the bulk of traditional teletraffic analysis methodologies unsuitable for modeling emerging high-speed networks.

Future telecommunication networks will transport diverse classes of services such as data, voice, image, and video of different Quality-of-Service (QoS) requirements and traffic characteristics. The issue of efficiently supporting applications with the above service classes remains an important and active research area.

In this paper, we will address the issue of playout delay of CBR sources in time slotted systems (e.g., ATM). Of particular focus will be the probabilistic analysis of playout buffer. A playout mechanism is used to reconstruct the periodic CBR flow departing from the statistical multiplexer and it is characterized by two components: delay (number of time slots to delay the first cell of the CBR flow before periodic playout) and buffer which stores cells before playout. It is shown that a playout buffer of size two cells guarantees no buffer overflow in a homogeneous environment. An insufficient playout delay results in buffer underflow (i.e., cell may not have arrived yet in the playout buffer at the time of its playout). We provide an exact probabilistic analysis of probability of buffer underflow in terms of system parameters.

Our result will indicate that a small playout delay maybe sufficient at a single node. Therefore, a playout delay at each node might be a good strategy, and hence, our results maybe directly applicable in a multiple node environment.

The queueing analysis of CBR networks has attracted considerable attention recently (see, for example, [2]). Also delay and delay jitter of CBR traffic are discussed, for example, in [3]-[4]. However, we are not aware of probabilistic playout analysis of such systems.

The outline for the remainder of the paper is as follows. In Sec. 2 we provide the mathematical model and the basic definitions and the problem statement. In Sec. 3, we cover the analysis. Finally, the numerical results and conclusions are provided in Sec. 4.

2 Mathematical Model

We assume a synchronous environment, where time is slotted and takes non-negative integer values $t = \{0, 1, 2, \cdots\}$. The time interval $[t-1, t)$ is referred to as slot t. We assume that sources produce fixed-length packets (cells) independently of each other. The cells are stored in a loss-free buffer (queue). It is assumed that the departures from the cell buffer take place at the beginning of slots, and the arrivals during a slot. We define:
q_t = queue length (in number of cells) at the end of t^{th} slot;
A_t = number of arrivals from all sources in the t^{th} slot, so that

$$q_{t+1} = \max(q_t - 1, 0) + A_t, \qquad (1)$$

In what follows, we describe the arrival process of individual flows. We identify two types of flows, namely tagged and background flows. The individual traffic source of interest (tagged flow) is assumed to be periodic with period T slots and cells arrive in slots $\tau_n,\ n \geq 0$ so that the n^{th} tagged cell is arrived in slot τ_n, and $\tau_n - \tau_{n-1} = T$, or

$$\tau_n = \tau_0 + nT \qquad (2)$$

Other flows are called background and assumed to be of *general* periodic type allowing multiple cells arriving in a frame. Background flow $j,\ j = 1, \ldots, N$ is defined by set of

random variables $A_t^{(j)},\ t = 1, 2, \ldots$ which have the follow properties:
1. $A_t^{(j)} \in \{0, 1\}$;
2. $A_{t+nT}^{(j)} = A_t^{(j)}$;
3. $A_1^{(j)}, A_2^{(j)}, \ldots, A_T^{(j)}$ are cyclically interchangeable and independent from random variables of other flows in the system;
4. $\sum_{t=1}^{T} A_t^{(j)} = M_j$.
It is easy to see, that these properties provides interchangeability of any set $A_t^{(j)}$, $A_{t+1}^{(j)}$, …, $A_{t+T-1}^{(j)}$ and that the $\sum_{\tau=t}^{t+T-1} A_\tau^{(j)} = M_j$ for any $t > 0$.

We consider a multiplexor transmitting one cell per time slot. The utilization ρ_j from background flow j is M_j/T. It is assumed that the buffer is assumed to be large enough (i.e., no loss) and the the total utilization of the system does not exceed unity. We have

$$\rho = \frac{1}{T} + \sum_{j=1}^{N} \frac{M_j}{T} \leq 1 \tag{3}$$

which, in particular gives obvious, but useful inequality for our analysis

$$N < T. \tag{4}$$

The cell transmission is assumed to be FIFO and one cell per slot is transmitted as long as the buffer is non-empty. The cells arriving in the same time slot enter the buffer *randomly*. The paper deals with the requirement of the playout buffer to reconstruct the periodic nature of the tagged flow departing from the multiplexor.

