Towards a Network Calculus for Bursty ON/OFF Traffic

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Abstract. Models of bursty traffic have attracted much attention in recent years. While there was considerable progress in understanding the performance implications of ON/OFF traffic in single-node bottleneck models, the generalisation to larger queueing networks is required for an end-to-end performance investigation. This paper presents first steps towards a new network calculus for probabilistic ON/OFF traffic, as generated by the so-called 1-Burst model. The approach is based on an ON/OFF description of output processes of the individual queues, and its computational feasibility results from the property that the state-space size to describe the ON/OFF traffic remains constant along the nodes of a tandem queue. The computation of the traffic parameters only requires matrix-analytic methods for the busy-period analysis of a single queue with such ON/OFF input. Numerical results obtained from analytic models confirm the validity of this approach.

Keywords: Performance model, ON/OFF traffic, Tandem queue, Network decomposition

1 Introduction

Upcoming real-time applications put demands on the underlying transmission networks which are formulated with respect to end-to-end performance metrics. Hence, performance modelling is also required in an end-to-end manner, taking the full network (multi-hop) into account. The deterministic network calculus [4] introduced the framework for worst case analysis, but cannot take into account the gain of statistical multiplexing of traffic sources in a network, leading to over-provisioning of services and resources for individual flows and to under-utilisation of network components. Resource provisioning for so-called ON/OFF traffic would particularly ignore its high potential for multiplexing gain. The statistical network calculus [2] tries to remedy these deficiencies to some extent. However, stochastic extensions based on effective bandwidths are usually only suitable to highly multiplexed traffic. For scenarios of single or few ON/OFF sources, alternative methods need to be investigated. This paper develops the first steps toward a network calculus
for ON/OFF traffic of low multiplex degree. Earlier results on ON/OFF traffic in single bottleneck queueing models [13] can thereby be extended to queueing networks.

Efficient and exact algorithms for the evaluation of queueing network models exist for networks with constraints on inter-arrival and service times distributions, such that the networks become separable and amenable to product-form solutions. Well known for this kind of network decomposition are the Jackson queueing networks. For more complex but still Markovian models, even moderately sized buffer capacities will often cause so-called state space explosion which may be tackled by state-space decomposition. In contrast, a traffic-based decomposition largely avoids the generation of state spaces altogether. Each node of the queueing network is analysed in isolation by (dedicated) algorithms, but good approximations of the departure processes by parsimonious models are crucial [8].

The output process of a single node is already analysed in early work, e.g., [3, 5]. In recent papers [7, 8, 12], it is shown that departure processes of MAP/PH/1 and MAP/MAP/1 queues are in general non-renewal, the inter-departure times can be exactly represented by a Markovian Arrival Process (MAP), and the correlation structure is approximated to a certain accuracy.

In the next section, the two-node tandem queue with an external ON/OFF traffic source is introduced as a first step towards a network calculus. The relevant traffic descriptors are explained, of which the so-called burstiness parameter is central. In a two-node tandem queue the down-stream queue is fed by the first queue during its busy period. Hence, the analysis of the first queue’s busy period and the subsequent synthesis of adequate input models for the second queue is of main importance. Section 3 shows our approach followed by some modifications in Section 4. In Section 5 the exact queueing behaviour of the second queue is compared with those resulting from the approximating input models. Section 6 provides an outlook on subsequent steps to obtain a full network calculus, before Section 7 summarises and concludes the paper.

2 Two-node tandem queue and first steps of a network calculus

In order to derive and explain the methodology to analyse ON/OFF traffic in larger networks, we first focus on an infinite-buffer, two-node feed-forward tandem queue, illustrated in Figure 1.

Fig. 1. Illustration of 2-node tandem queue with 1-Burst arrivals

The arrival process to the first queue, Q1, is a single-source ON/OFF process, called 1-Burst [13], with the following characteristics:

- The OFF periods are exponentially distributed with mean $Z$. No packet arrivals at Q1 occur during the OFF periods.
During ON periods, packets arrive according to a Poisson process with rate \( \lambda_p \).

The ON periods are matrix-exponential distributed with mean \( \overline{ON} \) (i.e., on average \( \pi_p := \lambda_p \overline{ON} \) packets arrive during an ON period).

