An argument for reduction of simulation complexity

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One way of creating Long Range Dependent (LRD) traffic is by the superposition of a large number of on-off sources. We show that a small number of sources is enough to generate LRD traffic thus addressing the scalability issue of the network simulation methodology. Our strategy lies in increasing the activity of each source thus reducing the required number of sources for producing target traffic. However, this impacts the burstiness of the source. We first build a Semi-Markov (SM) model for the arrival process and show that while the first moment of the traffic remains independent of the burstiness, the second order moments depend on it. This dependence directly influences the performance parameters. A Quasi Birth Death model (SM/M/1 queue) is used to study the impact of burstiness on the mean packet delay. We show the same dependency in the aggregate traffic and then define a criterion for reduction of simulation complexity. Our results show that 24 times reduction in the number of typical sources is easily possible.

1 Introduction

The Internet traffic is Long Range Dependent (LRD) in its nature [1,2]. LRD is attributed mainly to the session characteristics, or the user behavior. Various methods to generate LRD traffic have been proposed. A summary is presented in [3]. A sophisticated approach is the generation of traffic by the superposition of on-off sources [4]. If either, or both, the on- and off-times are heavy-tailed distributed then the resulting traffic is LRD in nature.

Large capacity networks carry large volumes of traffic. A large number of traffic sources would be required to simulate a high speed network. However, each source puts a demand on the computer resources (CPU and memory). Therefore, the available finite computing resources bound the number of sources that can be simulated. The scalability argument is true for every type of traffic source. A real scenario is considered to lay out the problem clearly and emphasize the importance of the solution. The more commonly available Linux kernel allows 4GBytes (8GBytes in future) of memory access in typical simulation platforms. We do not consider the “hard-disk swapping memory” as the simulation times become unacceptably large when it is used. The kernel and relevant applications/drivers/programs use 1GBytes of the total available memory leaving 3GBytes for the simulator. So the first obvious question is that how much traffic could be simulated with the available 3GBytes? The estimates show that each traffic source in our simulator consumes approx. 20KBytes of memory. This means that approx. 150,000 sources can be simulated. Each source is tuned to produce the traffic of a “typical” user which is approx. 12Kbps. This means that with 150,000 sources we can simulate up to 1.8Gbps of total traffic. That is the limit.

So how do we increase the capacity of the system to simulate bigger and faster networks? What are the possible solutions? A solution is to reduce the memory requirement of each source. We have dealt with that in [5]. This paper presents yet another solution based on the “reduction” of the number of sources required to produce the target traffic. For example, if we could some how double the amount of traffic produced by each source then we would need half the number of sources required to produce the same target traffic. In the context of the above discussion, if each source produced 24Kbps instead of 12Kbps then we could simulate 3.6Gbps of traffic with the same available memory! So theoretically if we could make the activity of each source 150,000 times higher then we would need just one source to produce 1.8Gbps of traffic!

However, this is not possible. There are limitations. We will show in this paper that the activity of a source can be increased and, therefore, a significant reduction in the total number of sources required to produce the same target traffic can be achieved up to a certain point without any significant deviations in the important traffic and performance parameters. However, beyond that point, increasing the activity of a source has a direct impact on the traffic statistics. It was observed in [6] that the Throughput (TP), Co-efficient of Variation (CV), Hurst parameter (H) and Mean Packet Delay (MPD) remain independent from the number of sources up to a certain point of aggregation but beyond that point H and MPD show dependence. However, no explanation for this phenomenon
was given. In this paper we explore the theoretical background for this transition while giving readers a deeper understanding of the LRD traffic. We will show that this is linked with the burstiness of the source.

The traffic produced by the source has long range correlations, or has a LRD character. We use a Semi-Markov (SM) model to describe the correlated arrival process. The SM model is used to quantify the first and second order moments of the arrival process. We show that the second order moments of the arrival process depend upon the burstiness of the source. We use plots of CV and Auto-Covariance (ACV) to show that reduced burstiness causes the arrival process to lose memory.