In what follows, the operation of the playout buffer is summarized. A buffer is identified by two parameters D and B. The goal is to plays out the tagged flow periodically, one cell every T slots indefinitely by delaying the 0^{st} cell of the flow by initial delay of D slots. A buffer of capacity B cells is used. Since in practice, the number of cells of the periodic flow is finite, our analysis provides an upper bound on both the required buffer size and initial playout delay.

We denote the departure time of tagged cell number n as t_n. With this convention, the n^{th} cell, if already arrived to the buffer, departs at slot $t_0 + D + nT$.

We are interested in the play out delay D (in slots) and buffer of size B (in cells) to guarantee no buffer underflow or overflow. Underflow occurs if at the play-out time of a cell, the cell has not departed from the node yet, i.e., it has not arrived in the playout buffer. It is assumed that a departing cell is available immediately for play-out as soon as it has arrives in the playout buffer. To avoid buffer underflow, the playout delay D should be chosen to be large enough and a few cells may have to be stored in a playout buffer before periodic play-out begins. If B is not sufficient, a buffer overflow will occur. Our approach is to find the probability of underflow or overflow for given playout delay D and buffer size B.

3 Analysis

As we defined above, A_t is the total number of cells arriving in slot t in multiplexor.

$$A_t = \delta_{t,\tau_n} + \sum_{j=1}^{N} A_t^{(j)}, \quad \text{where} \quad \delta_{t,\tau_n} = \begin{cases} 1, \text{if } t = \tau_n \text{ for some } n \\ 0, \text{else} \end{cases} \tag{5}$$

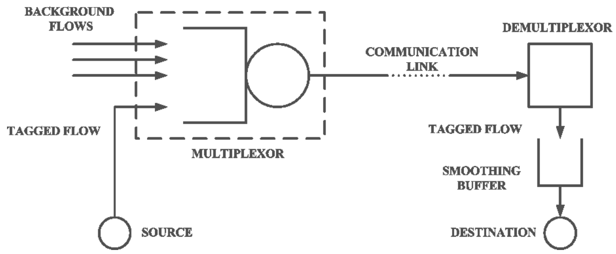

Fig. 1. System model.

Based on the description of the individual arrival flows, it is easy to see that the superposition process of all the flows is also periodic with period T, so that the total number of cells arriving in any slot $t \geq \tau_0$ satisfies

$$A_{t+nT} = A_t, \tag{6}$$

which yields $A_{\tau_0} = A_{\tau_n}\ \forall n$. Also it is easy to see that random variables $A_t, A_{t+1}, \ldots, A_{t+T-1}$ are cyclically interchangeable. Assuming that the multiplexor buffer is empty at $t = 0$, following τ_0 (the time of arrival of first cell of the tagged period flow), we have (see, for example, [1]) that

$$q_t = \max_{0 \leq i < T} \left(\sum_{j=t-i}^{t} A_j - i \right), \quad t \geq \tau_0,$$

which means that from the moment τ_0 queue q_t is periodic with period T:

$$q_{t+nT} = q_t,\ t \geq \tau_0 \tag{7}$$

In particular, we are interested in the random variable A_{τ_n} which represents the total number of arrivals in the arrival slots of the tagged flow. We can decompose this random variable into three parts of

$$A_{\tau_n} = 1 + b_n + c_n, \tag{8}$$

where 1 accounts for the cell from the tagged flow and b_n (c_n) denotes the number of background cells entering the queue before (after) the cell from the tagged flow.

