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An Efficient Solution to the Waiting Time Distribution in a Correlated Single Server Queue*

J. Kumaran [1], K. Mitchell [1], and A. van de Liefvoort [1]

[1] Department of Computer Science & Electrical Engineering,
School of Computing and Engineering,
University of Missouri–Kansas City,
Kansas City, MO 64110 USA.
jk343@umkc.edu, mitchellke@umkc.edu, and appie@umkc.edu

Abstract. In this paper, we compute the steady state waiting time distribution of a single sever queue where the arrival and the service processes are possibly correlated using a spectral decomposition of a coupling matrix. This approach is computationally more efficient than existing iterative approaches and it is possible, under certain conditions, to obtain analytical closed form solutions. This is illustrated in length by looking at some examples. For the special case of the *MEP/M/1* system, we present closed form expressions for the waiting time distribution in terms of the free parameters of the arrival process. The method has applicability in solving the inverse problem i.e., characterizing the arrival stream given the (moments of the) waiting time distribution and the service time distribution. Using empirical trace driven simulations, we provide insightful information about the effects of the source model parameters.
Keywords: Waiting time distribution, Spectral Decomposition, Matrix Exponential

1 Introduction

In modeling networks, analytic models that perform well in capturing the queueing behavior and at the same time are of smaller dimensions is a property that is highly desired. Network models based on renewal assumptions have been shown to under predict delay and losses when compared to actual traffic conditions. Near- and long-range dependent correlations have been observed in a wide range of networks that carry a wide variety of traffic. The purpose of this paper is two fold. First, we demonstrate a new approach to compute the waiting time distribution in a single server queue where both the arrival and the service processes can be correlated. The advantage of this approach is that it can be used to obtain closed form expressions for the waiting time distributions. Second, we propose to use the closed form expressions for the waiting time distribution to characterize traffic sources. The approaches that have been used to obtain the waiting time distribution often involve either the computation of the steady state system distribution by iteratively solving for the matrix quadratic equation to obtain the geometric generator $\boldsymbol{R}$ or a fixed point iteration of a non-linear matrix integral equation. Another method to compute $\boldsymbol{R}$ is the spectral decomposition of matrices. Mitrani and Mitra [1] use a spectral expansion method for solving a class of two-dimensional Markov random processes referred as a random walk on semi-infinite strips. Chakka [2] proposes a spectral expansion solution for a finite capacity multi-server system in a Markovian environment. Grassman [3] uses eigenvalue

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