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QoS Control for Multiuser Networks – A Joint Approach to Receiver Design and Resource Allocation

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Abstract. Resource allocation for wireless networks is strongly influenced by the effects of multiuser interference. Thus, a cross-layer approach is needed, which includes the power allocation as well as the design of joint receive strategies for interference mitigation. In this paper, we characterize the resulting QoS feasible region (the set of jointly achievable QoS). The impact of the receive strategy on the interference is modeled by a parameter-dependent matrix of coupling coefficients. The problem of feasibility is closely connected to the theory of non-negative matrices. Power allocation can be seen as an eigenvalue problem. We propose iterative algorithms which are able to achieve arbitrary QoS targets within the feasible region, including the boundary. This theoretical framework may serve as a basis for the development of algorithms for cross-layer resource allocation.

1 Introduction

The quality-of-service (QoS) is a measure for the performance and reliability of a communication link. Although its definition varies from case to case, it is always in some way determined by the effects of the propagation channel and by interference between the links. These effects are both captured by the signal-to-interference-plus-noise ratio (SINR). Thus, a common approach for resource allocation is to model the QoS as a one-to-one mapping of the SINR. Defining a bijective function $\phi : \mathbb{R}_+ \mapsto \mathbb{R}_+$, the QoS of the kth link can be defined as

$$QoS_k = \phi(\mathrm{SINR}_k), \quad k = \{1, 2, \ldots, K\} . \tag{1}$$

Some examples are the SINR itself: $\phi(x) = x$, BER: $\phi(x) = Q(\sqrt{x})$, MMSE: $\phi(x) = 1/(1+x)$, BER-slope for α-fold diversity: $\phi(x) = x^{-\alpha}$, queuing delay [1], or the information-theoretical capacity: $\phi(x) = \log(1 + x)$.

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Wireless networks are characterized by strong interdependencies between the communication links, which are typically coupled by interference. This results in a necessary tradeoff between the individual QoS. Increasing one users QoS generally comes at the cost of decreasing the QoS that can be achieved on other links. The set of jointly achievable QoS is referred to as the *feasible region*.

Resource allocation aims at finding a suitable tradeoff between the individual QoS. A common approach is to optimize the *network utility function*

$$f(QoS) = \sum_{k=1}^{K} \alpha_k \cdot QoS_k \ .$$

The functional $f(QoS)$ can be seen as measure of revenue for the network operator, who is paid according to the QoS he can deliver. The weighting factors $\boldsymbol{\alpha} = [\alpha_1, \dots, \alpha_K]$ depend on various parameters, like user requirements, traffic patterns, channel conditions, queuing lengths, or priority classes.

For some cases, the optimization of $f(QoS)$ can even be considered as the optimal strategy. For example, it was shown in the context of statistical queuing [2, 3], that this approach maximizes the stability region of a MIMO multiple access channel. An overview is given in [4].

Optimization of $f(QoS)$ always leads to a point on the boundary of the *QoS feasible region* $\mathcal{Q}$ (the set of jointly achievable QoS), as illustrated in Fig. 1. By adjusting $\boldsymbol{\alpha}$, it is

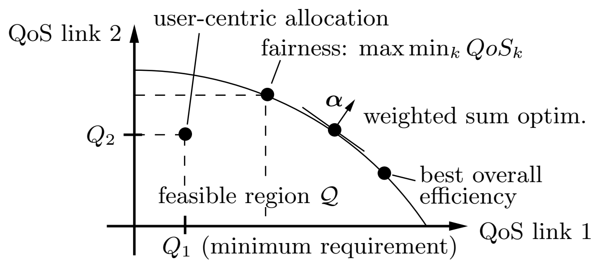

Fig. 1. Resource allocation over the QoS feasible region

possible to achieve arbitrary boundary points. Special cases of interest are the best overall efficiency (the optimum sum of QoS) and max-min fairness (all QoS balanced).

