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# Insensitive Traffic Splitting in Data Networks

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**Abstract.** Bonald et al. have studied insensitivity in data networks assuming a fixed route for each flow class. If capacity allocation and routing are balanced and the capacity of a given class is shared equally between the flows, the network state distribution and flow level performance are insensitive to any detailed traffic characteristics except the traffic loads. In this paper, we consider optimal insensitive load balancing executed at packet level so that the traffic of each flow may be split over several routes. Similarly to the case with fixed routing, the most efficient capacity allocation and traffic splitting policy can be determined recursively. We formulate the problem as an LP problem using either a set of predefined routes or arbitrary routes and present numerical results for two toy networks. Traffic splitting gives a clear performance improvement when compared to flow level balancing or fixed shortest path routing.
**Keywords:** Insensitivity, Traffic splitting, Whittle networks

## 1 INTRODUCTION

Load balancing has important applications in many computer and communication systems. Performance of a system is improved, if the service demands are divided between the servers in an efficient way. In static load balancing, the balancing does not depend on the system state and the optimal policy can be determined as a simple optimization problem. Better performance is obtained if the balancing depends on the system state. Dynamic load balancing problem is more complicated as the optimal policy is sensitive to detailed customer characteristics such as job size distribution. Optimal dynamic load balancing is a difficult problem and even simple systems can have nontrivial solutions [1].

In this paper, we discuss load balancing in data networks. In general, performance evaluation of data networks is difficult because detailed traffic characteristics such as flow size distribution affect the system behaviour. Data networks can be modeled as open processor sharing networks in which customers represent data flows. Recently, Bonald and Proutière have modeled networks using Whittle networks [2, 3]. If the capacity allocation is balanced and session arrivals are Poissonian, the steady state distribution is insensitive, i.e. does not depend on any traffic characteristics except the traffic loads on different routes. They introduced the concept of balanced fairness (BF) as the most efficient capacity allocation policy when static routing is used. The concept of balanced fairness has been generalized in order to analyze the performance of wireless ad hoc networks [4].

Load balancing can be executed at two different levels. Either an arriving flow is directed to a route and the same route is utilized until the flow is finished or a flow can be split between several routes. The first approach corresponds to flow level load balancing and the second to packet level load balancing. The feasibility of these methods depend on the studied network. Flow level balancing is a technically feasible solution in TCP based networks. Traffic splitting is not feasible in current TCP networks as different delays in different routes mix up the packet order. However, other approaches tolerate delay differences, for example protocols utilizing digital fountain codes do not depend on packet ordering [5].

Bonald and Proutière introduced the idea of insensitive flow level load balancing in [2]. When flow level balancing is used, an arriving flow is directed to one of the routes and the same route is utilized until the flow is finished. Optimal insensitive routing policies have been identified utilizing either local [6] or global [7] state information. Better performance is achieved if both capacity allocation and routing are optimized jointly [7, 8]. In this paper, we discuss insensitive load balancing at packet level. When balancing is executed at packet level, the packets of a flow can be divided among several routes.

The best balanced capacity allocations can be determined recursively. Starting from an empty network, the amount of utilized bandwidth is maximized in every state while satisfying two sets of constraints. The network imposes capacity constraints and insensitivity requires balance condition to be satisfied. The main contribution of this paper is the formulation of the maximization problem in the case of traffic splitting. When flows can be split onto multiple routes, the bandwidth maximization can be solved as a linear programming (LP) problem using a formulation based on network flows. In some special cases discussed in section 4, there is no need to solve the actual LP problem but the maximal allocation can be found using methods based on minimum cuts. The minimum cut approach can be used to determine bounds or approximations for the network performance.

We assume that the network has an access control policy that rejects an arriving flow if a minimum usable bandwidth cannot be provided. Blocking probability is then used as a performance metric in the numerical examples presented in section 5.

## 2 INSENSITIVITY IN DATA NETWORKS

### 2.1 Network Model

We consider a network consisting of a set of nodes $\mathcal{N}$ and links $\mathcal{L}$. The capacity of link $l \in \mathcal{L}$ is $C_l$. We assume that there are $\mathcal{K}$ classes of flows. The flows are elastic, i.e. the size of a transfer is fixed and the duration depends on the allocated bandwidth. The traffic load (bits/s) of class-$k$ is $\rho_k$. The state of the network is defined by the vector $x = (x_1, \ldots, x_{\mathcal{K}})$, where $x_k$ is the number of class-$k$ flows in progress.

As discussed in [3], the modeling approach allows the flows to be generated within sessions. A session is composed of a random number of flows and think times. The flow sizes and think time durations can have arbitrary distributions and may be correlated.

