High-throughput high-performance OFDM via carrier interferometry approaches for complex signaling

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Abstract: Carrier Interferometry (CI) approaches, including Carrier-Interferometry Orthogonal Frequency Division Multiplexing (CI/OFDM) and Pseudo-Orthogonal Carrier-Interferometry OFDM (PO-CI/OFDM), have been proposed to enhance OFDM’s performance and reduce OFDM’s peak-to-average power ratio (PAPR). However, the existing high-throughput, high-performance PO-CI/OFDM system is limited for real signaling. This paper first proposes to use Inverse Fast Fourier Transform (IFFT)/Fast Fourier Transform (FFT), instead of vector-matrix multiplication, to implement the spreading/despreading in CI/OFDM and PO-CI/OFDM. Secondly, PO-CI/OFDM system is proved not applicable for complex signaling. Based on the analysis of CI spreading codes, PO-CI spreading codes and CI/OFDM system, a novel OFDM system architecture, named CI-POCI/OFDM, is introduced to provide double-throughput, high-performance for complex signaling. Furthermore, PAPR problem of traditional OFDM is also eliminated in CI-POCI/OFDM system.

Keywords: OFDM, Carrier Interferometry (CI), pseudo-orthogonal (PO), CI/OFDM, PO-CI/OFDM, CI-POCI/OFDM, complex signaling.

1. INTRODUCTION

Carrier Interferometry (CI) approaches, including Carrier-Interferometry Orthogonal Frequency Division Multiplexing (OFDM) (CI/OFDM) and Pseudo-Orthogonal Carrier-Interferometry OFDM (PO-CI/OFDM), have been proposed to enhance OFDM’s performance [1][2] and reduce OFDM’s peak-to-average power ratio (PAPR) [3][4]. (PO-)CI/OFDM (Carrier-Interferometry and Pseudo-Orthogonal Carrier-Interferometry are abbreviated to (PO-)CI) uses (PO-)CI spreading codes to make each parallel data stream spread over all of the OFDM sub-carriers, thereby outperforming OFDM due to the full frequency diversity. (PO-)CI spreading codes also ensure that when one symbol’s sub-carriers

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add coherently, other symbols’ sub-carriers do not add coherently, which eliminates the large power peaks [5]. Furthermore, using the pseudo-orthogonality property of PO-CI spreading codes, the throughput or capacity of PO-CI/OFDM system can be doubled in the case of real signaling [2].

However, the existing high-throughput, high-performance PO-CI/OFDM system is limited for real signaling. But most of the practical OFDM systems are complex signaling systems. In this paper, we first propose to use Inverse Fast Fourier Transform (IFFT)/Fast Fourier Transform (FFT), instead of vector-matrix multiplication, to implement the spreading/despreading in CI/OFDM and PO-CI/OFDM. Secondly, we prove that PO-CI/OFDM system is not applicable for complex signaling. Then based on the analysis of (PO-)CI spreading codes and CI/OFDM system, we propose a novel OFDM system architecture, named CI-POCI/OFDM, to provide high-throughput, high-performance via CI approaches for complex signaling.

The remainder of this paper is organized as follows. The analysis of (PO-)CI spreading codes is presented in section 2. The CI-POCI/OFDM system architecture is proposed in section 3. The results of bit-error rate (BER) performance and PAPR simulations are presented in section 4, and the conclusions are drawn in section 5.

2. (PO-)CI SPREADING CODES

2.1. Implementation of (PO-)CI spreading/despreading

It has been proved that for real signaling, the CI spreading codes \( \{e^{j0}, e^{j(\Delta \theta)}, \ldots, e^{j((N-1)\Delta \theta)}\} \), \( k = 0,1,\ldots,N-1 \) are orthogonal to each other with \( \Delta \theta_k = 2\pi k/N \) [5], where \( N \) is the number of sub-carriers. Let \( s = [s_0, s_1, \ldots, s_{N-1}] \) be the result of spreading the real data sequence \( a_{CI} = [a_0, a_1, \ldots, a_{N-1}] \) with CI spreading codes, where

\[
 s_i = \sum_{k=0}^{N-1} a_k e^{j\frac{i\pi k}{N}}, i = 0,1,\ldots,N-1. \tag{1}
\]