We let $Q(\tau_n)$ denote the number of cells seen in the buffer by the n^{th} tagged cell arriving at time τ_n. We note that in the event of multiple arrivals at time τ_n, $Q(\tau_n)$ includes those cells entering the buffer ahead of the tagged cell:

$$Q(\tau_n) = q_{\tau_n} + b_n, \tag{9}$$

which implies that n^{th} tagged cell leaves multiplexor (and enters the playout buffer) at time

$$t_n = \tau_n + q_{\tau_n} + b_n.$$

We will make the natural assumption that in the event of multiple cells arriving in a time slot, the cell from the tagged flow will enter the buffer uniformly. Therefore, conditioning on A_{τ_n}, the random variables $b_n,\ n = 0, 1, \ldots$ have a (discrete) uniform distribution:

$$\Pr\{b_n = i \mid A_{\tau_n} = k+1\} = \frac{1}{k+1},\ 0 \le i \le k,\ \forall n \tag{10}$$

Since the orders of arrivals from all the flows in slots $\tau_n,\ n = 0, 1, \ldots$ are independent of each other, therefore random variables $b_n,\ n = 0, 1, \ldots$ are also *conditionally independent* given A_{τ_n}.

To find distribution of random variable A_{τ_n} in our model, we need the following:
Lemma: Let random variables $A_1^{(j)}, A_2^{(j)}, \ldots, A_T^{(j)}$ have properties:
1. $A_t^{(j)} \in \{0, 1\}$,
2. $\sum_{t=1}^{T} A_t^{(j)} = M$,
and let τ be a random variable with (discrete) uniform distribution in closed interval $[1, T]$, then

$$\Pr\{A_\tau^{(j)} = 1\} = \frac{M_j}{T}. \tag{11}$$

Proof of this lemma is obvious and omitted to shorten the text.

Therefore, from this lemma it is easy to see that background flow j with probability M_j/T makes a contribution of 1 cell to value of A_{τ_n}, and with probability $1 - M_j/T$ do not make any contribution independently from other background flows. If we assume that all M_j are equal to some constant M (we will refer to this situation as *symmetric case*), then for A_{τ_n} we have binomial distribution with parameters M/T and N:

$$p_k = \Pr\{A_{\tau_n} = 1{+}k\} = \binom{N}{k}\left(\frac{M}{T}\right)^k\left(1{-}\frac{M}{T}\right)^{N-k} \tag{12}$$

For the non-symmetric case, the distribution $p_k = \Pr\{A_{\tau_n} = 1{+}k\},\ k = 0, \ldots, N$ can be obtained numerically by convolution of N binary distributions.

3.1 Deterministic results

Let us note, that some of presented deterministic results can be found using general network algebraic methods from [5], but our approach is much simpler.

In order for not having an underflow, n^{th} tagged cell should arrive at playout buffer not later the than the time slot that it is due to be played out it should leave which is equal to $t_0 + D + nT = \tau_0 + q_{\tau_0} + b_0 + D + nT$, so we get inequality

$$\tau_n + q_{\tau_n} + b_n \le \tau_0 + q_{\tau_0} + b_0 + D + nT$$

which leads to simple inequality

$$b_n \le b_0 + D \tag{13}$$

as condition for n^{th} tagged cell not resulting in buffer underflow. If it is true for all n, it implies that underflow will never happen. Recall that N denotes the number of background flows. Since $A_{\tau_n} \le 1 + N$, then $b_n \le N,\ n = 0, 1, \ldots$ and from (13) we have *for* $\rho \le 1$ $D = N$ *guarantees no underflow*. So we assume that $D \le N$ in our below analysis.

As far as buffer overflow is concerned, we note that overflow can take place only at the moments of cell arrival in the playout buffer. Therefore, if we analyze a buffer overflow situation for buffer size of cell, i.e., $B = 1$, we need to get the conditions for some arriving cell to find inside the buffer 1 previous cell. Cell n arriving to the buffer at the moment $t_n = \tau_n + q_{\tau_n} + b_n$ will not find there cell $n-1$, if departure time of $(n-1)^{\text{th}}$ cell $\tau_0 + q_{\tau_0} + b_0 + D + (n-1)T$ is not more then t_n, so we get inequality

$$\tau_n + q_{\tau_n} + b_n \geq \tau_0 + q_{\tau_0} + b_0 + D + (n-1)T$$

which leads to another simple inequality

$$b_n \geq b_0 + D - T \tag{14}$$

as condition for n^{th} tagged cell do not find previous cell in the buffer, which is equivalent to condition for n^{th} cell do not result in overflow with buffer with size $B = 1$. Taking under consideration, that $b_0 \leq N$, we get our second result: *for $\rho \leq 1$ if $N \leq T - D$, then $B = 1$ provides no overflow.*

Now, it is easy to see that the condition for no overflow of buffer with size $B = 2$ is

$$b_n \geq b_0 + D - 2T \tag{15}$$

and taking under consideration that $b_0 \leq N$, $D \leq N$ and $N < T$, we have third result: *for $\rho \leq 1$ buffer of size 2 provides no overflow.* In what follows, we assume that $B \leq 2$.