Then, assuming a markovian service process, we use a SM/M/1 model - a Quasi Birth Death (QBD) process - to calculate the MPD for a typical user. We show that the MPD of a typical web-user is well bounded by an SM/M/1 queue. This acts as a bench mark. Then we show that the simulation complexity reduction strategy causes the queuing to transit from a QBD type SM/M/1 model to a Birth Death (BD) type M/M/1 model - the arrival processes becomes memoryless. The simulation reduction strategy causes the correlation in the arrival process to disappear so that M/M/1 becomes a valid queue model at the other extreme. The queue occupancy in M/M/1 type of a system is much lower than that of a SM/M/1 type thus MPD in the earlier case are much smaller. Then we study the first and second order moments of the aggregate traffic (multiples sources) and show that the same dependency exists between the burstiness of the source and the correlations in the traffic. We will use $H$ parameter that measures the degree of LRD. A weaker $H$ indicates a weaker correlation structure. Based on these observations, we formulate a simulation complexity reduction rule. We reduce the number of sources only to a point that keeps the correlation structures in the arrival process intact leading to typical MPD’s. Beyond this point the MPD’s would be smaller as LRD in the arrival process becomes weaker as compared to the LRD of a typical user - the simulation complexity reduction strategy will not be correct any more.

In section 2, we give an insight into the simulation complexity reduction strategy. In section 3, a brief intro to the Matrix Exponential (ME) type of distributions is given. We will discuss the heavy-tailed Truncated Power Tail (TPT) distribution in this section that would also facilitate the discussion in the following sections. In section 4, the SM arrival process for a single source is presented. Section 5. contains the Quasi Birth Death (QBD) model and its solution methodology. In the last section, we discuss the simulation setup and then the results.

2 Simulation complexity reduction argument

We briefly discuss the details of the HTTP-TCP source [7] that is used in the context of this discussion. It is modeled as an on-off source. It generates very realistic web-traffic as the actual transport of HTTP web-pages is done by TCP. On getting a user request, the HTTP protocol at the session level fetches the requested web page and passes it to TCP. TCP transports the web page from the web-server to the user. Because of its prevalence, HTTP-1.1 is considered at the session layer. It sends the web pages through a persistent connection. This means that a single connection is used to transport all the files in the web page. Let $V$ be the average file size that the web server generates. Let $Z$ be the average number of objects per web page. If $F$ denotes the average web-page size then $F = V \times Z$. The transmission of an average web page by TCP level is done on average in $T_{on}$ time. After the download of a web page, the connection is closed and the user remains inactive for an average $T_{off}$ time, the user think-time, before making the next request. Each web user cycles through this on and off behavior. The on-time is heavy-tailed distributed because of the heavy-tailed web-page sizes distribution. We assume that the off-time is negative exponentially distributed. Assuming that in an appropriately dimensioned network there are negligible packet losses, in [7] the authors came up with the following expression for the throughput of a single traffic source.

$$TP = \frac{F}{N \times RTT + T_{off}}, \quad (1)$$

where $RTT$ is the Round Trip Time and $N$ is the average number of $RTT$’s required to transport an average web page of size $F$. Therefore, the source on-time, $T_{on} = N \times RTT$. Let $M$ be the total number of sources. Then the aggregate throughput (TP) resulting from $M$ sources, or typical users, is given as:

$$TP = M \times \frac{F}{N \times RTT + T_{off}}. \quad (2)$$

The idea of reduction in the complexity of simulation is that by reducing $T_{off}$, the activity of one source can be increased so that a smaller number of sources is required to produce the target traffic. Let $M \leq W$ be the reduced number of sources. We define $Z$ as the aggregation level. Then:
\[ T_{\text{off},w} = \frac{T_{\text{off}}}{Z}, \quad W = \frac{M}{Z}, \]  

where \( T_{\text{off},w} \) is the scaled user mean think-time. Assuming \( T_{\text{off}} \gg N \ast \text{RTT} \), the aggregate TP from \( W \) sources can be written as,

\[
TP = W \ast \frac{F}{N \ast \text{RTT} + T_{\text{off},w}}.
\]  

