ITC19/ Performance Challenges for Efficient Next Generation Networks 1325-1334
LIANG X.J. and XIN Z.H.(Editors)
V.B. IVERSEN and KUO G.S.(Editors)
Beijing University of Posts and Telecommunications Press

Access Delay of the IEEE 802.11 MAC Protocol under Saturation

Taka Sakurai ⋆ and Hai Le Vu ⋆

ARC Special Research Centre for Ultra-Broadband Information Networks (CUBIN), Department of Electrical and Electronic Engineering, The University of Melbourne, Melbourne, VIC 3010, Australia.
{tsakurai,hlv}@ee.mu.oz.au

Abstract. In this paper, we analyze the access delay of the IEEE 802.11 MAC protocol for the case when all stations are saturated. We obtain explicit expressions for the first two moments, and the generating function. We show via comparison with simulation that the model accurately predicts the mean, standard deviation, and distribution of the access delay. The distribution is obtained by numerically inverting the generating function.
Keywords: IEEE 802.11, performance analysis, MAC delay, generating function.

1 Introduction

Products based on the IEEE 802.11 family of standards have acquired the lion's share of the burgeoning wireless local area network (WLAN) market. As in the wider Internet, the majority of traffic carried on a typical IEEE 802.11 WLAN today consists of non-realtime applications such as web browsing and email. However, in the near future, it is expected that a significant proportion of the traffic on WLANs will consist of new realtime services such as voice over IP [1]. To understand the potential for IEEE 802.11 WLANs to support such delay-sensitive applications, performance models for evaluating delay characteristics are needed. This paper is a step in this direction. We study the *access delay* of the MAC layer when there are multiple wireless stations in ideal channel conditions, where each station always has a packet available for transmission.

Formally, we define the access delay as the time interval between the instant when the packet reaches the head of the transmission queue and begins contending for the channel, and the time when the packet is successfully received at the destination station. The IEEE 802.11 MAC layer employs a channel access mechanism called the distributed coordination function (DCF). The DCF is a backoff protocol, and therefore, the access delay is a stochastic quantity. Closely related to the access delay is the packet inter-departure time. In fact, the packet inter-departure time and the access delay differ only by a small deterministic quantity; namely, the time to receive a MAC acknowledgement packet.

⋆ This work was supported by the Australian Research Council. CUBIN is an affiliated program of National ICT Australia.

Exact delay analysis of backoff protocols has so far proved elusive [2], so approximation techniques are typically used. The approximate analysis of the access delay or inter-departure time in IEEE 802.11 WLANs has been the subject of several papers. Carvalho and Garcia-Luna-Aceves [3] develop approximate formulae for both the mean and variance of the inter-departure time; however, comparisons with simulation results presented in the paper reveal that these formulae lack accuracy. In a recent paper, Zhai, Kwon and Fang [4] derive the generating function of the probability mass function of the packet inter-departure time. The generating function is derived from an approximate Markov chain model of the DCF introduced in the seminal paper by Bianchi [5]. Tickoo and Sikdar [6] derive a different expression for the generating function of the inter-departure time using probabilistic arguments.

In this paper, we use probabilistic arguments to obtain an approximate expression for the access delay in terms of constituent random variables. From this expression, we derive explicit expressions for the mean, standard deviation, and generating function. Our generating function differs from those in [4] and [6]. We use numerical transform inversion [7] to obtain values of the complementary cumulative distribution function (ccdf) from the generating function. With the aid of the popular *ns*-2 simulator [8], we demonstrate that our analytical formulae for the mean and standard deviation are very accurate. Significantly, we also find that the ccdf values obtained by numerical inversion are in remarkably good agreement with the simulation results.

In principle, it should be possible to obtain all moments of the the access delay by repeated differentiation of the generating function followed by taking the appropriate limit. However, the generating function in question is complicated, making differentiation and limit taking extremely tedious.

The rest of this paper is organized as follows. In Section 2, we describe the IEEE 802.11 Medium Access Control (MAC) protocol. In Section 3, we present our analysis of the access delay, and we compare our analytical results with simulation in Section 4.