Now let us discuss the follow question: is it possible for some given realization of background traffic and tagged flow in appropriate situation ($D < N$, $N > T - D$ and $B = 1$) to have both buffer underflow and overflow of playout buffer? To have both underflow and overflow, one b_i should violate (13) condition, and another b_j should violate (14) condition. Taking under consideration that all $b_n \in [0, N]$, it is easy to get necessary condition for the value of b_0:

$$\begin{cases} b_0 + D \ < \ N \\ b_0 + D - T > 0 \end{cases}$$

which is incompatible, because of $N < T$. Therefore, we have the following deterministic result: *for $\rho \leq 1$, for any realization if underflow is possible, then overflow is impossible and vise verse* .

3.2 First underflow or overflow

To formulate the probabilistic analysis of playout process, let us introduce some notations:
event $\mathcal{U}$ is the event of underflow (at least by one cell) for given D and B;
$n^{(u)}$ is a random value equal to the cell number resulting in underflow for given D and B;
event $\mathcal{U}_n$ is defined such that $n^{(u)} = n$;
event $\mathcal{O}$ is the event of overflow (at least by one cell) for given D and B;
$n^{(o)}$ is a random value equal to the cell number resulting in overflow for given D and B;
event $\mathcal{O}_n$ is defined such that $n^{(o)} = n$.

Note, that last result of above analysis yields $\Pr\{\mathcal{U} \cap \mathcal{O}\} = 0$.

Theorem 1: 1. For $\rho \leq 1$ and $D < N$

$$\Pr\{\mathcal{U}\} = \sum_{k=D+1}^{N} p_k \left(1 - \frac{D+1}{k+1}\right) \tag{16}$$

$$\Pr\{\mathcal{U}_n \mid \mathcal{U}\} = \frac{1}{\Pr\{\mathcal{U}\}} \sum_{k=D+1}^{N} \frac{p_k}{(1+k)^{n+1}} \sum_{m=D}^{k-1} (m+1)^{n-1}(k-m) \tag{17}$$

$$\mathbf{E}(n^{(u)} \mid \mathcal{U}) = \frac{1}{\Pr\{\mathcal{U}\}} \sum_{k=D+1}^{N} p_k \sum_{m=1}^{k-D} \frac{1}{m} \tag{18}$$

2. For $\rho \leq 1$ if $B = 1$ and $N > T - D$ then

$$\Pr\{\mathcal{O}\} = \sum_{k=T-D+1}^{N} p_k \left(1 - \frac{T-D+1}{k+1}\right) \tag{19}$$

$$\Pr\{\mathcal{O}_n \mid \mathcal{O}\} = \frac{1}{\Pr\{\mathcal{O}\}} \sum_{k=T-D+1}^{N} \frac{p_k}{(1+k)^{n+1}} \sum_{m=1}^{k-T+D} (k+1-m)^{n-1} m \tag{20}$$

$$\mathbf{E}(n^{(o)} \mid \mathcal{O}) = \frac{1}{\Pr\{\mathcal{O}\}} \sum_{k=T-D+1}^{N} p_k \sum_{m=1}^{k-T+D} \frac{1}{m}. \tag{21}$$

Proof of this theorem is omitted to shorten the text.

