of setting up a connection has been worked out so far. The methods which have

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appeared in the literature are limited to switching networks with single-rate traffic \([7, 8]\) or to switching networks with multi-rate traffic but without bandwidth reservation \([9]\).

In this paper, a new method PGBNBR (Point-to-Group Blocking probability calculation in multi-stage multi-service switching networks with \(N\) attempts of setting up a connection and Bandwidth Reservation) has been proposed. This method is based on the concept of the effective availability \([7,8,10–12]\).

The remaining part of this paper is organised as follows. Section 2 describes the reservation algorithms in switching networks. The next section is devoted to the elaboration of the PGBNBR method. In Section 4, the calculation results have been compared with the simulation results. Section 5 concludes the paper.

## 2 Reservation algorithms in switching systems

Let us consider a switching system with multi-rate traffic. Let us assume that the system services call demands having an integer number of the so-called BBUs (Basic Bandwidth Units) \([3]\). The system is offered \(M\) independent classes of Poisson traffic streams with the intensities: \(\lambda_1, \lambda_2, \ldots, \lambda_M\). A class \(i\) call requires \(t_i\) BBUs. The holding time for calls of particular classes has an exponential distribution with the parameters: \(\mu_1, \mu_2, \ldots, \mu_M\). The mean traffic offered to the system by the class \(i\) traffic stream is equal to \(a_i = \lambda_i / \mu_i\).

### 2.1 Basic recurrence relations

The multi-rate systems can be determined by the multi-dimensional Markov process. This process cannot be used for practical calculations because of an excessive number of states in which the process can be found. However, the multi-dimensional process can be approximated by the one-dimensional Markov chain, which can be described by the so-called generalised Kaufman-Roberts recursion \([1,10]\):

\[
nP(n) = \sum_{i=1}^{M} a_i t_i \sigma_i(n - t_i) P(n - t_i),
\]

where \(P(n)\) is the probability of \(n\) BBUs being busy in the system and \(\sigma_i(n)\) is the probability of admission of the class \(i\) call to the service when the system is found in the state \(n\). The blocking probability \(b(i)\) for the class \(i\) stream can be written as follows:

\[
b(i) = \sum_{n=0}^{V-t_i} P(n)[1 - \sigma_i(n)] + \sum_{n=V-t_i+1}^{V} P(n),
\]

where \(V\) is the capacity of the system.

If \(\sigma_i(n) = 1\) for all states, (1) is reduced to the recurrent Fortet-Grandjean formula \([13]\), which is generally known as the Kaufman-Roberts recursion \([2]\):

\[
P(n) = \sum_{i=1}^{M} a_i t_i P(n - t_i).
\]

Formula (3) determines the occupancy distribution in the full-availability group (FAG) with different multi-rate traffic streams \([3]\). The blocking probability in FAG is equal to:

\[
b(i) = \sum_{n=V-t_i+1}^{V} P(n).
\]
2.2 The Full-Availability Group with Reservation (FAGR)

In the systems with reservation the reservation threshold $Q_i$ for each traffic class is designated. The parameter $Q_i$ determines the borderline state of a system, in which servicing calls of a given class is still possible. According to the equalisation rule [1,3,4], the blocking probability in FAG will be the same for all calls if the threshold $Q_i$ for all traffic classes is identical and equal to the difference between the group capacity and the number of BBUs required by a call with maximum demands: $Q_i = Q = V - t_{\text{max}}$. All states higher than $Q_i$ belong to the reservation space $R_i = V - Q_i$, in which class $i$ calls are blocked. The occupancy distribution in FAG with reservation [1] is determined by (1), in which

$$ \sigma_i(n) = \begin{cases} 0 & \text{for } n > Q, \\ 1 & \text{for } n \leq Q. \end{cases} $$