# 3 OPTIMAL INSENSITIVE TRAFFIC SPLITTING

If the traffic can be split onto different routes, more capacity can be allocated to the flows than with fixed routes. The splitting problem can be defined in two ways. Either there is a predefined set of routes for each traffic class or the routes are arbitrary. In both cases, the maximal amount of allocated capacity is unambiguous, but the there can be several ways to provide the capacity to the traffic classes. As an example, several parallel links limit the amount of traffic, but the traffic classes can be split onto the links in different ways.

## 3.1 Problem with Predefined Routes

We assume that class-$k$ flows can be split onto routes $r \in R_k$. Each route $r$ consists of a set of links $r \subset \mathcal{L}$. The bandwidth allocated for class-$k$ flows on route $r$ is denoted $\phi_k^r(x)$. The total bandwidth allocated for class-$k$ traffic is $\phi_k(x) = \sum_{r \in R_k} \phi_k^r(x)$. The allocations have to satisfy the capacity constraints

$$\sum_k \sum_{r \in R_k : l \in r} \phi_k^r(x) \leq C_l \quad \forall x, l. \tag{5}$$

The allocated capacity is maximized recursively for all the states. The problem of finding the maximal capacity allocation in a given state $x$ while satisfying the balance condition (1) and the feasibility condition (5) can be formulated as a linear optimization problem. To simplify the notation, we define $u = \Phi(x)^{-1}$ for a given state $x$. The formulation is

$$\max_{u, \phi_k^r} u \tag{6}$$

$$\text{s.t.} \sum_{r \in R_k} \phi_k^r(x) = u \Phi(x - e_k) \quad \forall k : x_k > 0, \tag{7}$$

$$\sum_k \sum_{r \in R_k : l \in r} \phi_k^r(x) \leq C_l \quad \forall l, \tag{8}$$

$$\phi_k^r(x) \geq 0 \quad \forall k, r, \tag{9}$$

where (7) is the balance condition and (8) represents the capacity constraints. The problem can be solved using standard LP algorithms. If there is only one route per class the optimal allocation is identical with the ordinary balanced fairness and can be solved using recursion formula (3).

## 3.2 Problem with Arbitrary Routes

A more general problem can be formulated by not assuming predefined routes. Capacity is utilized more efficiently, if all possible routes can be utilized instead of a set of predefined ones. In each state, the amount of traffic is maximized over all possible routes while satisfying the capacity and balance constraints.

Similarly with the problem with predefined routes, the problem can be formulated and solved as an LP problem. The amount of class-$k$ traffic on the link from node $i$ to node $j$

so that the fraction $uD_k$ of each flow is transfered. In our balanced splitting problem, the demands are $D_k = \Phi(x - e_k)$ and $u = \Phi(x)^{-1}$.

The concurrent multicommodity problem can be formulated and solved as an LP problem as seen in section 3.2. However, specialized network algorithms are significantly faster than general LP solvers in many specific problem classes. Several maximum flow problems can be solved using minimum cuts. The seminal work of Ford and Fulkerson showed that the maximum flow always equals the capacity of the minimum cut separating the source from the destination in the single commodity maximum flow problem [12]. The concept of minimum cut can be generalized for multicommodity flows as

$$\rho^* = \min_{S \subset \mathcal{N}} \frac{\sum_{i,j \in \mathcal{N}:|S \cap \{i,j\}|=1} C_{ij}}{\sum_{k \in K:|S \cap \{s_k,t_k\}|=1} D_k}. \tag{18}$$

The minimum cut equals the maximum flow $u$ for 2-commodity flows. In general, the maximum flow can be smaller than the minimum cut as Figure 1 illustrates. With more than two commodities, the equality holds for networks with a single source and multiple sinks. In networks with undirected links, the equality holds also with a single sink and multiple sources. The maximum flow minimum cut equality has been proven for many special classes of networks, see e.g. [13–15]. If the equality holds for a given network, it is sufficient to find the minimum cut in order to determine the constant $u$. This is a more straightforward approach than to solve the corresponding LP problem and leads to a recursion similar to balanced fairness defined in (3). The recursion is $\Phi(0) = 1$ and

$$\Phi(x) = \max_{S \subset N} \frac{\sum_{k \in K:|S \cap \{s_k,t_k\}|=1} \Phi(x - e_k)}{\sum_{i,j \in \mathcal{N}:|S \cap \{i,j\}|=1} C_{ij}}. \tag{19}$$