For symmetric case it is easy to get asymptotic formula for probability of underflow as the number of sources approaches to infinity. In this case, we substitute Poisson distribution instead of binomial p_k and limit when $T \to \infty$ and $N \to \infty$, $NM/T \to \lambda$ is

$$Pr\{\mathcal{U}\} \to 1 - \sum_{k=0}^{D} e^{-\lambda} \frac{\lambda^k}{k!} - \frac{D+1}{\lambda} \left(1 - \sum_{k=0}^{D+1} e^{-\lambda} \frac{\lambda^k}{k!}\right).$$

We now concentrate on terms (18) and (16). It is easy to show that

$$\mathbf{E}(n^{(u)} \mid \mathcal{U}) = (D+2) \frac{1 + \frac{1}{p_{D+1}} \sum\limits_{k=D+2}^{N} p_k \sum\limits_{m=1}^{k-D} \frac{1}{m}}{1 + \frac{D+2}{p_{D+1}} \sum\limits_{k=D+2}^{N} p_k \frac{k-D}{k+1}} \tag{22}$$

For $\mathbf{E}(n^{(o)} \mid \mathcal{O})$ we get

$$\mathbf{E}(n^{(o)} \mid \mathcal{O}) = (T-D+2) \frac{1 + \frac{1}{p_{T-D+1}} \sum\limits_{k=T-D+2}^{N} p_k \sum\limits_{m=1}^{k-T+D} \frac{1}{m}}{1 + \frac{T-D+2}{p_{T-D+1}} \sum\limits_{k=T-D+2}^{N} p_k \frac{k-T+D}{k+1}} \tag{23}$$

In symmetric case, for example, where $p\ N = M\ N/T = \rho - \frac{1}{T} < 1$, it is easy to see that for as k grows, then p_k decreasing sub-exponentially and the partial harmonic series inside the numerator summation has a limit of $\log k$ for large k, and $(k-D)/(k+1)$ in denominator summation goes to 1 with increasing k. So we can conclude that in both cases results are rather insensitive to N and M, and the behavior of $\mathbf{E}(n^{(u)} \mid \mathcal{U})$ vs. D and $\mathbf{E}(n^{(o)} \mid \mathcal{O})$ vs. $T-D$ are (almost) linear as we will see in numerical results.

3.3 Second playout

Once a buffer underflow occurs, we may subject the "late" cell to an additional playout delay so that the rest of the flow can be played out periodically.

One option is to do nothing, and try to continue to transmit cells from playout buffer at the moments $t_0 + D + nT$. It is easy to see that from conditional independence of all b_n's, the probability distribution for number of transmitted cells until second buffer underflow will be the same as for the first one.

As far as the second playout parameters are concerned, we can show that a buffer of two cells is sufficient for no buffer overflow.

For underflow, we consider the strategy that if $n^{(u)} = n^*$ and cell n^* arrives in buffer at moment t_{n^*} and resulted in underflow, we delay it before transmission for some extra delay of $D^{(A)}$ and the $n \geq n^*$th cell is transmitted at time $t_{n^*} + D^{(A)} + (n - n^*)T$, if it has already arrived.

To formulate theorem about probabilistic analysis of such strategy, we introduce some notations:
event $\mathcal{U}^{(\mathcal{A})}$ denotes underflow for second playout with parameters D, B (parameters of first playout) and additional playout delay$D^{(A)}$;
$n^{(ua)}$ is a random variable denoting the cell number resulting buffer underflow with second playout;
event $\mathcal{U}_\backslash^{(\mathcal{A})}$ defined such that $n^{(ua)} - n^{(u)} = n$;
event $\mathcal{O}^{(\mathcal{A})}$ is defined as overflow after second playout for given D,$D^{(A)}$ and $B = 1$;
$n^{(oa)}$ is a random value equals to the cell number, which first results in overflow after the first playout for given D,$D^{(A)}$ and $B = 1$;
event $\mathcal{O}_n^{(A)}$ is defined such that $n^{(oa)} - n^{(u)} = n$.