Comparing (1) with the definition of IFFT

\[
 x(i) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{-j\frac{2\pi i k}{N}}, \tag{2}
\]

we can see that (ignoring the coefficient)

\[
 s = \text{ifft}(a_{CI}). \tag{3}
\]

According to the spreading, the results of despreading, defined as data sequence \( a'_{k} \), can also be described as

\[
 a'_{k} = \sum_{i=0}^{N-1} s_i e^{-j\frac{2\pi i k}{N}}, k = 0,1,\ldots,N-1. \tag{4}
\]

Similarly, comparing (4) with the definition of FFT

\[
 X(k) = \sum_{i=0}^{N-1} x(i) e^{-j\frac{2\pi i k}{N}}, \tag{5}
\]

we can also see that
\[ a'_{CI} = \text{fft}(s) = a_{CI}, \]  

where \( a'_{CI} = [a'_0, a'_1, ..., a'_{N-1}] \).  

Equation (3) and (6) denote that the spreading and despeeeding in CI/OFDM system can be implemented by IFFT and FFT respectively.  

As for PO-CI spreading codes, in the case of real signaling, [2] has proved that two different sets of orthogonal CI spreading codes demonstrate a minimal amount of interference when the phase offset between them is selected as \( \theta = \pi/N \). These two sets of CI spreading codes with phase offset \( \theta = \pi/N \) make up a set of PO-CI spreading codes \( \{e^{j\Delta \theta}, ..., e^{j(N-1)\Delta \theta}\} \), where \( \Delta \theta_k = \pi k/N, k = 0, 1, ..., 2N-1 \).  

Define the vector \( s = [s_0, s_1, ..., s_{N-1}] \) as the result of spreading the real data sequence \( a_{PO-CI} = [a_0, a_1, ..., a_{2N-1}] \) with PO-CI spreading codes, where  

\[ s_i = \sum_{k=0}^{N-1} a_k e^{j(\pi \Delta \theta_k)}, i = 0, 1, ..., N-1. \]  

Letting \( M=2N \), we can rewrite (7) as:  

\[ s_i = \sum_{k=0}^{N-1} a_k e^{j(\pi \Delta \theta_k)}, i = 0, 1, ..., N-1. \]  

From (5) we define  

\[ s' = \text{iff}(a_{PO-CI}), \]  

where  

\[ s' = [s_0, s_1, ..., s_{N-1}, s_{N}, ..., s_{2N-1}], \]  

\[ s = s'(0: N-1). \]  

That means \( s \), as a part of \( s' \), can be obtained through 2N-point IFFT.  

Similarly, the despeeeding of the PO-CI spreading codes can also be subscribed as  

\[ a'_k = \sum_{i=0}^{N-1} s_i e^{-j(\pi \Delta \theta_k)}, k = 0, 1, ..., 2N-1. \]  

Substituting \( M=2N \) into (12)  

\[ a'_k = \sum_{i=0}^{N-1} s_i e^{-j(\pi \Delta \theta_k)} = \sum_{i=0}^{M-1} r_i e^{-j(\pi \Delta \theta_k)}, k = 0, 1, ..., 2N-1, \]  

where \( r_i \) is given by  

\[ r_i = \begin{cases} s_i, & i = 0, 1, ..., N-1 \\ 0, & i = N, N+1, ..., 2N-1 \end{cases}, \]  

We define that  

\[ r = [r_0, r_1, ..., r_{2N-1}], \]
which can be gotten via appending zeros in the back of $s$. From (13) and (5), we have
\[ a_{PO-CI}' = \text{fft}(r). \] (16)
where $a_{PO-CI}' = [a_0', a_1', ..., a_{K-1}'].$

Therefore, the output of PO-CI/OFDM receiver $a_{PO-CI}'$ can also be obtained through 2$N$-point FFT. Due to the pseudo-orthogonality property of PO-CI spreading codes, the real part of $a_{PO-CI}'$ is approximate to $a_{PO-CI}$ as long as $N$ is large enough [2]. The flow chart of PO-CI spreading/despreading can be drawn as Figure 1.