The nonlinear dependency of QoS on channel conditions and powers, as well as on physical layer designs, are still main challenges of solving utility maximization problems in wireless networks. In particular, the impact of *multiuser reception* strategies for interference mitigation needs to be better understood. The choice of the receiver design determines the shape of the QoS feasible region, and thus has impact on the resource allocation. For example, the region $\mathcal{Q}$ was shown to be convex for certain classes of QoS mappings [5–8] and algorithms for optimizing $f(QoS)$ are proposed [9].

These results hold under the assumption of a *fixed* receiver design. The extension to the case of *adaptive* receiver designs, which will be studied in the following, is not straightforward. In this paper, we investigate the QoS feasible region $\mathcal{Q}$ under the assumption

Here, the receiver z_k is chosen so as to minimize the interference, and thus to maximize the respective SINR

$$\mathrm{SINR}_k(\boldsymbol{p}_k) = \frac{\boldsymbol{p}_k}{\mathcal{I}_k(\overline{\boldsymbol{p}})}, \quad k = \{1, 2, \ldots, K\} . \tag{5}$$

In general, the optimal receive strategy in (4) does not completely null out the interference. This is because there is a tradeoff between interference suppression and noise enhancement. Mostly, it is better to let the transmitted signals interfere with each other in a controlled way.

The non-linear interference $\mathcal{I}_k(\boldsymbol{p})$ of the kth link depends on *all* transmission power $\boldsymbol{p}$. Thus, all SINR (5) are tightly intertwined. In the following we will develop allocation strategies, which take into account this interdependency. The optimal receiver design is implicitly included by this model.

Our analysis is based on the following properties, which are all fulfilled by the interference function (4).

A1: $\mathcal{I}_k(\overline{\boldsymbol{p}})$ is non-negative on $\mathbb{R}_+^{K+1}$

A2: $\mathcal{I}_k(\mu\overline{\boldsymbol{p}}) = \mu\mathcal{I}_k(\overline{\boldsymbol{p}})$ for all $\overline{\boldsymbol{p}} \in \mathbb{R}_+^{K+1}$ and $\mu > 0$.

A3: $\mathcal{I}_k\left(\left[\begin{smallmatrix}\boldsymbol{p}^{(1)}\\ 1\end{smallmatrix}\right]\right) \geq \mathcal{I}_k\left(\left[\begin{smallmatrix}\boldsymbol{p}^{(2)}\\ 1\end{smallmatrix}\right]\right)$ if $\boldsymbol{p}^{(1)} \geq \boldsymbol{p}^{(2)}$.

A4: $\mathcal{I}_k\left(\left[\begin{smallmatrix}\boldsymbol{p}\\ a\end{smallmatrix}\right]\right) > \mathcal{I}_k\left(\left[\begin{smallmatrix}\boldsymbol{p}\\ b\end{smallmatrix}\right]\right)$ if $a > b$.

This axiomatic framework A1-A4 is similar to the concept of *standard interference functions* [11], thus we know that feasible target SINRs can be achieved by a fixed-point iteration, which converges to a unique optimal power allocation. But we can also exploit the matrix structure of our interference model (4), which is a special case of the more general concept A1-A4. By exploiting the theory of non-negative matrices, we will develop new algorithms for QoS allocation. A detailed problem formulation will be given in the next section.

2.2 QoS-Based Resource Allocation Policies

The achievable sum-power constrained QoS region, i.e., the set of all jointly achievable $Q = (Q_1, \ldots, Q_K) > 0$ is

$$\mathcal{Q}(P) = \{(Q_1, \ldots, Q_K) : C(P, Q) \geq 1\} , \tag{6}$$

where

$$C(P, Q) = \max_{\boldsymbol{p} \geq 0} \Big(\min_{1 \leq k \leq K} \frac{\phi\big(\mathrm{SINR}_k(\boldsymbol{p})\big)}{Q_k} \Big) \quad \text{subject to } \|\boldsymbol{p}\|_1 \leq P . \tag{7}$$

The region (6) is illustrated in Fig. 1 for the 2-user case, assuming that we seek to maximize the QoS functions (otherwise max and min must be interchanged). The boundary is the subset for which $C(P, Q) = 1$. Sometimes, the sum power constraint $\|\boldsymbol{p}\|_1 \leq P$ in (7) is replaced by individual power constraints $\boldsymbol{p}_k \leq P_k$, $1 \leq k \leq K$.