The following two parameters can be derived:

\[
\text{Overall average packet rate, } \lambda = \lambda_p \frac{\overline{ON}}{\overline{ON} + Z}, \text{ and burstiness [11], } b := 1 - \frac{\lambda}{\lambda_p}.
\]

For \( b \to 0 \) the ON/OFF process converges to constant-rate Poisson arrivals, while for \( b \to 1 \), the ON/OFF process converges to a bulk-arrival model. Hence, in the following, we will use the parameter set \( \langle \lambda, b, Z \rangle \) for the parameterisation of 1-Burst models at Q1. For later characterisation of '1-Burst-type processes' at Q2 we will use the same identifiers, but indicate the queue number as superscript, i.e. \( \langle \lambda^{(2)}, b^{(2)}, Z^{(2)} \rangle \).

The service times in Q1 and Q2 are both exponentially distributed with rates \( \nu_1 \) and \( \nu_2 \), respectively. Hence, the first queue, Q1, is a 1-Burst/M/1 queueing system, which has an MMPP/M/1 representation. In the scope of this paper, we are interested in steady-state performance parameters, in particular queue-length distributions, i.e. the joint distribution of the number of packets \( N_i \) at Queue \( i \). For \( \nu_2 > \nu_1 \), the first queue is the bottleneck in this setting, and the performance behaviour of the tandem system would be predominantly determined by the first queue (with well known previous results [13]). Consequently this paper mainly focuses on settings with \( \nu_1 \geq \nu_2 > \lambda \).

When the system occupancy \( N := N_1 + N_2 \) is truncated at some value \( N_{\text{max}} \), which corresponds to discarding the additional 1-Burst arrivals at Q1 while \( N > N_{\text{max}} \), the tandem system is a finite state-space Markov process as described in the appendix. Due to the special structure of this Markov process, optimised algorithms as in [6] can be used for the computation of the steady-state probabilities. Hence, lower bounds for the queue occupancy can be obtained via direct computation of this truncated tandem system. For increasing \( N_{\text{max}} \), these lower bounds converge to the exact solution, at the cost of a quickly growing state-space.

The output process of a 1-Burst/M/1 queue (here Q1) in principle also follows an ON/OFF pattern: During busy periods, packet departures occur with rate \( \nu_1 \), while during idle periods, no departures are generated. Hence, an approach for a network calculus using traffic composition via modified 1-Burst processes is discussed in the following: The overall average packet rate at Q2 is \( \lambda \) for a loss-less system. The packet rate during ON periods at Q2 is given by the service rate of Q1, while the average duration of the ON period is determined by the busy period of Q1:

\[
\lambda^{(2)} = \lambda, \quad \lambda_p^{(2)} = \nu_1, \quad \overline{ON^{(2)}} = BFP_1.
\]

The average duration of the busy period of the 1-Burst/M/1 queue can be computed using Matrix-algebraic methods, see [9, 10]. It follows that the other parameters of Q2 are:

\[
b^{(2)} = 1 - \frac{\lambda}{\nu_1} =: 1 - \rho_1, \quad Z^{(2)} = BFP_1 \cdot \left( \frac{\nu_1}{\lambda} - 1 \right) = BFP_1 \frac{1 - \rho_1}{\rho_1}.
\]

Note that only the average OFF period duration at Q2, \( Z^{(2)} \), depends on actual queueing behaviour at Q1 (via its busy period).

In order to apply this decoupling approach, three major questions have to be addressed:
1. What *distribution* for the ON periods at Q2 is appropriate?
2. A busy period contains at least one packet departure. An MMPP ON/OFF model would also allow for 'empty' ON periods, without any packet arrivals to Q2 during ON periods. Is this deficiency relevant?
3. What is the impact of the decoupling on relevant performance metrics (e.g. jitter)?

These questions are addressed in the subsequent sections.

### 3 Analysis of Busy Period Duration

When using a finite buffer MMPP/M/1/K approximation the duration of the busy period has a straightforward matrix-exponential representation, namely the corresponding part of the generator matrix of the queue that corresponds to levels 1,...,K. This exact (for finite K) representation of the busy period duration can be used in the 1-Burst process at Q2, however this would lead to a large state-space increase, which is not acceptable for a network calculus. Consequently, adequate approximations need to be investigated.

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**Fig. 2. Complementary Distribution function of Busy period duration:** The log-log graph shows the tails of the distribution for the M/M/1/K and the 1-Burst/M/1/K queue. A fitted exponential distribution (dotted) and a 3-moment fit of a Hyp-2 (dashed) is shown in comparison. The left graph corresponds to a utilisation of 45% (using K = 100), while the right graph reflects a large utilisation of 95% (using K = 200).

Figure 2 shows the complementary distribution function of the Busy Period (BP) duration of a M/M/1/K and 1-Burst/M/1/K queue and two different utilisation values $\rho = 0.45$ (left) and $\rho = 0.95$ (right). The exponential approximation (dotted line) clearly does not reflect the tail-behaviour, in particular for large $\rho$ at which Power-Tail like behaviour (straight line on log-log scale) can be observed. A 3-moment fit of a 2-phase Hyper-exponential distribution (Hyp-2, dashed line) on the other hand yields a much better fit. Hence, in the following the three moment fit of a Hyp-2 distribution is used.