Thus, blocking probabilities for all call classes can be calculated as follows:

$$ b(i) = \sum_{n=Q+1}^{V} P(n). $$

2.3 The Limited-Availability Group with Reservation (LAGR)

A limited-availability group (LAG) is a model of separated transmission links [12]. LAG is the group divided into $\nu$ identical links, each of the capacity equal to $f$ BBUs. The total capacity of the system is equal to $V = \nu f$. The system services a call – only when this call can be entirely carried by the resources of an arbitrary single link. According to [12], the occupancy distribution in LAG can be approximated by (1), in which

$$ \sigma_i(n) = F(V - n, \nu, f, 0) - F(V - n, \nu, t_i - 1, 0)/F(V - n, \nu, f, 0), $$

where $F(x, \nu, f, t)$ is the number of arrangements of $x$ free BBUs in $\nu$ links, calculated with the assumption that the capacity of each link is equal to $f$ BBUs and each link has at least $t$ free BBUs:

$$ F(x, \nu, f, t) = \sum_{i=0}^{\left\lfloor \frac{x}{f-t+1} \right\rfloor} (-1)^i \binom{\nu}{i} \frac{x - \nu(t-1) - 1 - i(f-t+1)}{\nu - 1}. $$

To simplify further discussion, the occupancy distribution in LAG, determined by (1), (7) and (8), was designated $OD0$.

The blocking equalisation rule elaborated for FAGR is not effective when considering LAGR because of its structure and state-dependent processes occurring in the system. In this section three reservation algorithms in LAGR are presented [6].

Algorithm 1. In Algorithm 1, the common reservation threshold $Q$ is determined for the whole group. The threshold $Q$ is designated for all call classes except for the oldest class $M$ (we assume: $t_M = t_{\text{max}}$). This means that only calls of the class $M$ can be serviced by the group in states belonging to the reservation space. Thus, $\sigma_i(n)$ in (1) is defined as follows:

$$ i \neq M \Rightarrow \sigma_i(n) = \begin{cases} (7) & \text{for } n \leq Q, \\ 0 & \text{for } n > Q, \end{cases} $$

$$ i = M \Rightarrow \sigma_i(n) = (7) \text{ for each } n. $$
According to (2) and (9), the blocking probability for class $i \neq M$ calls:

$$b(i) = \sum_{n=0}^{V-Q} P(n)[1 - \sigma_i(n)] + \sum_{n=V-Q+1}^{V} P(n).$$

(11)

The blocking probability for class $M$ calls is determined directly by (2). To simplify further discussion, the occupancy distribution in LAGR with Algorithm 1, was designated OD1.

**Algorithm 2.** In this algorithm, the threshold $Q$ is introduced for all traffic classes (also for the oldest class calls). Thus, $\sigma_i(n)$ is equal to:

$$\sigma_i(n) = \begin{cases} (7) & \text{for } n \leq Q, \\ 0 & \text{for } n > Q. \end{cases}$$

(12)

Blocking probabilities for all streams are calculated by (11). The occupancy distribution in LAGR with Algorithm 2, determined by (1) and (12), was designated OD2.

**Algorithm 3.** Algorithms 1 and 2 assume independent strategies of links occupation. They require one reservation mechanism for the group. It is possible to determine other reservation algorithms. In Algorithm 3 the threshold $Q$ is introduced into all $\nu$ links of the group. The value of $Q$ is the same in each link and it is equal to $f - t_{\max}$. The occupancy distribution in the case of Algorithm 3 can be approximated by the distribution OD2 [6].

### 2.4 Distribution of available links in LAG and LAGR

The distribution of available links (DAL) determines the probability $P(i, w)$ of an event in which each of arbitrarily chosen $w$ links can carry the class $i$ call [12]. In order to calculate $P(i, w)$, it is indispensable to know the conditional distribution of available links $P(i, w|x)$. This distribution determines the probability of an arrangement of $x$ ($x = V - n$) free BBUs, in which each of $w$ arbitrarily chosen links has at least $t_i$ free BBUs, while in each of the remaining ($\nu - w$) links the number of free BBUs is lower than $t_i$:

$$P(i, w|x) = \frac{(\nu)^w}{w!} \sum_{u=wt_i}^{\Psi} F(u, w, f, t_i) F(x - u, \nu - w, t_i - 1, 0) F(\nu, x, f, 0),$$

(13)

where: $\Psi = wf$, if $x \geq wf$, $\Psi = x$, if $x < wf$. On the basis of $P(i, w|x)$:

$$P(i, w) = \sum_{n=0}^{V} P(n) P(i, w|V - n),$$

(14)

where $P(n)$ is the distribution OD0. The distribution of available links in LAG, determined by (13), (14) and the distribution OD0, was designated DAL0.