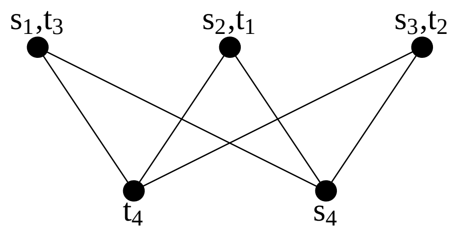


**Fig. 1.** A graph with min-cut 1 and max-flow 3/4. All demands and capacities are one. [15]

Max-flow min-cut results can be used to derive bounds for the concurrent multicommodity problem. According to our knowledge, the tightest lower bound for constant $u$ is [16]

$$u \geq \frac{1}{c\lceil \log k^* \rceil} \rho^*, \tag{20}$$

where $c$ is a constant and $k^*$ is the cardinality of the minimal vertex cover of the demand graph, i.e. the minimum number of nodes that include either the source or the sink of every source-sink pair. The lower bound can be used to determine performance bounds for insensitive traffic splitting, if the max-flow min-cut equality does not hold.

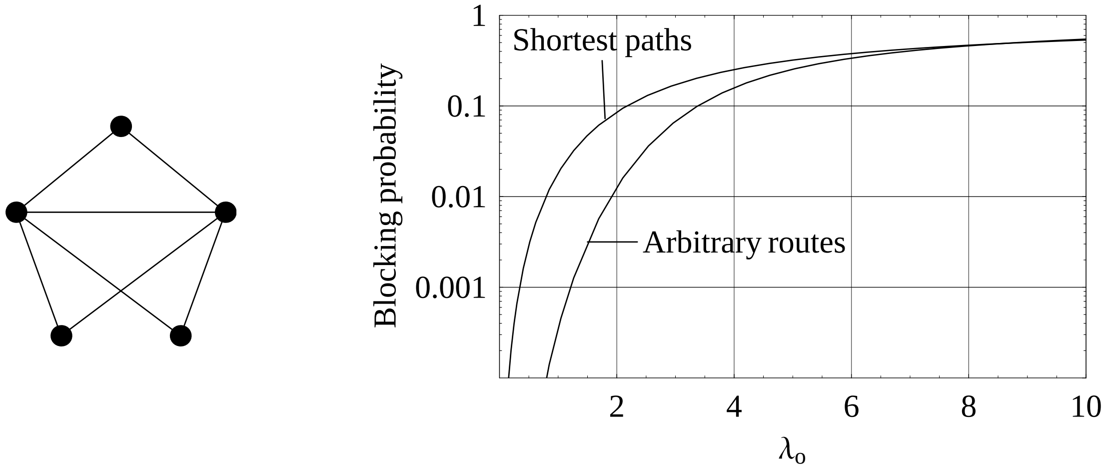


**Fig. 3.** Example network with five nodes and blocking probabilities with fixed routes and traffic splitting.

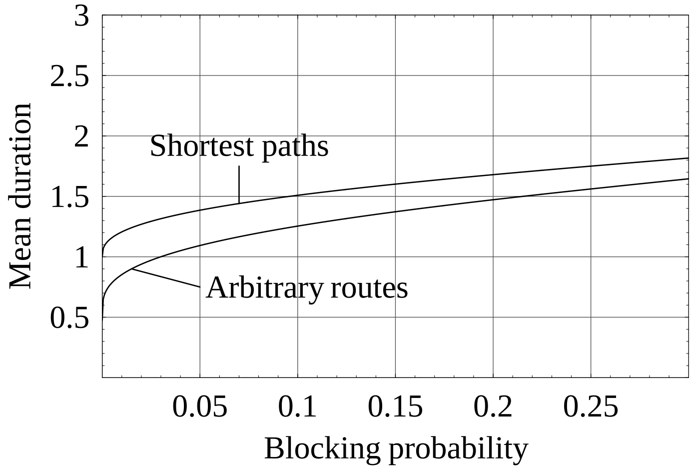


**Fig. 4.** Mean duration of an accepted flow.

minimum bandwidth is $b^{\min} = 1/3$. The blocking probabilities using shortest path routing and traffic splitting are illustrated in Figure 3.

Applying Little's formula, we get the mean transmission duration of an accepted flow

$$\mathrm{E}[T] = \frac{\mathrm{E}[|X|]}{\lambda_a} = \frac{\mathrm{E}[|X|]}{(1-B)\lambda_o}, \tag{21}$$

where $\mathrm{E}[|X|]$ is the mean number of active flows, $\lambda_a$ is the accepted flow arrival intensity and $B$ is the blocking probability. Figure 4 illustrates the mean duration of an accepted flow as a function of the blocking probability. It should be noted that a network utilizing traffic splitting carries more traffic with a given blocking probability. For comparison, Figure 5 illustrates the mean duration of an accepted flow as a function of accepted flow arrival intensity. In both cases, traffic splitting reduces the durations significantly. The advantage decreases as the amount of traffic increases.

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