For example, if \( Z = 2 \) then the mean off-time is reduced by half and \( W = M/2 \) sources would be required to produce the same target TP. In other words one source would be doing the job of two. Equation (4) hides the fact that reducing the number of sources by scaling the off-time impacts the correlation structures thus leading to different queue occupancies. In the coming sections, we will build a SM model for the on-off traffic source to express the second order moments. We will later show the loss of correlation structures when \( T_{\text{off}} \) is reduced. We now introduce \( b \), the burstiness of source as:

\[
b = \frac{T_{\text{off},w}}{T_{\text{on}} + T_{\text{off},w}}, \quad 0 \leq b \leq 1
\]  

For \( T_{\text{off},w} \gg T_{\text{on}} \), \( b \approx 1 \) that indicates high burstiness of the source. For \( T_{\text{off},w} = 0 \), \( b = 0 \) i.e., the source does not have any burstiness. It is important to mention here that for the rest of the paper, we will use burstiness in connection with off-time of the on-off source. We will use this parameter to argue for a minimum number of sources required for simulation.

### 3 Matrix Exponential distributions

We are interested in finding the mean response time for a typical web user. We want to study the QBD type SM/M/1 queue. We start with an introduction to ME distributions and give an example of a typical distribution like negative exponential and then discuss TPT distribution which will be used in the context of our discussion. Following this introduction we will model the on-off type of a source with an SM type of process leading to an analysis of a SM/M/1 queue. Phase type distributions were introduced by Neuts in 1975 [8] which were later generalized to ME distributions [9].

Consider a system composed of many states. We want to quantify the time spent by the customer in the system. The transition probabilities of going from one state to another in the system are given by the matrix \( P \). The completion rate matrix \( M \) is a diagonal matrix of each state’s leaving rates. Given that a customer started at state \( i \), let \( Q \) be a vector whose component \( Q_i \) gives the mean time to leave the system. Then,

\[
Q' = M^{-1}e' + PQ' = [M(I - P)]^{-1}e' = Ve',
\]

where \( e' \) is a column vector of all 1’s and \( I \) is an identity matrix of appropriate dimensions. Equation (6) is a sum of the mean time in a given state and the time in the system after the transition. The matrix \( V=M(I-P)^{-1} \) is called the service time matrix whose component \( V_{ij} \) gives the total mean time spent at state \( j \) until the customer who started in state \( i \) leaves the system. The inverse of service time matrix is the service rate matrix \( B=[M(I-P)] \).

There is one more detail and that is the entrance probabilities. A row vector \( p \) is the entrance vector whose component \( p_i \) gives the probability that the customer will go to the state \( i \) after entering the system. At this point, we consider the simple case that the new customer enters the system independently of any previous customer. This will be modified in the case of Semi-Markov (SM) process where there is a dependence structure.

The pair \( <p,B> \) completely describes the system and gives rise to Reliability Function for the time spent by the customer in the system: \( R(t) = p_0 \exp(-tB)e' \). The moments for a random variable \( X \) are then given by:

\[
E(X^n) = n!pV^n e'.
\]  

We now give examples of some ME distributions, specially the heavy-tailed TPT distribution which will be used in the later discussion. An exponential distribution can be represented by a single state in the system. Then \( p=1, B=\mu \), where \( \mu \) is the mean rate of the exponential distribution. Also, \( V=\frac{1}{\mu} \).
Truncated Power Tail distribution: Consider Fig. 1. Let $T$ be the truncation level, or the number of states in the TPT distribution. This is a hyper-exponential distribution in which the probability of entering state $i$ is given as $g_i$ and the leaving rate of state $i$ is given as $r_i$:

$$g_i = \frac{\theta^{i-1}(1-\theta)}{1-\theta^T}, \quad i = 1, ..., T, \quad r_i = \frac{1}{\gamma^i} \frac{1-\theta}{1-\theta^T} \frac{1-(\theta\gamma)^{T-1}}{1-\theta\gamma} E(X) \quad i = 1, ..., T. \quad (8)$$