Theorem 2: 1. For $D + D^{(A)} < N$

$$\Pr\{\mathcal{U}^{(\mathcal{A})} \mid \mathcal{U}\} = \frac{1}{\Pr\{\mathcal{U}\}} \sum_{k=D+D^{(A)}+2}^{N} \frac{p_k}{k+1} \sum_{m=D+D^{(A)}+1}^{k-1} \left(1 - \frac{1+m}{1+k}\right) \tag{24}$$

$$\Pr\{\mathcal{U}_n^{(A)} \mid \mathcal{U}^{(A)}\} = \frac{1}{\Pr\{\mathcal{U}^{(A)}\}} \sum_{k=D+D^{(A)}+2}^{N} \frac{p_k}{(k+1)^{n+2}} \sum_{i=0}^{k-D-D^{(A)}-2} \sum_{m=i+D+D^{(A)}+1}^{k-1} (m+1)^{n-1} (k-m) \tag{25}$$

$$\mathbf{E}(n^{(ua)} \mid \mathcal{U}^{(A)}) = \frac{1}{\Pr\{\mathcal{U}^{(A)}\}} \sum_{k=D+D^{(A)}+2}^{N} \frac{p_k}{k+1} \sum_{i=0}^{k-D-D^{(A)}-2} \sum_{m=i+D+D^{(A)}+1}^{k-1} \frac{1}{k-m} \tag{26}$$

2. If $B = 1$ and $D + D^{(A)} \leq N$ then

$$\Pr\{\mathcal{O}^{(A)} \mid \mathcal{U}\} = \frac{1}{\Pr\{\mathcal{U}\}} \sum_{k=T-D^{(A)}+1}^{N} \frac{p_k}{k+1} \sum_{i=0}^{k-D-1} \left(1 - \frac{\max(i+D+1, T-D^{(A)}+1)}{k+1}\right) \tag{27}$$

$$\Pr\{\mathcal{O}_n^{(A)} \mid \mathcal{O}^{(A)}\} = \frac{1}{\Pr\{\mathcal{O}^{(A)}\}} \times$$

$$\times \sum_{k=T-D^{(A)}+1}^{N} \frac{p_k}{(k+1)^{n+2}} \sum_{i=0}^{k-D-1} \sum_{m=\max(i+D+1, T-D^{(A)}+1)}^{k} (k-m-D^{(A)}+T+1)^{n-1} (m+D^{(A)}-T) \tag{28}$$

$$\mathbf{E}(n^{(oa)} \mid \mathcal{O}^{(A)}) = \frac{1}{\Pr\{\mathcal{O}^{(A)}\}} \sum_{k=T-D^{(A)}+1}^{N} \frac{p_k}{k+1} \sum_{i=0}^{k-D-1} \sum_{m=\max(i+D+1, T-D^{(A)}+1)}^{k} \frac{1}{m+D^{(A)}-T} \tag{29}$$

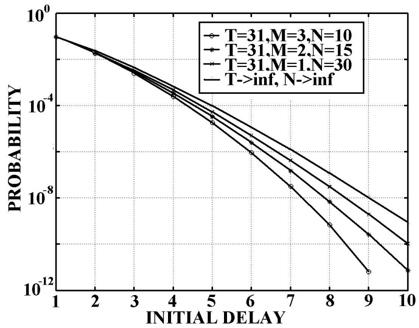

Fig. 2. Underflow probability. Symmetric case, $\rho = 1$.

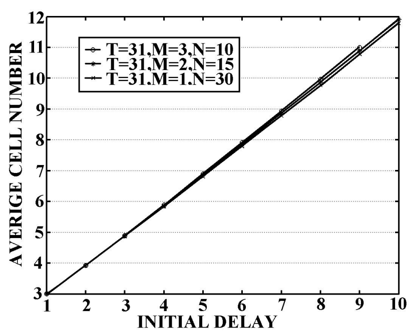

Fig. 3. Average cell number before first underflow. Symmetric case, $\rho = 1$.

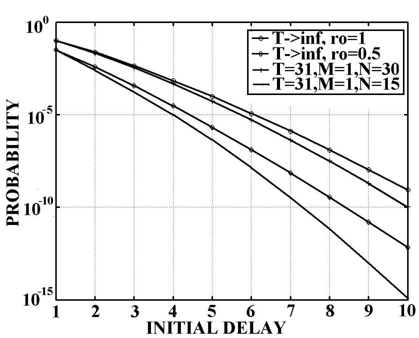

Fig. 4. Underflow probability asymptotic for $T \to \infty, \rho \to \text{const}$. Symmetric case.