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3 The truncation level $K$ can be picked such that the probability that the busy period reaches queue-length $K$ is below a certain threshold; this probability can be computed with matrix-algebraic methods.
to approximate the busy period duration of Q1. The moments can be computed without large computational effort from the Matrix-Exponential representation of the busy period duration. Using this Hyp-2 distribution, the state-space increase along the tandem queue is avoided.

4 The problem of empty bursts: MAP vs MMPP

As described before, the basic approach to derive a network calculus, in the first step for tandem queues, relies on approximating the output process of each queue by another 1-Burst process. In order to avoid increasing state-space sizes along the nodes of the tandem queue, the ON time distribution of these 1-Burst processes is chosen as a moment-fitted Hyp-2 distribution. However, a busy period in the first queue causes at least one arrival in the second queue, while an ON period in a 1-Burst MMPP may not contain any arrivals at all: For a Hyp-2 distribution with rates $\mu_1$ and $\mu_2$ and corresponding state entrance probabilities $p_1$ and $1 - p_1$, the probability that an empty ON period results (at Poisson arrival rate $\nu_1$ to Q2 during ON) is:

$$\Pr(\text{empty burst}) = p_1 \frac{\mu_1}{\mu_1 + \nu_1} + (1 - p_1) \frac{\mu_2}{\mu_2 + \nu_1}.$$  

This probability can be significantly large, and these empty bursts lead to prolonged OFF periods, hence the traffic effectively becomes more bursty. As a consequence, the performance is worse in the MMPP/M/1 queue, Q2, than in the comparable tandem setting (see the next section).

Fig. 3. Illustration of 1-Burst MMPP (left) and MAP (right): Generated arrivals are indicated by dashed arrows. In the MAP version, the transition to the OFF state is associated with an arrival.

In order to avoid empty bursts, the 3-state 1-Burst MMPP is modified to a MAP as shown in Figure 3: The transition from the ON states to the OFF state is associated with an arrival; the Poisson arrival rates during the ON states have to be modified accordingly to keep the overall packet rate during ON at $\nu_1$. Hence, these Poisson rates are reduced by the state leaving rates $\mu_1$ and $\mu_2$ respectively. In order to avoid negative Poisson departure rates, another boundary condition on the Hyp-2 distribution results: $\mu_i \leq \nu_1$, $i = 1, 2$. In case the 3-moment Hyp-2 fit does not fulfil these conditions on $\mu_i$, $\mu_1$ is set to $\nu_1$ and the remaining $\mu_2$ and $p_1$ are obtained from a 2-moment fit.

Because of its paramount impact on queueing behaviour the coefficient of variation, $C$, and the correlation between subsequent inter-arrival times are both computed analytically using matrix algebraic methods for the different approximating input processes for Q2.
Fig. 4. Squared Coefficient of Variation (left) and Correlation lag 1 (right) of different departure process approximations: $C^2$ and $r(1)$ are plotted for different burstiness, $b$, of the 1-Burst input process to Q1. For larger values of $b \geq 0.5$, the 3-moment fit could be applied in the MAP model, while for smaller $b$ the boundary conditions of the MAP model mandated a 2-moment fit.

Figure 4. shows $C^2$ and lag-1 correlation, $r(1)$, of four different MAP processes and three different MMPPs:

- The exact MAP representation of the output process of Q1 when truncated at level $K = 300$ (exact MAP, solid).
- Approximations with a 1-Burst MMPP process (dashed) using the exact Busy Period distribution (MMPP BP), an exponential (MMPP Exp), and a moment fitted Hyp-2 (MMPP Hyp-2) approximation.
- The corresponding MAP extensions (solid line) of the 1-Burst process, avoiding empty bursts $b$, also in three versions: MAP process using the exact Busy Period distribution (MAP BP), exponential BP (MAP Exp), and moment-fitted Hyp-2 (MAP Hyp-2) approximation.

**Observations and discussion:** At the left end, $b = 0$, the 1-Burst arrival process at Q1 converges to a Poisson process, hence (for infinite $K$), the output process is also Poisson, i.e. $C^2 = 1$. The MAP approximations show the same 'correct' convergence properties, while the MMPP approximations (because of increased burstiness due to empty bursts) converge to a value $C^2 > 1$. When using an exponential approximation for the ON period duration at Q2, both the MAP and MMPP versions (MAP Exp, MMPP Exp) show zero correlation (renewal process), which is of course not appropriate for cases with $b > 0$ at Q1 when in fact there is correlation in the output process. The 1-Burst approximations, both MMPP and MAP, with non-exponential ON periods always yield positive correlation.