2 The 802.11 MAC protocol

The principal access mechanism of the IEEE 802.11 MAC layer is the distributed coordination function (DCF) [9]. Under DCF, nodes contend for the channel using a carrier sense multiple access mechanism with collision avoidance (CSMA/CA). In the case when stations always have packets backlogged – the so-called 'saturated' case that we study in this paper – the operation of the DCF is simplified.

The first thing that a station does is defer transmission for a guard period known as the distributed interframe spacing time DIFS, during which the channel must be sensed idle. This is followed by a random backoff interval. Backoff intervals are slotted, and stations are only permitted to commence their transmissions at the beginning of slots. When backoff is initiated, a random integer backoff time is selected with a uniform distribution from the range $[0, CW-1]$, where CW is the so-called contention window. The backoff time represents the number of idle slots that must pass before the next packet can be transmitted. At the first transmission attempt, CW is set equal to W, the minimum contention window. The backoff time counter is decremented as long as the channel is sensed idle. It is *frozen* when a packet transmission is detected on the channel, and

conditioned on a certain number of collisions taking place, and the mixing probabilities are the respective probabilities of the number of collisions.

In an analysis of the throughput of the 802.11 MAC, Bianchi [5] introduced a key enabling approximation that each packet collides with constant and independent probability p, and he showed that this leads to a fixed point formulation for p. The fixed point formulation, modified for our context where there is a limit m on the number of window doublings and a limit K on the total number of transmissions, consists of two formulae. The first is an expression for the overall average backoff window W_{bo}, namely

$$W_{bo} = \frac{\eta W(1-(2p)^m)}{2(1-2p)} - \frac{1-p^m}{2(1-p^K)} + \frac{(2^m W - 1)(p^m - p^K)}{2(1-p^K)}, \tag{2}$$

where $\eta = (1-p)(1-p^K)^{-1}$. The second formula gives p in terms of W_{bo}:

$$p = 1 - (1 - 1/W_{bo})^{n-1}. \tag{3}$$

Equations (2) and (3) establish a fixed point formulation from which the collision probability p can be computed using a numerical technique.

From the assumptions made for p, it follows that the probability of no collision is η, of one collision is ηp, and so on up to $K-1$ collisions, which has probability ηp^{K-1}. We therefore have that

$$A = \begin{cases} A^{(0)} & \text{w.p.} \quad \eta \\ A^{(1)} & \text{w.p.} \quad \eta p \\ \vdots & \\ A^{(i)} & \text{w.p.} \quad \eta p^i, \\ \vdots & \\ A^{(K-1)} & \text{w.p.} \quad \eta p^{K-1}, \end{cases} \tag{4}$$

where 'w.p.' means 'with probability'. The component r.v. $A^{(i)}$ is comprised of $i+1$ backoff intervals and i collisions. We can write

$$A^{(i)} = \sum_{j=0}^{i} B_i^{(j)} + \sum_{j=1}^{i} C_{ij}, \tag{5}$$

where it is understood that if $i = 0$, the value of the second sum is zero. The r.v.'s C_{ij} account for the channel occupancies of collisions involving the tagged user, while the $B_i^{(j)}$ represent backoff intervals of the tagged station and interruptions by non-tagged stations. Clearly, the r.v.'s C_{ij}, $i = 1, \ldots, K-1$ and $j = 1, \ldots, i$, are i.i.d.. The r.v.'s $B_i^{(j)}$, $i = 0, \ldots, K-1$ and $j = 0, \ldots, i$, are independent, and for j fixed, they are i.i.d. in the index i.