The probability $g_i$ of entering into a state $i$ decays geometrically by a factor $\theta < 1$ (in this paper we used $\theta = 0.5$ as given in [10]). The state holding time (inverse of $r_i$) grows geometrically by a factor $\gamma = 1/\theta^{1/\alpha}$, where $\alpha$ is the shape-parameter of the TPT distribution. The reliability function for such a distribution is given as:

$$R_T(x) = \frac{1}{1-\theta\gamma} \sum_{j=0}^{T-1} \theta^j e^{-(\theta^j\gamma x)}$$

This reliability function shows power-law behavior $R(x) \sim x^{-\alpha}$ for several orders of magnitude before it drops exponentially [11]. The higher the number of states, or the truncation level, the later the drop-off occurs. This imparts the LRD character to the traffic. The relevant matrices are given as:

$$p = \frac{1-\theta}{1-\theta^T} [\theta^0, ..., \theta^{T-1}], \quad B = \mu \begin{bmatrix} \frac{1}{\gamma^0} & 0 & \cdots & 0 \\ 0 & \frac{1}{\gamma^1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \frac{1}{\gamma^{T-1}} \end{bmatrix}, \quad V = \frac{1}{\mu} \begin{bmatrix} \gamma^0 & 0 & \cdots & 0 \\ 0 & \gamma^1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \gamma^{T-1} \end{bmatrix} \quad (10)$$

4 Semi-Markov process

A typical user is modeled as an on-off source as discussed in section 2. The on-off model is shown in Fig. 2. The TPT distribution considered in the previous section is used as the on-times distribution (right hand side, same as Fig. 1.). Heavy-tailed sized pages are then produced in the on-phase. These pages are transported to the user through packets which, under our assumption, have negative exponentially distributed inter-arrival times, or the packet generation process is poissonian. Then a user stays in the on-phase for a time that is a function of the web-page size chosen from the TPT distribution and $r$, the packet generation rate in the on-state. As defined in section 2. the user is in the on-state for $T_{on}$ mean time. The user goes into the off-state after downloading the page. As defined in section 2. the user is in the off-state for $T_{off}$ mean time. We consider the off-phase as negative exponential distributed (left hand side of Fig. 2). The off-time is tied to the burstiness of the source through (5).

There are several approaches to model an on-off source, however, because of the mathematical tractability of ME algebra we choose to model the source as an SM process that captures the concept of correlations in traffic [12] i.e., the idea that the next customer enters the system in a state that depends upon the leaving state of the last customer. We will use the ME algebra discussed in the previous section to generate the first and second order moments for the on-off type of traffic source. However, the assumption of independence considered in the last section has to be modified that means that entrance vector $p$ must be replaced.
Another matrix \( L \) is introduced to capture the behavior of the customer after the departure. The component \( L_{ij} \) gives the departure rate from state \( i \) after which the next customer arrives in state \( j \) of the process. The matrix \( B \) introduced in the previous section represents the behavior of the customer before departure. \( L \) and \( B \) must be consistent, i.e. \( Le' = Be' \) because both represent the customer departures.

In steady state, the normalized left eigenvector \( p_{sm} \) of \( VL \) for the eigenvalue of 1 is the steady-state substitute for \( p \) vector of the renewal process. The components of \( p_{sm} \) give the probabilities that the new customer will give to the respective state when the process is in the steady-state. The process has \( T+1 \) states. The process goes from off-state (state 1) to one of the sub-states of the \( T \) on-states (2,..,\( T+1 \)) with, \( P_{c1+1}^c = g_i \). The probability that chain moves from one of the TPT sub-states to the off-state is 1, \( P_{c1}^c = 1 \), \( k = 2,.., T+1 \). The subscript \( c \) indicates the underlying chain. The state departure rate matrix \( M^c \):

\[
M^c = \text{diag}\left(\frac{1}{\gamma^0}, \frac{\mu}{\gamma^0}, ..., \frac{\mu}{\gamma^{T-1}}\right)
\]

The packet generation process is poissonian. Now, there are two ways the process can leave a sub-state of TPT: through state completion rates given by matrix \( M^c \) and through the packet generation process given by \( L \) matrix.