2.2. Property Analysis
In this section, we analyze the correlation of CI spreading codes and the PO-CI spreading codes in the case of complex signaling respectively.

For CI spreading codes, the cross correlation between $e^{i\theta_m}$ and $e^{i\theta_n}$ is
\[
R_{m,n}(\Delta \theta) = R(\Delta \theta) = \frac{1}{\Delta f} \sum_{t=0}^{N-1} e^{i\Delta \theta},
\] (17)
where $\Delta \theta = \Delta \theta_m - \Delta \theta_n$. By the use of a limited series expansion, (17) can be rewritten as
\[
R_{m,n}(\Delta \theta) = \frac{1}{\Delta f} \frac{\sin(N \frac{1}{2} \Delta \theta)}{\sin(\frac{1}{2} \Delta \theta)} e^{i\frac{1}{2} \Delta \theta}.\] (18)

Equation (18) demonstrates that the cross correlation is $R_{m,n}(\Delta \theta) = 0$ with $\Delta \theta = \Delta \theta_m - \Delta \theta_n = 2\pi k/N$, $k = 1, 2, \cdots, N-1$. That is, the CI spreading codes $\{e^{i\theta_0}, e^{i\theta_1}, \cdots, e^{i(N-1)\theta_0}\}$, $k = 0, 1, \cdots, N-1$ are orthogonal to each other and make up of a set of orthogonal CI spreading codes with $\Delta \theta = \Delta \theta_k = 2\pi k/N$. Therefore, CI/OFDM system is also applicable for complex signaling.

As for PO-CI spreading codes, in the case of complex signaling, the root-mean-square cross correlation that exists between two sets of orthogonal CI spreading codes is given by
\[
R_{rms} = \left[ \frac{1}{N^2} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} |R_{1,2}(m,n)|^2 \right]^{1/2},
\] (19)
where $R_{1,2}(m,n)$ refers to the cross correlation between the $m$th CI spreading code in orthogonal CI spreading code set 1 and the $n$th CI spreading code in orthogonal CI spreading code set 2. Referring to (17), the cross correlation is given by
\[ R_{1,2}(m,n) = \frac{1}{\Delta f} \sum_{\nu=0}^{N-1} e^{i(\Delta \theta - \Delta \theta_0 + \nu \theta)} = \frac{1}{\Delta f} \sum_{\nu=0}^{N-1} e^{i\left(\frac{2\pi \nu}{N} - \frac{2\pi \nu \theta}{N}\right)}, \quad (20) \]

where \( \theta \in (0,2\pi/N) \). It is easy to show that

\[
\sum_{m=0}^{N-1} |R_{1,2}(m,n)|^2 = \sum_{m=0}^{N-1} |R_{1,2}(m,n')|^2, n \neq n'. \quad (21)
\]

So that we can rewrite (19) as

\[
R_{rms} = \left[ \frac{1}{N} \sum_{m=0}^{N-1} |R_{1,2}(m,0)|^2 \right]^{1/2}. \quad (22)
\]

Substituting (20) into (22)

\[
R_{rms} = \left[ \frac{1}{N} \sum_{m=0}^{N-1} \frac{1}{\Delta f} \sum_{\nu=0}^{N-1} e^{i\left(\frac{2\pi \nu}{N} - \frac{2\pi \nu \theta}{N}\right)} \right]^{1/2} = \frac{1}{\Delta f} \left[ \frac{1}{N} \sum_{m=0}^{N-1} \sum_{\nu=0}^{N-1} e^{i\left(\frac{2\pi \nu}{N} - \frac{2\pi \nu \theta}{N}\right)} \right]^{1/2}. \quad (23)
\]