Next we develop a representation for $B^{(j)}$ as a combination of other random variables (for simplicity, we drop the index i from the notation). The scope of $B^{(j)}$ is defined by a backoff interval that takes a discrete uniform distribution. Each slot of the backoff interval can, with certain probabilities, be interrupted at the start of the slot by a successful

3.2 Mean and standard deviation

To derive the mean E[D] and standard deviation StdDev[D] of the access delay, we start by writing down expressions for E[D] and StdDev[D] in terms of the means and variances of constituent random variables. Referring to (1), since A and T are independent, we trivially have that

$$\mathrm{E}[D] = \mathrm{E}[A] + \mathrm{E}[T], \tag{14}$$

$$\mathrm{StdDev}[D] = (\mathrm{Var}[A] + \mathrm{Var}[T])^{1/2}. \tag{15}$$

Recall that the distribution of A is a simple mixture (see (4)), which is a special case of a conditional distribution. The mean and variance are readily obtained:

$$\mathrm{E}[A] = \eta \sum_{i=0}^{K-1} p^i \, \mathrm{E}[A^{(i)}].$$

$$\mathrm{Var}[A] = \eta \sum_{i=0}^{K-1} p^i (\mathrm{Var}[A^{(i)}] + (\mathrm{E}[A^{(i)}] - \mathrm{E}[A])^2).$$

The mean and variance of $A^{(i)}$, $i = 0, 1, \ldots, K-1$, follow in a straightforward way from (5):

$$\mathrm{E}[A^{(i)}] = \sum_{j=0}^{i} \mathrm{E}[B^{(j)}] + i\, \mathrm{E}[C], \qquad \mathrm{Var}[A^{(i)}] = \sum_{j=0}^{i} \mathrm{Var}[B^{(j)}] + i\, \mathrm{Var}[C].$$

Next, we derive the mean and variance of $B^{(j)}$. From (6), we see that $B^{(j)}$ is a *random sum*, where the number of terms is uniformly distributed. It follows that

$$\mathrm{E}[B^{(j)}] = \mathrm{E}[U^{(j)}](t_{slot} + \mathrm{E}[Y]),$$

$$\mathrm{Var}[B^{(j)}] = \mathrm{E}[U^{(j)}]\, \mathrm{Var}[Y] + (t_{slot} + \mathrm{E}[Y])^2\, \mathrm{Var}[U^{(j)}].$$

It is straightforward to show that

$$\mathrm{E}[U^{(j)}] = \begin{cases} (2^j W - 1)/2 & \text{for } j = 0, \ldots, m-1, \\ (2^m W - 1)/2 & \text{for } j = m, \ldots, K-1, \end{cases}$$

$$\mathrm{Var}[U^{(j)}] = \begin{cases} (2^{2j} W^2 - 1)/12 & \text{for } j = 0, \ldots, m-1, \\ (2^{2m} W^2 - 1)/12 & \text{for } j = m, \ldots, K-1. \end{cases}$$

As can be seen from (11), the distribution of Y is a simple mixture. The mean and variance are given by

$$\mathrm{E}[Y] = q(1 - q_c)\, \mathrm{E}[T^*] + q q_c\, \mathrm{E}[C^*],$$

$$\mathrm{Var}[Y] = q(1 - q_c)(\mathrm{Var}[T^*] + (\mathrm{E}[T^*] - \mathrm{E}[Y])^2) + q q_c (\mathrm{Var}[C^*] + (\mathrm{E}[C^*] - \mathrm{E}[Y])^2).$$

For constant length packets, we infer from the previous section that

$$\mathrm{E}[T] = t_{data} + t_{difs},$$

$$\mathrm{E}[C] = \mathrm{E}[C^*] = \mathrm{E}[T^*] = t_{data} + t_{sifs} + t_{ack} + t_{difs},$$

$$\mathrm{Var}[T] = \mathrm{Var}[C] = \mathrm{Var}[C^*] = \mathrm{Var}[T^*] = 0.$$

Finally, from (11) we obtain

$$\widehat{Y}(z) = q(1-q_c)\widehat{T^*}(z) + qq_c\widehat{C^*}(z).$$

In the case of constant length packets,

$$\widehat{T}(z) = z^\alpha, \qquad \widehat{T^*}(z) = \widehat{C}(z) = \widehat{C^*}(z) = z^\beta,$$

where α and β are integer constants given by

$$\alpha = (t_{difs} + t_{data})/\delta, \qquad \beta = (t_{data} + t_{sifs} + t_{ack} + t_{difs})/\delta.$$