Then, for the SM process:

\[
M = M^c + L.
\]

Leading to state transition matrix:

\[
P = \begin{bmatrix}
0 & p_0 \\
\text{j}' & 0
\end{bmatrix},
\]

where \( p_0 = [g_1,..,g_T] \) and \( j = \frac{\mu/\gamma^i-1}{\mu/\gamma^{i-1}+r}, i = 1,..,T \). The vector \( j \) gives the modified state transition probabilities, considering that the inter-state transitions occur if the state time (\( M^c_{ii} \) for state \( i \)) is smaller than the packet generation time (rate \( L_{ii} \) for state \( i \)). Knowing \( P \) and \( M \), it is straightforward to find \( B = [M(I-P)] \). Also, as discussed above the vector \( p_{sm} \) replaces the \( p \) vector, leading to characterization of the SM process as \( \langle p_{sm}, B \rangle \) pair. The \( n^{th} \) moments of the SM process come out as:

\[
E(X^n) = n!p_{sm}V^n e'.
\]

The autocovariance function for lag-\( k \) [10]:

\[
r(k) = p_{sm}V((VL)^k - e'p_{sm})Ve' \quad k = 1,2,..
\]

We show theoretical results for CV and ACV. Packet generation rate is set as \( r = 120 \text{packets/sec} \). The typical mean user think-time \( T_{off} = 40 \text{sec} \) and is negative exponentially distributed and the web-page sizes distribution is heavy-tailed distributed with an average web-page size of 60KBytes [20]. For Maximum Segment Size (MSS) =1460Bytes, 60KBytes = 42packets. This makes \( T_{on} = \frac{42\text{packets}}{120\text{packets/sec}} = 0.35\text{secs} \). For \( \alpha = 1.5 \) and \( T=20 \) the TPT distribution shows LRD for 4 orders of magnitude.

Fig. 3. and Fig. 4. show the impact of reduced off-time on the CV and ACV structure(s). In other words the impact of reduced burstiness on second order moments. As off-time is reduced, \( b \) is reduced - the SM process
starts to lose dependence structures. At \( b = 0 \), the correlations disappear and the process becomes memoryless (\( CV = 1 \)).

### 4.1 Some additional comments

Whereas the regular assumption is that in the sub-states of the TPT distribution packets are produced according to a Poisson process, we point out that in reality there are correlations in the packet generation process. The basic reason behind is that in real networks close to 90% packet are transported by TCP which does introduce burstiness on its own. In [5] we modeled TCP as an on-off source which emits a window of packets in a RTT. We showed that while LRD structure comes from the heavy-tailed file-size distribution, TCP introduces multi-fractal structures in the small time scales [13] through the RTT distribution. This aspect of the traffic is not captured by the above model. It only captures the mono-fractal or the LRD behavior in the scope of this paper.

At this point we also briefly discuss the Hurst parameter \((0.5 < H < 1)\). The long range dependence is the same as the second order asymptotic self-similarity. As \( H \to 1 \), it indicates a greater degree of LRD. \( H = 1 \) indicates a purely fractal process. \( H = 0.5 \) indicates the absence of LRD and the process is called Short Range Dependent (SRD). Poisson process has \( H = 0.5 \) and is SRD. The most popular way to estimate \( H \) is based on the wavelet transform (WT) method [13]. It is to be noted that the scale parameter \( \alpha \) of a heavy-tailed distribution and \( H \) are tied through the following expression [14]:

\[
H = 3 - \frac{\alpha}{2}.
\]

