avoidance mechanisms with no performance analysis. Riedi et al.[16, 17] developed a multiscale queuing analysis in the case of tree-based multiscale input models. Gao et al simulated queues fed by multiplicative multifractal processes in [7] but provided no analytical results. In contrast to these results we consider general multifractal in the wavelet domain. And derive analytical results for the queue tail asymptotic.

Our aim is to contribute to the queuing theory of multiscale queues and also to the traffic engineering implications. In this paper we present an efficient and accurate analysis of multifractal queues in the wavelet domain, including the tail asymptotic, special case, and practical applications. With the scaling nature of wavelet, the queue result is much better than the traditional analysis.

## 2. QUEUEING MODEL

All we consider a simple queuing model: a single server queue in continuous time, the serving principle for offered work is defined to be FIFO(First In, First Out), the queue has infinite buffer and constant service rate $c$, Denote by $K(t)$ the total size of work arriving to the queue from time instant $-t$ in the past up to this moment, time instant 0. The so called workload process $W(t)$ is the total amount of work stored in the buffer in time interval $(-t,0)$, that is:

$$W(t) = K(t) - ct \tag{2}$$

Our interest, however, is the current buffer length of the queue, denoted by $Q$. This is the queue length in the equilibrium state of the queue when the system has been running for a long time and the initial queue length has no influence. If this state of the system does exist, i.e., stationary and ergodicity of the workload process hold, and the stability condition for the system is also satisfied, i.e., $\lim \sup_t E[K(t)]/t < s$ then:

$$Q = \sup_{t \geq 0} W(t) \tag{3}$$

Where $W(0)$ is assumed to be 0. This equation is also referred to as Lindley's equation.

The input process $X(t)$ is considered as a general multifractal process which is defined by (1), this definition, presented by Mandelbrot et.al. in [10], describes multifractal process in terms of moments which leads to a more intuitive understanding of multifractality.

In [20], the main proposition for the analytical approximation for queue tail probabilities is presented as following:

queue size at time instant $t$. Denote by $K_r$ the aggregate traffic arriving between time instants $-r+1$ and 0. In the following, we refer to $K_r$ as representing data at time-scale $r$. We set $K_0 := 0$. Considering the equation (3), we can compute as following:

$$Q_0 = \max(Q_{-1} + K_0 - c, 0) = \max(Q_{-2} + K_2 - 2c, K_1 - c, 0) \tag{8}$$

Processing (5) iteratively, at last we can obtain the common equation:

$$Q_0 = \max(Q_{-r} + K_r - rc, ...., K_0) \tag{9}$$

Since $Q_{-r} \geq 0$ for all $r$ we must have $Q_0 \geq \sup_{r\in Z_+}(K_r - rc)$. Denoting by $-j$ the last instant the queue was empty before time instant 0, we obtain $Q_0 = K_j - jc \leq \sup_{r\in Z_+}(K_r - rc)$. Thus if the queue was empty at some time in the past,

$$Q_0 = \sup_{r\in Z_+}(K_r - rc) \tag{10}$$

We will study the quantity $\tilde{Q}_0$ obtained by taking the supermum in (7) over a finite subset of $Z_+$ corresponding to powers of 2 values for $n$—the dyadic time scale, which is easy for wavelet analysis, that is:

$$\tilde{Q}_0 := \sup_{m\in\{0,...,n\}}(K_{2^m} - c2^m) \tag{11}$$

for some fixed $n \in Z_+$. Clearly, $\tilde{Q}_0 \leq Q_0$.

We will approximate the desired tail queue probability by $P(\tilde{Q}_0 > b) \approx P(Q_0 > b)$.

Here we will concentrate on the tail queue probability of the models through wavelet analysis. The Haar wavelet scaling coefficients on the branch are related to the quantities $K_{2^m}$, that is:

$$K_{2^{n-i}} = 2^{-i/2}U_{i,2^i-1} \quad \text{for} \quad i = 0,1,..,n \tag{12}$$

obtained from the variance-time plot using time scales 512ms to 262.144s is H = 0.86. We perform our simulation and analysis under a single queue with constant service rate and AUCK traffic input. We process the real traffic simulation, the analysis estimation from (4) and estimation based on wavelet separately to validate the queue analysis simulation and prove the wavelet method present better results.

To compute the (4), we choose the time unit as dyadic ones, that is:

$$E[|X^{2^m\Delta t}|^q] = c(q)(2^m\Delta t)^{\tau_0(q)} \qquad m = 1,2,3...$$

Choose the $\Delta t$ as the time unit, then:

$$\log E[|X^{2^m\Delta t}|^q] = \tau_0(q)\log m + \log c(q) \quad q>0 \tag{17}$$

by simple linear regression, we can compute the discrete $t(q)$ and $c(q)$, then the analysis estimation can be done.