By the use of a limited series expansion, (23) can be rewritten as

\[
R_{rms} = \frac{1}{\Delta f} \left[ \frac{1}{N} \sum_{m=0}^{N-1} \sin\left[N \cdot \frac{1}{2} \cdot \left(\frac{2\pi}{N} m - \theta\right)\right] e^{\frac{N}{2} \left(\frac{2\pi}{N} m - \theta\right)} \right]^{1/2} = \frac{1}{\Delta f} \left[ \frac{1}{N} \sum_{m=0}^{N-1} \sin\left[N \cdot \frac{1}{2} \cdot \left(\frac{2\pi}{N} m - \theta\right)\right] \right]^{1/2} \]

\[
= \frac{1}{\Delta f \sqrt{N}} \left[ \sum_{m=0}^{N-1} 1 - \cos\left(\frac{2\pi}{N} m - \theta\right) \right]^{1/2} = \frac{I(\theta, N)}{\Delta f}, \quad (24)
\]

where

\[
I(\theta, N) = \frac{1}{\sqrt{N}} \left[ \sum_{m=0}^{N-1} 1 - \cos\left(\frac{2\pi}{N} m - \theta\right) \right]^{1/2}. \quad (25)
\]

For (25) can not be simplified any more, we have to seek the result by computer. By the use of MATLAB (an interactive, integrated environment for numerical computations), we get the result that \( I(\theta, N) = \sqrt{N} \) with \( \theta \in (0,2\pi/N) \). The curve of the function \( I(\theta, N) \) is shown in Figure 2. Then

\[
R_{rms} = \frac{\sqrt{N}}{\Delta f}. \quad (26)
\]

Equation (26) demonstrates that, in the case of complex signaling, the value of the root-mean-square cross correlation between two sets of orthogonal CI spreading codes has nothing to do with the phase offset \( \theta \), and is proportional to \( \sqrt{N} \). That is to say, no matter how to select the phase offset \( \theta \) in the open interval \( (0,2\pi/N) \), the root-mean-square cross correlation between two sets of orthogonal CI spreading codes has a quite large value. Consequently, PO-CI/OFDM system is not applicable for complex signaling.
3. CI-POCI/OFDM

3.1. Implementation of CI/OFDM

As we know, modulating data $s_i, i = 0, 1, ..., N-1$ onto $N$ orthogonal sub-carriers equally spaced by frequency separation $\Delta f$ can get the complex baseband signal of CI/OFDM:

$$ S(t) = \sum_{i=0}^{N-1} s_i \exp(j2\pi \frac{i}{T_s} t) $$

(27)

where $T_s$ is the duration of CI/OFDM symbol and $T_s = 1/\Delta f$.

In order to guarantee the excellent time and frequency resolution of the complex baseband signal of CI/OFDM system, the time sample rate should be 2 times of the frequency of the highest frequency component. Then letting $t = nT_s/(2N), n=0, 1, ..., 2N-1$ we can get

$$ S = S(\frac{nT_s}{2N}) = \sum_{i=0}^{N-1} s_i \exp(j2\pi \frac{i}{T_s} \frac{nT_s}{2N}) = \sum_{i=0}^{N-1} s_i \exp(j2\pi \frac{i\cdot n}{2N}). $$

(28)

Substituting $M=2N$ into (28)

$$ S = \sum_{i=0}^{M/2-1} s_i \exp(j2\pi \frac{i\cdot n}{M}) = \sum_{i=0}^{M-1} r_i \exp(j2\pi \frac{i\cdot n}{M}) = \text{ifft}(r), $$

(29)

where $r_i$ and $r$ are defined as (14) and (15) respectively.

The receiver of CI/OFDM system performs the corresponding contrary operations. Thus the flow chart of CI/OFDM system can be drawn as Figure 3.

Figure 3. CI/OFDM system.
3.2. CI-POCI/OFDM system

From Figure 1 and Figure 3 we can see that, the OFDM modulation and demodulation parts of CI/OFDM system are the same with the PO-CI despreading and spreading parts respectively. Consequently, transmitting the real part of signal $S$ will result in the approximation between $a'_{CI}$ and $a_{CI}$. This can be described as Figure 4.

For two real signals can respectively act as the inphase and quadrature components of one complex signal, two systems described as Figure 4 can make up a novel OFDM system, named CI-POCI/OFDM system, with double throughput. As CI/OFDM system is applicable for complex signaling proved in section 2.2, CI-POCI/OFDM system is applicable for both real and complex signaling. This means that CI-POCI/OFDM system is more practical than PO-CI/OFDM system. Figure 5 illustrates the architecture of CI-POCI/OFDM system.