Subsequently, let us derive DAL for LAGR in which Algorithm 3 has been applied:

$$P^*(i, w) = \sum_{n=0}^{V} P^*(n) P^*(i, w|V - n).$$

(15)

In (15), $P^*(n)$ is the occupancy distribution OD2 and $P^*(i, w|x)$ is equal to:

$$P^*(i, w|x) = P(M, w|x) \quad \text{where } t_M = t_{\max}.$$  

(16)

The distribution of available links in LAGR, determined by (15), (16) and the distribution OD2, was designated as DAL1.
2.5 Reservation in switching networks

Let us consider a switching network with multi-rate traffic (Fig. 1). Let us assume that each of inter-stage links has the capacity equal to \( f \) BBUs and that outgoing transmission links create link groups called directions (a direction can be treated as the limited-availability group). The switching network under consideration operates with point-to-group selection and several attempts of setting up a connection. Thus, if the connection cannot be set up during the last allowed \( N \) attempt, a class \( i \) call is lost as a result of internal blocking. A call can be lost in \( k \)th attempt (\( 1 \leq k \leq N \)) if, for a given state of the switching network, \( k - 1 \) last-stage switches which have a free outgoing link exist. When none of outgoing links of the demanded direction of the switching network can service the class \( i \) call (i.e. does not have \( t_i \) free BBUs), a call is lost due to the phenomenon of external blocking.

In order to maintain the blocking equalisation for a certain number or for all call streams in the switching networks four reservation algorithms have been proposed:

1. **Algorithm D1.** The reservation mechanism is introduced exclusively into particular outgoing directions of the switching network. In order to ensure the blocking equalisation, Algorithm 1 has been used.
2. **Algorithm D3.** The reservation mechanism, according to Algorithm 3 is introduced into particular outgoing directions.
3. **Algorithm BD3.** Algorithm 3 is introduced into outgoing links and Algorithm FAGR is introduced into inter-stage links between stages 1-2.
4. **Algorithm BCD3.** Algorithm 3 is introduced into outgoing links and Algorithm FAGR is introduced into inter-stage links between stages 1-2 and 2-3.

3 Switching network calculations – PGBNBR method

In this section an approximate method PGBNBR (Point-to-Group Blocking probability calculation in multi-stage switching networks with \( N \) attempts of setting up a connection and Bandwidth Reservation) is presented. The presented considerations are based on PGBMTn method worked out in [9] for switching networks without reservation.
3.1 Blocking probability in an equivalent switching network

Let us consider a multi-stage multi-service switching network with point-to-group selection and several attempts of setting up a connection. Now, for the class $i$ calls, we can determine an equivalent switching network (Sec. 3.2) carrying a single-rate traffic [12]. Providing that the outgoing direction of the considered switching network has the capacity equal to $\nu$ links, let us consider one of possible states of the equivalent switching network (Fig. 2):

$$\begin{align*}
&x \quad \text{links available for the first-stage switch} \ \alpha \ \text{on the incoming link of which a call appears} \\
&\nu - x \quad \text{links non-available for the switch} \ \alpha \\
&w \quad \text{free links in a given direction} \\
&w - z \quad \text{free links in the set of} \ x \ \text{available links for the switch} \ \alpha \\
&w - z \quad \text{free links in the set of} \ \nu - x \ \text{non-available links for the switch} \ \alpha
\end{align*}$$

Fig. 2. One of possible states of the outgoing direction of the switching network

The internal blocking probability $P_b(k)$ for $k$ unsuccessful attempts of setting up a connection, in the equivalent switching network, is given by the following equation [9]:

$$P_b(k) = \sum_{x=0}^{\nu} P_b(k, x)P(x), \quad (17)$$

where $P(x)$ is the probability of the availability of $x$ outgoing links for the switch $\alpha$ and $P_b(k, x)$ is the probability of the internal blocking for $k$ unsuccessful attempts of setting up a connection, providing the outgoing group has $x$ available links for the switch $\alpha$:

$$P_b(k, x) = \sum_{z=0}^{w} \frac{\nu - z}{x - z} \frac{x - z}{w} \frac{(w - z)}{w} = \prod_{j=0}^{k-1} \left(1 - \frac{x}{\nu - j}\right). \quad (18)$$