3 The Boundary of the Unconstrained Region

The power minimization problem (9) aims at fulfilling certain QoS targets with optimal power efficiency. In this respect, it differs from the problem formulations (8) and (10), which only require a "best effort" solution, limited by the totally available power. Whereas problem (9) may become *infeasible*, i.e., no solution exists.

In order to decide whether or not some QoS targets $\boldsymbol{\Gamma}_{\mathrm{Q}}$ can be fulfilled, it is important to study the *unconstrained* QoS region, which is the ultimate limit for jointly achievable QoS.

Since the QoS is monotonically increasing in the transmission power, the boundary of this region is approached in the high-power regime, which means that noise can be neglected. In this case, the interference function is reduced to

$$\mathcal{I}_k(\boldsymbol{p}) = \min_{z_k \in \mathcal{Z}_k} \big[\boldsymbol{\Psi}(z)\, \boldsymbol{p}\big]_k \,, \quad k \in \{1, 2, \ldots, K\} \,, \tag{11}$$

and the SINR (5) becomes the signal-to-interference ratio (SIR).

Note, that (11) still fulfill the properties A1-A3. Property A4 is based on the existence of a positive noise component.

Let γ be the inverse function of ϕ, then

$$\gamma_k = \gamma(Q_k), \quad k \in \{1, 2, \ldots, K\} \,, \tag{12}$$

is the minimum SIR level needed by the kth user to satisfy the QoS requirement Q_k. Thus, the problem of achieving certain QoS requirements, carries over to the problem of achieving target SIR's

$$\boldsymbol{\Gamma}_{\mathrm{Q}} = \mathrm{diag}\{\gamma_1, \ldots, \gamma_K\} \,. \tag{13}$$

A target $\boldsymbol{\Gamma}_{\mathrm{Q}} > 0$ is achievable if and only if there exists a power allocation $\hat{\boldsymbol{p}} > 0$ such that $\mathrm{SIR}_k(\hat{\boldsymbol{p}}) \geq \gamma_k$, for all $k = 1, \ldots, K$, which is equivalent to $\max_k \gamma_k / \mathrm{SIR}_k(\hat{\boldsymbol{p}}) \leq 1$. Assuming some receive strategy z, the minimum achievable level is given by the min-max problem

$$C^{(z)}(\boldsymbol{\Gamma}_{\mathrm{Q}}) = \inf_{\boldsymbol{p}>0} \Big(\max_{1 \leq k \leq K} \frac{[\boldsymbol{\Gamma}_{\mathrm{Q}} \boldsymbol{\Psi}(z) \boldsymbol{p}]_k}{\boldsymbol{p}_k} \Big) \,.$$

That is, $C^{(z)}(\boldsymbol{\Gamma}_{\mathrm{Q}}) \leq 1$ is required in order to be able to support the SIR targets $\boldsymbol{\Gamma}_{\mathrm{Q}}$.

It is known from the literature (see e.g. [5, 6, 12]) that $C^{(z)}(\boldsymbol{\Gamma}_{\mathrm{Q}})$ equals the spectral radius of the coupling matrix $\boldsymbol{\Psi}(z)$ if $\boldsymbol{\Psi}(z)$ is *irreducible* (but the converse need not be true). This follows from the Perron/Frobenius theorem for irreducible matrices.

However, the assumption of irreducibility is not necessarily justified for the case at hand, where $\boldsymbol{\Psi}(z)$ depends on the parameter z. In particular, the receive strategy z may perform interference cancellation or nulling, and thus may render $\boldsymbol{\Psi}(z)$ *reducible*. This is especially true for the boundary, which is approached for $\|\boldsymbol{p}\| \to \infty$. Then, MMSE designs, for example, are equivalent to zeroforcing, i.e., resources are orthogonalized and $\boldsymbol{\Psi}(z) = \mathbf{0}$.