4. SIMULATIONS

4.1. BER performance

In all simulation systems, 16QAM modulation is assumed, the frequency separation between sub-carriers is $\Delta f = 50\text{kHz}$, the OFDM symbol duration is $T_s = 1/\Delta f = 20\mu s$, and the guard interval is $T_g/8 = 2.5\mu s$. The channel environment is assumed as a static frequency selective fading channel with AWGN, and the multipath parameters is given in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Path</th>
<th>Attenuation (dB)</th>
<th>Delay (µs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Frequency domain equalization is performed in OFDM receiver, and minimum mean-square error combining (MMSEC) [5] with the same fade parameters is performed in CI/OFDM and CI-POCI/OFDM receiver. In the case of the number of sub-carriers \( N=64 \), the OFDM and CI/OFDM systems transmit 64 16QAM symbols. In CI-POCI/OFDM system, however, 128 16QAM symbols are sent over \( N=64 \) sub-carriers.

Figure 6 illustrates the BER versus normalized signal-to-noise ratio \((E_b/N_0)\) for OFDM, CI/OFDM and CI-POCI/OFDM. Referring to Figure 6, the double throughput 64-sub-carrier CI-POCI/OFDM offers 5-dB gain over the 64-sub-carrier OFDM at a BER of \(10^{-3}\). While it loses 5.5 dB relative to the 64-sub-carrier CI/OFDM. This means that CI-POCI/OFDM outperforms OFDM due to the full frequency diversity achieved via CI spreading/despreading. At the same time, the interference caused by the pseudo-orthogonality property of PO-CI spreading codes degrades the BER performance of CI-POCI/OFDM relative to CI/OFDM. On the other hand, the interference between the PO-CI spreading codes decreases as \( N \) increases. Therefore the BER performance of CI-POCI/OFDM is close to that of CI/OFDM when \( N \) is very large, e.g., \( N=1024 \), as illustrated in Figure 6. So that CI-POCI/OFDM is an excellent choice for the practical OFDM systems with large number of sub-carriers.

4.2. PAPR

Figure 7 illustrates the Cumulative Distribution Function (CDF) of the PAPR for 16QAM, 64-sub-carrier OFDM, CI/OFDM and CI-POCI/OFDM, which determined over 100,000 OFDM symbols. (PO-)CI/OFDM systems have low PAPR due to the (PO-)CI spreading/despreading. With CI approaches, CI-POCI/OFDM system succeeds the PAPR benefits.

As shown in Figure 7, CI approaches improve the PAPR statistics of OFDM. For OFDM, it is not until \( x=15.9690 \) that \( \Pr(\text{PAPR} < x) = 100\% \), and the mean of PAPR is 5.5598. While for CI/OFDM \( \Pr(\text{PAPR} < x) = 100\% \) when \( x=9.0442 \), and for CI-POCI/OFDM \( \Pr(\text{PAPR} < x) = 100\% \) when \( x=9.1431 \). The means of PAPR for CI/OFDM and CI-POCI/OFDM are reduced to 3.8946 and 4.4840, respectively. Therefore, PAPR problem of traditional OFDM is also eliminated in CI-POCI/OFDM system.
5. CONCLUSIONS

In this paper, we present that the spreading/despreading of (PO-)CI spreading codes can be implemented by IFFT/FFT. PO-CI/OFDM system is proved not applicable for complex signaling. Moreover, based on the analysis of (PO-)CI spreading codes and CI/OFDM system, a novel OFDM system architecture, named CI-POCI/OFDM, is proposed to provide double-throughput via CI approaches, and it is applicable for both real and complex signaling. The BER performance and PAPR simulation results demonstrate that, in a frequency-selective fading channel, 128-symbol, 64-sub-carrier 16QAM CI-POCI/OFDM outperforms 64-symbol, 64-sub-carrier OFDM by 5 dB at a BER of $10^{-3}$, and the PAPR of CI-POCI/OFDM is reduced evidently relative to OFDM.

REFERENCES