In Equation (18), the following expression:

$$C_k(x) = \prod_{j=0}^{k-1} (\nu - x - j) = a_k x^k + a_{k-1} x^{k-1} + \ldots + a_1 x + a_0 \quad (19)$$

is a polynomial of degree $k$ with real coefficients $a_j$, where $0 \leq j \leq k$. As the $j$-order moment of the random variable $x$ is equal to:

$$M_j(x) = \sum_{x=0}^{\nu} x^j P(x), \quad (20)$$

and taking into account (19) and (20), (17) can be expressed as follows:

$$P_b(k) = \frac{1}{\prod_{j=0}^{k-1} (\nu - j)} [a_k M_k(x) + \ldots + a_0] = \frac{1}{\prod_{j=0}^{k-1} (\nu - j)} A_k(x), \quad (21)$$

where $A_k(x) = C_k(x)P(x)$ is a polynomial of the random variable of availability equal to $x$ with the same real coefficients as in $C_k(x)$ polynomial.
According to (19), we have the expression: $C_{k+1}(x) = C_k(x)(\nu - k - x)$, which can be rewritten as follows:

$$(a_k x^k + a_{k-1} x^{k-1} + \ldots + a_0) (\nu - k - x) = a'_{k+1} x^{k+1} + a'_k x^k + \ldots + a'_0.$$  

Equation (22) gives the direct recurrent relations between coefficients of a polynomial of degree $k$ and coefficients of a polynomial of degree $k + 1$:

$$\begin{cases} 
  a'_{k+1} = -a_k, \\
  a'_j = -a_{j-1} + a_j(\nu - k) \quad \text{for } 0 < j < k + 1, \\
  a'_0 = a_0(\nu - k),
\end{cases}$$  

where coefficient $a_0$ of a polynomial of degree zero is equal to one.

The analysis of (21) shows that the determination of the first $k$ moments of the random variable of the availability is essential to the internal blocking probability calculation in switching networks with $k$ unsuccessful attempts of setting up a connection. In the paper we have assumed that the first moment of availability is approximated by the effective availability parameter $d_e(i)$ for class $i$ calls (Section 3.2). We have also assumed that the effective availability distribution $P(x)$ is approximated by the binomial distribution:

$$P(x) = \binom{\nu}{x} p^x (1 - p)^{\nu - x},$$  

where $p = d_e(i)/\nu$ is the probability of the availability of an arbitrarily chosen outgoing link.

### 3.2 Effective availability parameter

The concept of the equivalent switching network [12] is the base for the effective availability calculation for the class $i$ traffic stream. Following this concept, the network with multi-rate traffic is reduced to an equivalent network carrying a single-rate traffic. Each link of an equivalent network is treated as a single-rate link with a fictitious load $b_j(i)$ equal to the blocking probability for a class $i$ stream in a link of the real switching network between stages $j$ and $j + 1$. If the reservation mechanisms are not introduced into the inter-stage links, $b_j(i)$ is calculated by (4). If these mechanisms are used, $b_j(i)$ is calculated on the basis of (6). The effective availability in a real $z$-stage switching network is equal to the effective availability $d_e(i)$ in an equivalent switching network and can be determined by the formula derived in [12]:

$$d_e(i) = [1 - \pi_z(i)]\nu + \pi_z(i)\eta Y_1(i) + \pi_z(i)[\nu - \eta Y_1(i)]b_z(i)\sigma_z(i),$$  

- $\pi_z(i)$ – the probability of non-availability of a stage $r$ switch for the class $i$ call; $\pi_r(i)$ is the probability of an event where a class $i$ connection path cannot be set up between a given first-stage switch and a given stage $r$ switch; the evaluation of this parameter is based on the channel graph of the equivalent switching network and the parameter can be calculated by the Lee method [14];
- $\nu$ – the number of outgoing links from the first stage switch;
Table 1. External blocking probability calculation in the PGBNBR method

<table>
<thead>
<tr>
<th>Reservation algorithm in an outgoing direction</th>
<th>Occupancy distribution</th>
<th>External blocking probability $B_{ex}(i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>without reservation</td>
<td>OD0</td>
<td>(2) for each $i$</td>
</tr>
<tr>
<td>Algorithm 1</td>
<td>OD1</td>
<td>(2) for $i = M$ and (11) for $i \neq M$</td>
</tr>
<tr>
<td>Algorithm 2</td>
<td>OD2</td>
<td>(11) for each $i$</td>
</tr>
<tr>
<td>Algorithm 3</td>
<td>OD2</td>
<td>(11) for each $i$</td>
</tr>
</tbody>
</table>