To numerically invert the generating function, we use the LATTICE-POISSON algorithm of Abate, Choudhury and Whitt [7], with parameters $l = 1$ and $r = 10^{-4/k}$, where k is the index of the inversion point. This should result in an inversion error no greater than 10^{-8}. The computational cost of inversion for large k can be high, especially if the lattice spacing δ is small. To alleviate the computational cost, we have found it prudent to tolerate some discretization error and settle for a larger δ than that required to meet the strict condition that all random variable values fall on the lattice points.

4 Numerical evaluation and discussion

The objective of numerical evaluation in this section is twofold: to verify our analytical model proposed in the previous Section with simulation, and to study the characteristics of the access delay as a function of number of nodes and packet sizes. The simulation was performed using the *ns*-2 simulator [8] (version 2.27) which has a built-in implementation of the IEEE 802.11 MAC. Tests revealed that the simulator contains several bugs that noticeably affect the output delay statistics, so these were remedied.

The network scenario that we simulate is a population of n saturated stations sending packets to an access point, in ideal channel conditions. The stations use the UDP protocol with a fixed packet size. We choose MAC and physical layer parameter values consistent with an 802.11b system. Table 1 lists the parameters, the symbols that we use for them, and their values.

Table 1. 802.11b MAC and PHYS parameters

Parameter	Symbol	Value	Parameter	Symbol	Value
Data bit rate	r_{data}	11 Mbps	Slot time	t_{slot}	20 μs
Control bit rate	r_{ctrl}	1 Mbps	SIFS	t_{sifs}	10 μs
PHYS header	t_{phys}	192 μs	DIFS	t_{difs}	50 μs
MAC header	l_{mac}	224 bits	Min CW	W	32
UDP/IP header	l_{udpip}	320 bits	Doubling limit	m	5
ACK packet	l_{ack}	112 bits	Retry limit	K	7

If we denote the UDP packet payload by l_{pay} bits, then the packet transmission times used in our analytic models are given by

$$t_{data} = t_{phys} + \frac{l_{mac} + l_{udpip} + l_{pay}}{r_{data}}, \qquad t_{ack} = t_{phys} + \frac{l_{ack}}{r_{ctrl}},$$

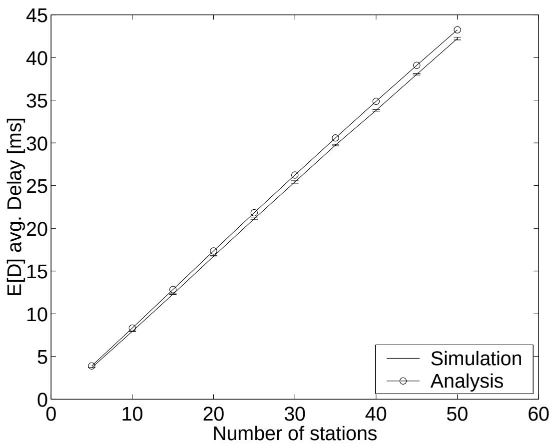

Fig. 1. Average access delay, UDP payload of 33 bytes.

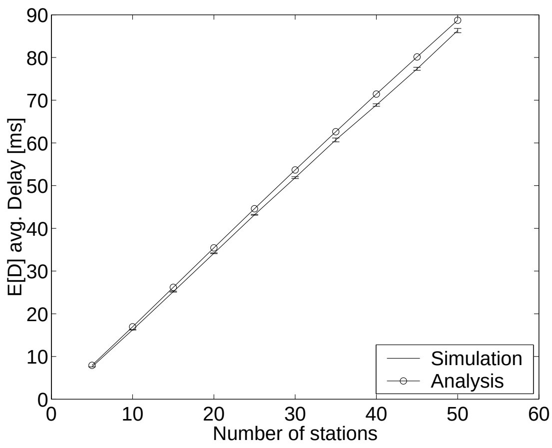

Fig. 2. Average access delay, UDP payload of 1000 bytes.