Through Discrete wavelet transform, we can get the needed parameter to perform our estimation (14).

In this paper, we present two simulation results in figure 1 and figure 2 separately. The Hurst parameter is 0.8 in figure 1, and 0.85 in figure 2. The X axis indicates the queue size in 100kb unit, Y axis indicates $P(Q > b)$. The upper dash-dotted line is the real traffic simulation, the middle solid line is the estimation based on wavelet, and the lower dotted line is the analysis estimation. Intuitively in both simulation results, the estimation in wavelet presents a better estimation.

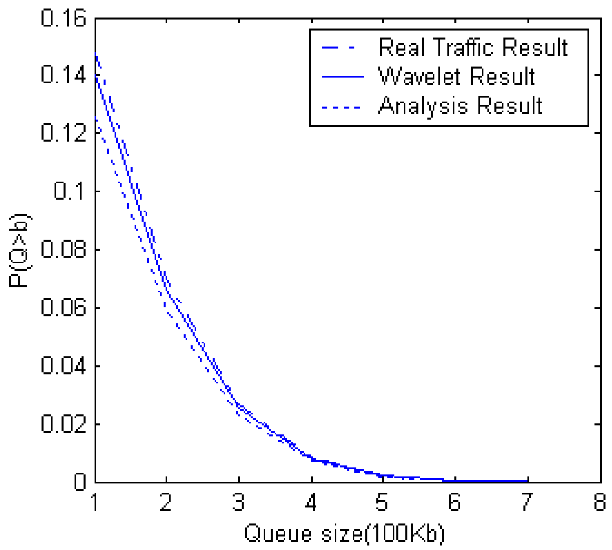


Figure 1 Queue Analysis Results

6. Feldmann, A. C. Gilbert, and W. Willinger. Data Networks as Cascades: Investigating the Multifractal Nature of Internet WAN Traffic. ACM Computer Communication Review, 28, September 1998:42–55.
7. J. Gao and I. Rubin. Multifractal modeling of counting processes of long-range dependent network traffic. In Proceedings SCS Advanced Simulation Technologies Conf., San Diego, CA, April 1999.
8. J. H¨usler and V. Piterbarg. Extremes of a certain class of Gaussian processes. Stochastic Process. Appl., 83, 1999:257–271.
9. T. G. Kurtz. Stochastic Networks: Theory and Applications, chapter Limit Theorems for Workload Input Models. Oxford University Press, 1996.
10. B. Mandelbrot, A. Fisher, and L. Calvet. A Multifractal Model of Asset Return. Yale University, 1997. Working Paper.
11. M. B. Marcus and L. A. Shepp. Sample behaviour of Gaussian processes. In Proceedings of the Sixth Berkeley Symposium, 1972.
12. S. Moln´ar and T. D. Dang. Scaling analysis of IP traffic components. In ITC Specialist Seminar on IP Traffic Measurement, Modeling and Management, Monterey, CA, USA, 18-20 September 2000.
13. S. Moln´ar and A. Vid´acs. On Modeling and Shaping Self-Similar ATM Traffic. In15th International Teletraffic Congress, Washington, DC, USA, June 1997.
14. Norros. A storage model with self-similar input. Queueing Systems, 16, 1994:387–396
15. V. J. Ribeiro, R. H. Riedi, M. S. Crouse, and R. G. Baraniuk. Multiscale queuing analysis of long-range-dependent network traffic. In Proceedings of IEEE INFOCOM 2000, Tel Aviv, Israel, March 2000.
16. R. H. Riedi, M. S. Crouse, V. J. Ribeiro, and R. G. Baraniuk. A multifractal wavelet model with application to network traffic. IEEE Trans. Inform. Theory,45(3), April 1999:992–1018.
17. R. H. Riedi and J. L´evy V´ehel. Multifractal properties of TCP traffic: a numerical study. INRIA research report 3129, Rice University, February 1997.
18. M. S. Taqqu, V. Teverovsky, and W. Willinger. Is network traffic self-similar or multifractal? Fractals, 5, 1997:63–73.
19. J.L´evi V´ehel and B. Sikdar. A multiplicative multifractal model for TCP traffic. In Proc., IEEE ISCC, Hammamet, Tunisia, July 2001.
20. S. Molnár, T. D. Dang, I. Maricza, On the Queue Tail Asymptotics for General Multifractal Traffic, NETWORKING 2002, Pisa, Italy, May 19-24, 2002