

## Non-exponential tail probabilities in queueing systems

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**Abstract.** Obtaining (tail) probabilities from a transform function is an important topic in queueing theory. To obtain these probabilities in discrete-time queueing systems, we have to invert probability generating functions, since most important distributions in discrete-time queueing systems can be determined in the form of generating functions. In this paper, we analyze the tail behavior of two particular random variables in priority queueing systems, by means of the dominant pole approximation. We show that obtaining this tail behavior can be complex and that the obtained tail probabilities are not necessarily exponential (as in most queueing systems). Further, we show the impact and significance of the various system parameters for the type of tail behavior. Finally, we compare our approximation results with simulations.

**Keywords:** priority queueing systems, tail probabilities, dominant pole, non-exponential behavior

## 1 Introduction

Many probability distributions of interest in (discrete-time) queueing systems can be determined in the form of probability generating functions (pgf's). It is widely recognized that pgf's are very useful to extract numerical results, e.g., to calculate moments. However, a seeming disadvantage of working with pgf's is that it is not always easy to explicitly calculate corresponding probability mass functions (pmf's). Often, we are only interested in the (asymptotic behavior of) tail probabilities (pmf for larger values), because these tail probabilities typically represent the 'exceptional' situations in a queueing system (or more generally, a communication network), of which we want to estimate the frequency of behavior. E.g. the probability that the delay is larger than a given value  $N$  or the packet loss are examples of interesting performance measures for which the calculation of the tail probability is usually sufficient.



















