A Study of Tariff Mechanisms for Mobile Phone

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Abstract: By putting forward the model of the effective traffic between the calling party and receiving party, and the model of the operator’s income based on the theory of demands and consumer behaviors, the paper analyzes the way how the different mobile tariff mechanisms influence the effective traffic and the income with the variety of the price per unit of traffic. The study finds out that there is a break-even point of the price between CPP (Calling Party Pays) and BPP (Both Parties Pays) mechanisms, and there is a lower limit price under CPP because of the extra cost for the income settlement existed. So it should be very cautious to transfer BPP to CPP in China.

Key words: mobile communication, tariff mechanisms, calling party pays, both parties pays, break-even point.

1. INTRODUCTION

Mobile phone tariff is always the hotspot of telecommunication regulation. In China, adopting CPP (Calling Party Pays) mechanism or remaining BPP (Both Parties Pays, another term is Mobile Parties Pays) mechanism is the dispute focus. Because that there is no an accepted theory to prove which mechanism is more reasonable at now in all of the world, there is no universal standpoint about the issue, so different countries have implemented their own tariff mechanism[1],[2],[3].

Based on the essential principle of economics and consumer behaviors, the paper establishes a mathematic model to analyze the movement rules of effective traffic and

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income of the operators with the variety of the price per unit of traffic under CPP and BPP respectively. The objective of the paper is to find out an optimal tariff mechanism and its corresponding conditions, which can give useful suggestions to the regulation organization in China.

2. HYPOTHESIS OF THE MODEL

A set of basic hypotheses below is made to establish a mathematic model.

1) Assuming the cost of mobile phone traffic per unit keeping unchanged within a time period, and adopting the cost-plus method to price mobile phone traffic in unit of time, say $p$. At this assumption, it is obvious that $p$ is dependent on the cost and average profit rate, but independent of the tariff mechanisms. Here, by tariff mechanisms we mean mainly CPP and BPP, which only determine the proportions of $p$ paid by the calling party and receiving party.

2) According to the theory of demand [4], supposing average calling traffic per unit of time is a linear function about $p$ [5] without losing generality, and the average accepted rate at receiving party is also a linear function about $p$.

3) If only one party pays, there exists an extra cost for the income settlement, say $C$, inside an operator and between the mobile operators, otherwise no such cost.

3. THE BASIC MODEL OF TARIFF MECHANISMS

According to the above three assumptions, a basic model is set up as shown in Figure 1.

In Figure 1, parameter $\alpha$ is the proportion of $p$ paid by the calling party. In practice, $\alpha$ have three values: 0, 1/2, 1. $\alpha=0$ means receiving party pays (RPP), $\alpha=1/2$ means both parties pays (BPP), and $\alpha=1$ indicates calling party pays (CPP).

Let $Q$ be the traffic demand of calling party in unit of time. Obviously $Q$ is the function of $\alpha$ and $p$. Given $\alpha$, $Q$ is only the linear function of $p$ as below,
\[
Q(p) = \begin{cases} 
q_1 & \alpha = 0 \\
q_1 - \frac{1}{2}k_qp & \alpha = 1/2 \\
q_1 - k_qp & \alpha = 1 
\end{cases}
\] (3-1)

Where, \(q_1\) is the expected maximum calling traffic per unit of time; \(k_q\) is the slope of demand line for CPP, half of \(k_q\) is for BPP (without losing generality, we also assuming that calling party has a linear response to the price), and obviously 0 for RPP.

Let \(R\) be the accepted rate of receiving party when call arrivals. Obviously \(R\) is also the function of \(\alpha\) and \(p\). Given \(\alpha, R\) is only the linear function about \(p\) as below,

\[
R(p) = \begin{cases} 
q_1 - k_r p & \alpha = 0 \\
q_1 - \frac{1}{2}k_r p & \alpha = 1/2 \\
q_1 & \alpha = 1 
\end{cases}
\] (3-2)

Where, \(r_1\) is the expected maximum accepted rate; \(k_r\) is the slope of accepted rate line for RPP, half of \(k_r\) is for BPP, and obviously 0 for CPP.

Let \(N\) be the effective traffic in unit of time that is accepted by the receiving party, according to the definitions above, we have,

\[
N(p) = Q(p) \times R(p) = \begin{cases} 
N_0 = q_1 \times (q_1 - k_r p) & \alpha = 0 \\
N_{1/2} = (q_1 - \frac{1}{2}k_q p) \times (q_1 - \frac{1}{2}k_r p) & \alpha = 1/2 \\
N_1 = (q_1 - k_q p) \times r_1 & \alpha = 1 
\end{cases}
\] (3-3)

Obviously, only those calls accepted by the receiving party make sense to the both communication parties, and make money for the operators, so we call \(N(p)\) the effective traffic function.