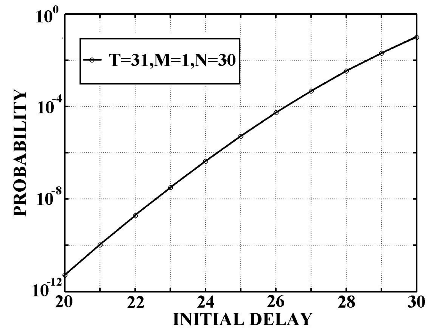

Fig. 5. Overflow probability for $B = 1$. Symmetric case, $\rho = 1$.

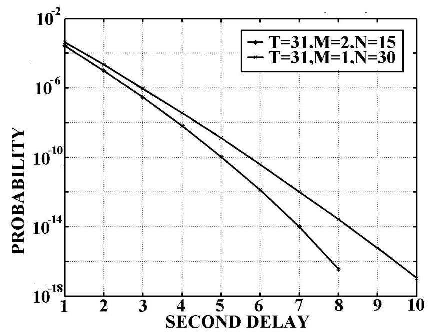

Fig. 6. Underflow probability after second play out. Symmetric case, $\rho = 1$, initial delay $D = 7$.

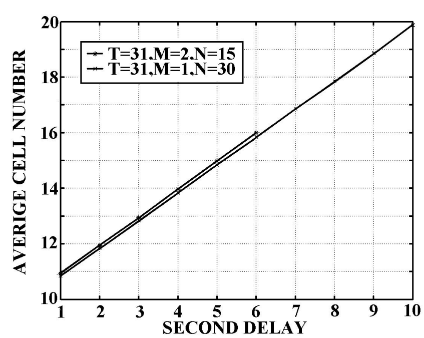

Fig. 7. Average cell number before underflow after second play out. Symmetric case, $\rho = 1$, initial delay $D = 7$.

Proof of this theorem is omitted to shorten the text.

Again, it is easy to show that $\mathbf{E}(n^{(ua)} \mid \mathcal{U}^{(A)})$ can be rewritten as

$$\mathbf{E}(n^{(ua)} \mid \mathcal{U}^{(A)}) = (D + D^{(A)} + 3) \frac{1 + \frac{D+D^{(A)}+3}{p_{D+D^{(A)}+2}} \sum_{k=D+D^{(A)}+3}^{N} \frac{p_k}{k+1} \sum_{i=0}^{k-D-D^{(A)}-2} \sum_{m=i+D+D^{(A)}+1}^{k-1} \frac{1}{k-m}}{1 + \frac{(D+D^{(A)}+3)(D+D^{(A)}+2)}{p_{D+D^{(A)}+2}} \sum_{k=D+D^{(A)}+2}^{N} \frac{p_k}{k+1} \sum_{m=D+D^{(A)}+1}^{k-1} \left(1 - \frac{1+m}{1+k}\right)} \quad (30)$$

but $\mathbf{E}(n^{(oa)} \mid \mathcal{O}^{(A)}, \mathcal{U})$ cannot be represented the same way, because its inner summations have more complex form.

4 Numerical results and conclusions

So, we provided an exact probabilistic analysis of buffer underflow and overflow for homogeneous environment in terms of system parameters.

For our specific numerical experimentations, we assumed a $T = 31$ (except in Figs. 2 and 4 which asymptotic behavior is also depicted) and $B = 2$, unless otherwise stated. In Fig. 2, we show that in the worst case, a playout delay of 9 time slots will result in a probability of underflow of about 10^{-8}. A total utilization of unity is assumed.

In Fig. 3, we depict the average number of cell arriving until the first underflow occurs (conditioning that an underflow has occurred). As noted above, this is almost linear and quite insensitive to the parameters of N and M defined in the arrival model in the paper.

Fig. 4 provides the probability of buffer underflow for various system utilizations. Fig. 5 shows the buffer underflow with insufficient B of a one cell (instead of two used in other examples). Finally, Fig. 6 depicts the probability of buffer underflow vs. second playout delay. A first playout delay of $D = 7$ is used.

Numerical results show that in the homogeneous case an initial playout delay of few time slots provide a small underflow probability. We also show that the expected number of cells before a first underflow is (almost) proportional to the size of the playout delay.

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