### 5 Quasi Birth Death process

We are interested in finding the mean number of customers in the system as well as the mean response time for a typical web user. We want to study the SM/M/1 queue, however, we discuss a more general ME/ME/1 queue which can then be easily adapted to any arrival and service processes. Fig. 5. is a model for general ME/ME/1 type of queue where both arrival and service processes are of ME type. Its generator is given as [15]:

\[
Q = \begin{bmatrix}
A_{0,0} & A_{0,1} & 0 & 0 & 0 & \cdots \\
A_{1,0} & A_1 & A_0 & 0 & 0 & \cdots \\
\vdots & 0 & A_2 & A_1 & A_0 & \cdots \\
\vdots & \vdots & 0 & A_2 & A_1 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots 
\end{bmatrix}.
\]

The left most column for the states with queue-length 0, the next for queue-length 1, etc. The upper diagonal consists of forward transition rates, while the lower one gives the backward transition rates. The middle diagonal gives the local transitions of the state. \( A_i \) and \( A_{i,j} \) are all matrices which take their form from the arrival and service processes. \( A_0 \) expresses the forward transitions, \( A_1 \) the local transitions and \( A_2 \) the backward transitions.

We just assume at this stage to develop a more general solution:

* arrival process is represented by a ME type of distribution given by \( \langle p_n, B_A \rangle \).
*service process* is represented by a ME type of distribution given by $<p_a, B_s>$. The generator’s matrices are obtained [16]: $A_{0.0} = B_A, A_{0.1} = (B_A^0 p_a) \otimes p_s, A_{1.0} = B_A \otimes B_s$, $A_{0} = (B_A^0 p_a) \otimes B_s, A_{1} = (B_A^0 p_a) \otimes (B_A^0 p_s) + B_A \otimes B_s, A_{2} = B_A \otimes (B_A^0 p_s)$, $B_A^0 = e - B_A e$ and $B_s^0 = e - B_s e$, where $\otimes$ is the Kroneker product.

A general methodology for the solution of the model is as follows. It is assumed that the generator is irreducible and positive recurrent. The target is to find the stationary probability vector $\pi$ which is partitioned as $\pi = [\pi_0 \pi_1 ...]$, where $\pi_i$ stationary probability vector of the $i^{th}$ state. The solution is possible because of the matrix geometric relation [15] that exists in the steady-state probabilities of the process’s states. The solution is based on a matrix $R$ which is used for the computation of steady-state vector $\pi$ and then the performance measures.

\[ \pi_i = \pi_1 R^{i-1}, \quad \forall i \geq 1, \quad (19) \]

where $R$ is the solution of the matrix equation,

\[ A_0 + A_1 R + A_2 R^2 = 0. \quad (20) \]

Equation (20) is obtained from the repeating states of (18). The solution is obtained through a simple iterative algorithm. Faster approaches, for example the Logarithmic Reduction (LR) algorithm, are given in [17].

The boundary states give the following system of linear equations [18]:

\[ [\pi_0 \pi_1] \begin{bmatrix} B_{0.0} & B_{0.1} \\ A_1 + RA_2 \end{bmatrix} = (0, 0), \quad (21) \]

where 0 is a null matrix of appropriate dimension. However the system of (21) is not enough to get a solution therefore another equation is required. This is the normalization equation:

\[ \sum_{i=0}^{\infty} \pi_i e = \pi_0 e + \pi_1 \sum_{i=0}^{\infty} R^i e = 1, \quad (22) \]

Equations (21) and (22) give the solution of boundary states.

Since we are interested in the mean packet delay we mention the two relevant measures. $E[N]$, the average number of packets at the time of packet arrivals in the system (queue and server) [19]:

\[ E[N] = (p_a V e') \pi R (I - R)^{-1} Le' \quad (23) \]

The average response time, $E[R]$:

\[ E[R] = (E[N] + 1)/C \quad (24) \]
where $C$ is the capacity of the server. A packet that arrives in the system sees $E[N]$ packets already in the system. Now it is a straightforward to plug in the matrices of the SM process of the section 4. as the arrival process, and the exponential distribution as the service process in the QBD construction of (18) to obtain the performance measures.