4. ANALYSIS OF THE EFFECTIVE TRAFFIC MODEL

From the point of psychics of consumer behavior, the calling party is an active party with his/hers own purpose, so he/her should pay for calls; but the receiving party is a passive party, if \(p\) is paid totally by the receiving party, i.e. RPP is adopted, lot of calls will be rejected and the effective traffic will be lower than that CPP is adopted. In the matter of fact, only very limited telephone services adopt RPP all of the world, such as a very small part of international toll telephone service and 800 services. So we have,

**Axiom 1**: CPP dominates RPP when \(p > 0\), i.e., \(N_1 > N_0, \forall p > 0\).

**Consequence 1**: \(q_1k_r > r_1k_q\).

Consequence 1 means that the receiving party is much more sensitive to the price than the
calling party, generally we have \( q_1 < r_1 \leq 1 \), so we get \( k_r >> k_q \).

Comparing \( N_{1/2} \) and \( N_0 \), we have,

\[
N_{1/2} - N_0 = \left( q_1 - \frac{1}{2} k_q p \right) \times \left( r_1 - \frac{1}{2} k_r p \right) - q_1 (r_1 - k_r p) = \frac{1}{4} k_q k_r p^2 + \frac{1}{2} (q_1 k_r - r_1 k_q) p \geq 0 \quad (4-1)
\]

That gives,

**Consequence 2**: BPP dominates RPP when \( p > 0 \).

So we will not discuss RPP afterwards. Comparing the effects of BPP and CPP with regards to the variety of \( p \), we have,

\[
N_{1/2} - N_1 = \frac{1}{4} k_q k_r p^2 + \frac{1}{2} (q_1 k_q - r_1 k_r) p \quad (4-2)
\]

Let \( N_{1/2} - N_1 = 0 \), then we get two roots,

\[
p_1 = 0, \quad p_2 = \frac{2(q_1 k_r - r_1 k_q)}{k_q k_r} > 0 \quad (4-3)
\]

By specifying proper values of \( k_q \) and \( k_r \), we can get \( N_i > 0 \) at the point \( p_2 \), then we have \( 2r_1 k_q > q_1 k_r \) by substituting \( p_2 \) into \( N_1 \). The curves of effective traffic under different tariff mechanisms are shown with solid lines in Figure 2.

![Figure 2 The effective traffic under different tariff mechanisms](image)

In Figure 2, \( p_2 \) is a break-even point at which \( N_{1/2} - N_1 \) cross zero. That means CPP dominates BPP when \( p_1 < p < p_2 \), and BPP dominates CPP when \( p > p_2 \).

This conclusion is derived from the psychics of the consumer behavior. Obviously, the consumers like CPP at a lower price. But, what is the behavior of operators and what tariff mechanism they would like to choose.
5. ANALYSIS OF INCOME MODEL FOR OPERATORS

Which mechanism the mobile operators will adopt relies on the service income that they could earn. Obviously, the price and effective traffic are main factors to influence the income when the running cost keeping unchanged.

According to the analysis of the effective traffic model, we only discuss BPP and CPP mechanisms. Let $I$ be the service income in unit of time. Obviously $I$ is the function of $N, p$, and the income settlement cost $C$ as shown below,

$$I(N, p, C) = \begin{cases} 
I_{1/2} = (q_1 - \frac{1}{2}k_qp)(r_1 - \frac{1}{2}k_rp) \times p & \alpha = 1/2 \\
I_1 = (q_1 - k_qp)r_1 \times p - C & \alpha = 1 
\end{cases}$$  \hspace{1cm} (5-1)

Firstly, we analyze the characteristic of the income under CPP. Because there is a settlement cost in CPP, there exits a lower limit price $p_{1min}$ to keep $I_1 > 0$. And $I_1$ is a quadratic equation of $p$ and the second derivative of $I_1$ is negative, so the maximum value of $I_1$ exists at the point $p_{1max}$ as shown in Figure 3 (a).

Then we analyze the characteristic of the income under BPP. $I_{1/2}$ is a cubic equation of $p$, there are three roots as shown in (5-2) and Figure 3 (b),

$$p_{01} = 0, \quad p_{02} = \frac{2r_1}{k_r}, \quad p_{03} = \frac{2q_1}{k_q}$$ \hspace{1cm} (5-2)

Let the first derivative of $I_{1/2}$ equals zero, we obtain two extremums of $I_{1/2}$, and with the second derivative of $I_{1/2}$, we have,
\[
\begin{aligned}
\left\{
\begin{array}{l}
  p_{1/2_{\text{max}}} = \frac{(q_{1}k_{r} + r_{1}K_{q}) - \sqrt{(q_{1}k_{r} + r_{1}K_{q})^2 - 3q_{1}r_{1}k_{r}K_{q}}}{1.5k_{r}K_{q}} \\
  p_{1/2_{\text{min}}} = \frac{(q_{1}k_{r} + r_{1}K_{q}) + \sqrt{(q_{1}k_{r} + r_{1}K_{q})^2 - 3q_{1}r_{1}k_{r}K_{q}}}{1.5k_{r}K_{q}}
\end{array}
\right.
\end{aligned}
\tag{5-3}
\]

By the inequality \(2r_{1}k_{q} > q_{1}k_{r}\) we got above, we know that the positive range of \(I_{1/2}\) covers that of \(I_{1}\), but their maximums are not at the same point of \(p\), so we define \(\Delta I = I_{1/2} - I_{1}\) and have

\[
\Delta I = \frac{1}{4}k_{q}k_{r}p^3 + \frac{1}{2}(r_{1}k_{q} - q_{1}k_{r})p^2 + C
\tag{5-4}
\]