Even though the Perron/Frobenius theorem cannot be applied, it can be shown that the optimum of the min-max characterization can still be expressed in terms of the spectral

4 Power-Constrained Optimization

One important difference between problems (8)-(10) and resource allocation for fixed receiver designs (see e.g. [5,6]), is the non-linear interference function (4), which optimally adjusts the parameter z for each choice of transmission powers. Generally, the optimizer $\hat{z}$ does not even need to be unique. Also, convexity properties, as analyzed in [5, 6], are more difficult to show under the assumption of adaptive receiver design.

A useful way of characterizing the achievable region (6) is by means of the extended coupling matrix

$$\boldsymbol{\Phi}(z, P, Q) = \begin{bmatrix} \boldsymbol{\Gamma}_{\mathrm{Q}} \boldsymbol{G}(z) \\ \mathbf{1}^T \boldsymbol{\Gamma}_{\mathrm{Q}} \boldsymbol{G}(z)/P \end{bmatrix} , \tag{20}$$

where $\mathbf{1}$ is the K-dimensional all-one vector and P is the total transmission power.

4.1 Max-Min SINR Balancing

The matrix $\boldsymbol{\Phi}$ has a real maximum eigenvalue, which equals the spectral radius $\rho(\boldsymbol{\Phi})$. The inverse spectral radius can be interpreted as the maximum balanced SINR margin. It is monotonically increasing in the total transmission power P, which is ensured by the last row $\mathbf{1}^T \boldsymbol{\Gamma}_{\mathrm{Q}} \boldsymbol{G}(z)/P$. Thus, the max-min balancing problem (7) is equivalent to an eigenvalue optimization problem

$$C(P, Q) = \frac{1}{\min_{z \in \mathcal{Z}} \rho\big(\boldsymbol{\Phi}(z, P, Q)\big)} \tag{21}$$

and the QoS achievable region is

$$\mathcal{Q}(P) = \{Q : \min_{z \in \mathcal{Z}} \rho\big(\boldsymbol{\Phi}(z, Q, P)\big) \leq 1\} . \tag{22}$$

Let $\hat{Q}$ be a point on the boundary of $\mathcal{Q}(P)$, i.e., $\min_z \rho\big(\boldsymbol{\Phi}(z, \hat{Q}, P)\big) = 1$. Then the set of optimal receive strategies is given by

$$\mathcal{Z}_{\hat{Q}} = \{z : \rho\big(\boldsymbol{\Phi}(z, \hat{Q}, P)\big) = 1\} . \tag{23}$$

A receive strategy $\hat{z}$ achieves the boundary point $\hat{Q}$, i.e., $\hat{z} \in \mathcal{Z}_{\hat{Q}}$, if and only if the following properties hold jointly:

$$\hat{z}_k = \arg\min_{z_k \in \mathcal{Z}_k} \big[\boldsymbol{G}(z)\,\hat{\overline{\boldsymbol{p}}}\big]_k , \quad k \in \{1, 2, \ldots, K\} \tag{24}$$

$$\boldsymbol{\Phi}(\hat{z}, P, \hat{Q})\hat{\overline{\boldsymbol{p}}} = \hat{\overline{\boldsymbol{p}}}, \quad [\hat{\overline{\boldsymbol{p}}}]_{K+1} = 1 \tag{25}$$

The power allocation which fulfills targets Q together with a receive strategy $\hat{z} \in \mathcal{Z}^{opt}$ is given as the right-hand principal eigenvector of the extended coupling matrix $\boldsymbol{\Phi}(\hat{z}, Q, P)$, scaled such that its last component equals one.

Note, that due to the non-uniqueness of the optimal receiver, there may exist multiple "optimal" matrices $\boldsymbol{\Phi}$. However, it can be shown, that all have the same principal right eigenvector.

Motivated by the optimality conditions (24) and (25), we propose the following iterative algorithm to solve the weighted max-min problem (7):

QoS achievable region is adequately characterized by the spectral radius of the coupling matrix. The assumption of irreducibility is not required. In this respect, the paper extends previous work, which was based on the assumption of irreducibility.

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