### 6 Results

The simulation complexity reduction strategy is based on scaling of the mean off-time to increase the activity of each source thus leading to a reduced number of sources to produce the same aggregate traffic. This, however, impact the burstiness of the source. We showed earlier that as the off-time reduces the burstiness of each source reduces which leads to the decrease of correlations to the point that when $b = 0$ the SM arrival process becomes purely markovian, i.e. memoryless. We again choose $T_{on} = 0.35$ secs and make two simulation studies: one with with $Z = 1$ therefore $T_{off,w} = T_{off} = 40$ sec, therefore, $b = 1$, and then with $Z = \infty$, therefore $T_{off,w} = 0$ and $b = 0$. We show that for high burstiness SM/M/1 provides a good fit and when the source has no burstiness the M/M/1 provides a good upper limit. For simulations we use the Ptolemy Simulator extended for network simulations at our department. Fig. 6 shows the simulation setup. Propagation delays $d_1 = 10$ msec and $d_2 = 15$ msec. Capacities $C$ and $R$ are adjusted to keep their utilization around 50%.

#### 6.1 Impact of $T_{off}$ or $b$ on mean packet delay

**Large $T_{off,w}$ (or $b \approx 1$):** The settings for the SM arrival process given in the last sub-section are also valid here. It is to be noted that as $b \approx 1$, therefore, the source is highly bursty so long range correlations are expected to produce long MPD’s. Fig. 7 shows the results of simulation compared with the model. The SM/M/1 model provides a good upper bound for the mean packet delays for a typical user.

**$T_{off,w} = 0$ (or $b = 0$):** In the previous section, we had shown that for $T_{off,w} = 0$ the long range correlations disappear, and the arrival process becomes memoryless thus SM to markovian transition occurs. Fig. 8 gives the comparison of MPD’s of simulation with M/M/1 and M/D/1 queues. M/M/1 acts as an upper bound and M/D/1 acts as the lower. One should recall that the delays in case of deterministic delays should be half of those in the markovian case. The mean packet delays in this case are somewhere between the two bounds set by the two models as the packet sizes are neither constant nor markovian rather the packet sizes distribution has $0 < CV < 1$. This is caused by the last packet in the transmission of a file as it is not a complete packet. The end-of-file packets then give a non-deterministic character to the packet-sizes distribution, however, it is intuitive that measured results should be closer to the deterministic case. This is visible in the results.

#### 6.2 Simulation complexity reduction argument

Having stressed the importance of burstiness in determining the important traffic parameters as well as the mean packet delay, we now turn our attention to the complexity reduction argument. The target is to find a minimum value of off-time (indirectly, the burstiness of the source) with which all the important parameters remain unchanged, however, the total number of sources required for producing total aggregate traffic would be minimal.

We now use a more elaborate simulation setup, given in Fig. 6. Here instead of using the open-loop source producing packets at poissonian rate, we use multiple HTTP-TCP traffic sources. An HTTP-TCP source transports the packets via TCP protocol. Thus the multi-fractal structures are expected to be present with their impact on the traffic. This means that we expect to see a difference in the TCP based traffic when $b = 0$. Whereas the SM model predicts evaporation of LRD for $b = 0$, we expect to see some correlations because of the presence of TCP. Server
in Fig. 6 now depicts multiple HTTP-TCP sources, the client similarly means multiple clients. TCP-Reno was used as the transport protocol. The core link has capacity $C = 50Mbps$ and $R = 100Mbps$. Propagation E2E delay = (10+15msec) 25msec. Buffer size was kept as 1000Packets. For HTTP-TCP settings tuned to a typical web user, we followed [20] that gives the average web-page size of 60Kbytes. We set shape parameter $\alpha = 1.5$ for the TPT distribution. Typical user think-time as approximately $T_{off,w} = T_{off} = 40$ sec (for $Z = 1$) and negative exponentially distributed [20].