- $Y_j(i)$ – an average value of the fictitious traffic served by the stage $j$ switch: $Y_j(i) = lb_j(i)$;
- $l$ – the number of outgoing links from the last stage switch;
- $\eta$ – a portion of the average fictitious traffic from a switch of the first stage, which is carried by the direction in question;
- $\sigma_z(i)$ – the so-called secondary availability coefficient [12] which is the probability of an event in which a connection path of the class $i$ connection passes through directly available switches of intermediate stages:

$$\sigma_z(i) = 1 - \prod_{r=2}^{z-1}[\pi_r(i)].$$

(26)

### 3.3 Blocking probability in a multi-rate switching network

Having determined the internal blocking probability $P_b(k)$ for $k$ ($k \leq N$) unsuccessful attempts of setting up a connection in an equivalent switching network (determined for class $i$ calls), the internal blocking probability for class $i$ calls when $N$ attempts of setting up a connection are allowed can be calculated:

$$B_{in}(i, N) = \frac{1}{1 - P(w = 0)} \sum_{w=1}^{N} P_b(k)P(w), \quad \text{where } k = \begin{cases} w & \text{for } w < N \\ N & \text{for } w \geq N \end{cases}$$

(27)

and $P(w)$ is the distribution of free outgoing links of a given direction in an equivalent switching network. In a real switching network this is the distribution of free links for class $i$ calls. In the proposed method, this distribution is approximated by DAL0 distribution – if the reservation mechanism is not introduced into outgoing links (or introduced according to Algorithm 1 or 2) – and by DAL1 distribution – if the reservation mechanism is introduced into outgoing links, according to Algorithm 3.

External blocking probability $B_{ex}(i)$ depends on the reservation algorithm adopted. A list of the occupancy distributions in relation to the reservation algorithms used in outgoing groups is presented in Table 1.

Total blocking probability $B(i, N)$ for the class $i$ call is a sum of external and internal blocking probabilities. Assuming the independence of these blocking events:

$$B(i, N) = B_{ex}(i) + B_{in}(i, N)[1 - B_{ex}(i)].$$

(28)

### 4 Calculation and simulation results

In order to confirm the assumptions adopted in the PGBNBR method, the results of calculations were compared with the results of simulation of the switching network (Fig. 1)
with the following parameters: $\nu = 4$, $f = 30$; $t_1 = 1$, $t_2 = 2$, $t_3 = 6$. The system was offered three traffic classes in the proportions: $a_1 : a_2 : a_3 = 6 : 3 : 1$.

The analytical and simulation results are presented in Figs. 3(a) and 3(b) in relation to the value of traffic $a = \sum_{i=1}^{M} \frac{a_i t_i}{V}$, whereas in Fig. 4 the results are presented in relation to the reservation space $R$. The results of the simulation are shown in the charts in the form of marks with 95% confidence intervals. In many cases the value of the confidence interval is lower than the height of the sign used to indicate the value of the simulation experiment. Figure 3(a) shows the results of blocking probability in the switching network without reservation, whereas Figs. 3(b) and 4 show the results obtained in the case of the algorithms BCD3 and D1, respectively.

To determine the influence of the point-to-group selection with several attempts of setting up a connection on traffic characteristics of switching networks, Fig. 5 shows the percentage changes in blocking probability in relation to the number of attempts of setting up a connection for the switching network with Algorithm D3.
5 Conclusions

In the paper the PGBNBR method has been derived for the switching networks with several attempts of setting up a connection and with bandwidth reservation. A simulation study has confirmed fair accuracy of the proposed analytical method. The obtained accuracy is comparable to that in the switching networks without bandwidth reservation [7–9]. The simulation research carried out by the authors proves that the proposed method ensures fair accuracy of calculations for switching networks with any number of attempts of setting up a connection, for various network structures, for various traffic structures and for different reservation algorithms. The proposed PGBNBR method can be used to determine the most effective number of attempts of setting up a connection in multi-service switching networks with bandwidth reservation.

References