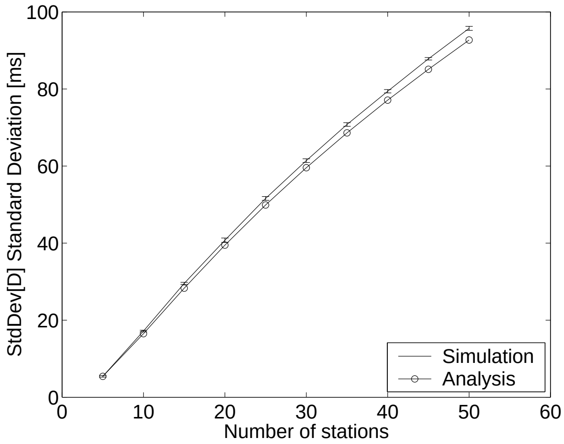

Fig. 3. Standard deviation of access delay, UDP payload of 33 bytes.

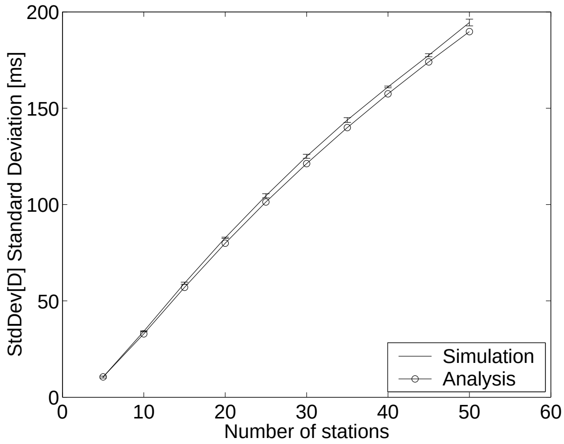

Fig. 4. Standard deviation of access delay, UDP payload of 1000 bytes.

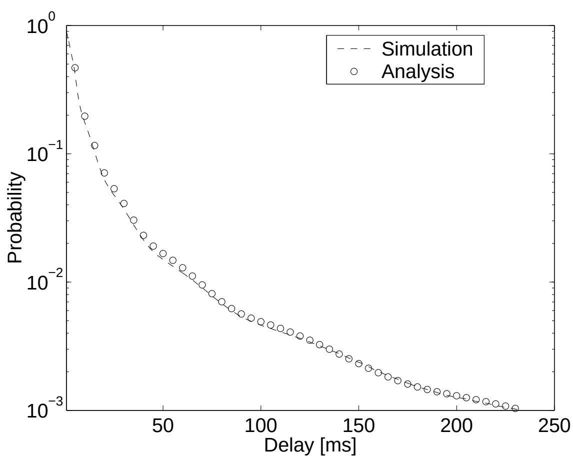

Fig. 5. CCDF of access delay, $n = 10$ and UDP payload of 33 bytes.

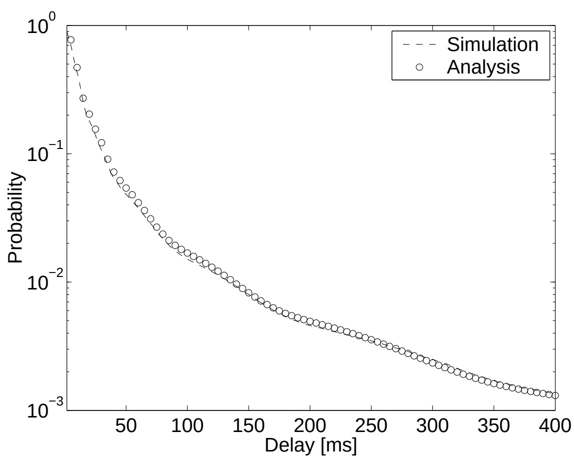

Fig. 6. CCDF of access delay , $n = 10$ and UDP payload of 1000 bytes.