We chose a target aggregate traffic of 35Mbps on the core link. This approximately translates to 2880 sources. The TP, CV, MPD and H were measured. We then decreased the number of sources by decreasing the off-time of each source and increasing its activity. Table I gives the idea. For example, when each source is set with a typical $T_{off,w} = T_{off} = 40$ sec then 2880 sources are needed to produce 35Mbps of traffic. Each source produces 12.3Kbps of IP packet traffic approximately. When off-time is reduced to $T_{off,w} = 20$ sec then only 1440 sources are required. We changed the aggregation level $Z$ to 2,4,.. etc and measured the above given traffic parameter. We show the impact of reduced number of sources on the important parameters.

**TP:** All settings yielded approx. 35Mbps traffic, thus this parameter remains independent of the burstiness of the source.

**CV (arrival and departure):** Both parameters remain independent of the off-time and thus the burstiness of the source. For all settings CV (arrival)$\approx 1.82$ and CV(departure)$\approx 1.57$. We have not modeled the CV of aggregate traffic. However, for our scope of work in this paper we observe that both the parameters remain independent of the burstiness of the source and thus are unimportant for simulation complexity reduction argument.

**MPD and Hurst parameter:** We expect the biggest impact on these two parameters. The mean packet delays are given in Fig. 9. The MPD’s remain unaffected for $T_{off,w} \geq 5$sec. We also refer to Fig. 10 in which H parameter (a measure of persistence of LRD) is also unaffected for $T_{off,w} \geq 5$sec. This means the number of required sources can be reduced 8 times without any worries of an impact on the important traffic parameters.

If one allows a variation of approx. 10% in the MPD then $T_{off,w} \geq 1.25$ is acceptable. For this setting the number of sources can be reduced 24 times. This is a huge saving in terms of memory usage. However further reduction in the burstiness of the source starts influencing the LRD structure and hence we see reduced Hurst parameter measurements. We explained and showed in earlier sections that autocovariance structures begin to weaken as $T_{off,w} \rightarrow 0$, leading to weaker correlations. The arrival process tends to being memoryless. It is clear that our strategy is flawed in this region.

One notices that H is not 0.5 for $T_{off,w} = 0$ rather $H \approx 0.61$. This is a clear indication that there are correlations in the traffic, albeit weaker, contrary to the prediction of our model. The origin of these structures is suspected to be the TCP. Whereas in our modeling approach, we had assumed that the packet generating process in the on-phase is poissonian, this is not the case as packets are transported by TCP. As indicated earlier, TCP introduces burstiness
in small time scales through its congestion control mechanisms. We have not modeled this aspect of the traffic. However, the reader should get an ample feel of the impact of burstiness on LRD through the models that we have presented.

**How many sources?** It is not trivial to model superposition of large number of SM processes (consider 2880 sources!). That approach would give ACV and CV expressions leading to a methodology for figuring out the minimum number of sources required for realistic simulations. We, however, take a simpler approach based on the burstiness factor. The last row in Table I indicates $b$, the burstiness of the source. We note that for $T_{off,w} \geq 5$ the burstiness remains approximately same thus showing little impact on the MPD and Hurst parameter. Thus we can reduce the off-time to a point where the change in the burstiness of source is not significant. Allowing a deviation of 10% in the MPD, we can tolerate up to 20% decrease in the burstiness. Note that $b = 0.8$ for $T_{off,w} = 1.25$. It is a simple exercise to calculate $W$, the reduced number of sources, from equations (3) and (5) for a given $T_{on}$ time.

This approach is not rigorous, however, it is obvious that it is the burstiness of each source that finally impacts the second order moments of the aggregate traffic. If we do not allow the burstiness to decrease significantly despite scaling the off-time and increasing the activity of each source, the important traffic parameters would not show much impact.

7 Conclusion

We have given an explanation of how many sources are enough to make a realistic simulation. It is clear that a large number of sources only increase the complexity of simulation and are not required at all. We used a SM model for the arrival process and a QBD model for the queue to show that a reduced number of sources do not make a huge impact on the traffic and performance parameters. We have shown that 24 times reduction in simulation complexity can be easily achieved. This complexity reduction strategy opens the room for high speed networks’ simulations.

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