Letting

\[
\frac{\partial \Delta I}{\partial p} = \frac{3}{4}k_{q}k_{r}p^2 + (r_{1}k_{q} - q_{1}k_{r})p = 0
\tag{5-5}
\]

We get

\[
p^* = \frac{4(q_{1}k_{r} - r_{1}k_{q})}{3k_{q}k_{r}} > 0
\tag{5-6}
\]

It is easy to see that the second derivative of \(\Delta I\) is positive at \(p^*\). Substituting \(p^*\) into equation (5-4), we get the minimum value of \(\Delta I\), say \(\Delta I_{\text{min}}\),

\[
\Delta I_{\text{min}} = \frac{1}{6}(r_{1}k_{q} - q_{1}k_{r})(p^*)^2 + C
\tag{5-7}
\]

Let \(\Delta I_{\text{min}} > 0\) at \(p^*\), then we have,

\[
C > \frac{(p^*)^2(q_{1}k_{r} - r_{1}k_{q})}{6}
\tag{5-8}
\]

In this case, BPP totally dominates CPP as shown in Figure 4 (a).

Let \(\Delta I_{\text{min}} < 0\) at \(p^*\), then we have,

\[
C < \frac{(p^*)^2(q_{1}k_{r} - r_{1}k_{q})}{6}
\tag{5-9}
\]

In this case, we have \(p_3\) and \(p_4\) as roots of equation \(\Delta I = 0\). So CPP dominates BPP only within the field \((p_3, p_4)\), and BPP dominates CPP within the fields \((0, p_3)\) and \((p_4, \infty)\) as shown in Figure 4 (b).
6. CONCLUSIONS

According to the analyses of the effective traffic and income of operators with two models, we get several conclusions as follows.

1) RPP is totally dominated by CPP and BPP. That is why only very limited communication services adopt RPP mechanism.

2) There is a break-even point of price between CPP and BPP from the effective traffic point of view. It means that BPP mechanism is a reasonable choice at a higher price, otherwise CPP mechanism is suitable. The practice demonstrates that true, the fixed telephone service adopts CPP in most of the world, and the mobile phone service adopts BPP mostly at the initial time of the service.

3) Because of existing an extra cost of income settlement inside an operator and between the operators, the range of the price that CPP dominates BPP may be dwindled, and there exists a lower limit price to keep operators financial balance.

4) So at the very low price due to competitions or technical innovation, BPP becomes suitable even in the fixed telephone service. That is also true in practice. The monthly fixed tariff is adopted for more and more operators in different countries and regions now, which is a kind of BPP in fact.

5) To sum up the above, we achieve that it should be very cautious to transfer BPP to CPP for mobile phone service in China.

Some continued studies are desired:

1) Parameters $q_1$, $r_1$, $k_q$, $k_r$ need to be estimated from the practical data, then the some important points of price, such as $p_2$, $p^*$, $p_3$, $p_4$, can be obtained.

2) Traffic demand function $Q$ can be defined more delicate. In this paper, $Q$ is a linear demand function by a virtual lumped consumer.
7. A CASE STUDY

In the reference [5], Professor Ping Xin-qiao has calculated a curve about consumer demand on the mobile phone according to the operation data of China Mobile in 2001. Combining the model hypothesis, we can get the calling party demand curve under CPP as below,

\[ N_1 = \frac{1}{9}(84 - p_h) \quad (7-1) \]

Where, \( N_1 \) is measured in minutes per busy hour, so \( p_h \) is a price for one hour. Dividing the equation (7-1) by 60 minutes, we get \( N_1 \) in traffic as unit (i.e. in Erl) and \( p \) is a price for one minute, as shown below,

\[ N_1 = (q_1 - k_q p) \cdot r_1 = (0.1556 - 0.1111 \cdot p) \cdot r_1 \quad (7-2) \]

Letting \( r_1 = 1 \), we can get \( q_1 = 0.1556 \text{Erl}, k_q = 0.1111 \). From Consequence 1, \( q_1 r_q > r_1 k_q \), we can get \( k_q > 0.711 \). BPP was adopted in China in 2001, considering the monthly fixed fee and prepay service in that year, we can estimate the price for both parties, \( p = 1.0 \text{ RMB Yuan/per minute} \), so obtain \( N_1 = 0.044 \text{Erl} \) in busy hour, which is a reasonable value in network planning. It is easy to see that \( k_q \in [0.8, 1.0] \) is a reasonable area when \( p = 1 \) from the accepted rate function (3-2). According to the equation (4-3),

\[ p_2 = \frac{2(q_1 k_q - r_q k_q)}{k_q k_r} = 2 \left( \frac{q_1}{k_q} - \frac{1}{k_r} \right) \quad (7-3) \]

we can get the break-even point \( p_2 \in [0.3, 0.8] \).

Now the real price of per minute for mobile phone, either China Mobile or Unicom, is more than 0.3, so it should be very cautious to transfer BPP to CPP in China.

REFERENCES