In summary, the recommended approach using a modified 1-Burst with forced arrival at the end of the ON period and with a Hyp-2 approximation of the busy period duration of Q1 for the ON period distribution for Q2 shows the following properties:

- It correctly models the variance ($C^2$) for both limits, $b \to 0$ when $C^2 \to 1$ and for $b \to 1$. In the intermediate region, it underestimates the variance of the departure process of Q1 (Fig. 4, left).

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4 For $K=300$, the probability that Q1 reaches at least level $K=300$ during the BP, is smaller than $1e^{-17}$. 
It correctly models the correlation at the right end \( b \to 1 \) (at several lags, although only lag 1 is shown in the figure), while it over-estimates correlation for \( b < 1 \) including in particular the non-bursty case \( b \to 0 \), when the departure process is Poisson.

5 Analysis of Queueing Behaviour

The purpose of a network calculus for bursty traffic is to reflect/approximate the performance behaviour, here measured in terms of Queue-length distribution, more specifically average queue-lengths (which correspond via Little’s theorem to average delay). So the main interest is in an investigation of how the different approximation models (MAP/M/1 and MMPP/M/1 queues) reflect the correct tandem queue-lengths.

![Graph showing comparison of average Queue-length at Q2 as predicted by different approximations](image)

Fig. 5. Comparison of average Queue-length at Q2 as predicted by different approximations: The MAP and MMPP approximations using the full ME representation of the busy period of Q1 (not shown in this figure) were very close to the corresponding MAP/MMPP models using the fitted Hyp-2 distributions.

Figure 5 shows the average queue-length at Q2 for different approximations of the arrival process at Q2. As it can be seen in the figure, the MAP approximation using the Hyp-2 ON period approximation (solid, MAP Hyp-2) predicts the correct queueing behaviour rather accurately, with slight over-estimation at both ends, \( b \to 0 \) (Poisson case) and \( b \to 1 \) (bulk-arrivals at Q1). A MAP approximation utilising exponential ON periods provides an exact value at the (less interesting) left end, but largely under-estimates the queueing behaviour in the more bursty case. The MMPP models (dashed curves) on the other hand tend to over-estimate strongly (MMP Hyp-2) or are inconclusive (MMP Exp). Similar conclusions were obtained for other choices of the average rate \( \lambda \) (not shown here).

6 Towards a network calculus with ON/OFF traffic

In this paper, the first important steps towards a new network calculus for bursty ON/OFF traffic as generated by the 1-Burst process are presented. Simple equations are sufficient for the computation of most parameters of the output process at a node. Only the moments of the distribution of the busy period are computationally more demanding, but
can be obtained efficiently via well-known matrix-algebraic methods. Also, the queueing behaviour at each node is determined by a MAP/M/1 queue for which numerical results can be obtained. The following steps are still necessary to develop a full network calculus:

- **Flow aggregation:** Multiple traffic flows may merge at the input of a specific node. If all these flows show 1-Burst characteristics, the resulting aggregated process is of the type of aggregated ON-OFF sources, called N-Burst in previous papers [13]. The computation at each node becomes somewhat more computationally demanding, but performance results for this type of queueing system are also known. The output process can still be approximated by a modified 1-Burst MAP, but accuracy aspects would need to be investigated.

- **Feedback Loops:** There are two types of feedback: (1) Output traffic flows may be partially directed back to earlier nodes. Such a feedback can in principle be described by flow aggregation as above, but fix-point iteration may become necessary to solve for the queueing behaviour. (2) Performance behaviour may impact source traffic parameters as e.g. for TCP traffic. See e.g. [14] for modifications of ON/OFF sources to include TCP behaviour. Again, fix-point iteration may be necessary, but details need to be investigated in future work.

- **Correlation between queues:** Cross-correlation of the queue-length processes at different queues in the network will in particular affect parameters such as delay variations (jitter): the variance of the occupancy of the complete tandem system is the sum of the variance of the individual queues, corrected by covariance terms. Figure 6 shows example calculations, illustrating that the correlation is positive and becomes non-negligible for larger burstiness \( (b > 0) \) of the original arrival process at Q1.

![Fig. 6. Variance of the queue-lengths and coefficient of correlation between the queues: For bursty input traffic, the queue-lengths at Q1 and Q2 are positively correlated, hence a decoupling approximation underestimates the variance of the Queue-lengths, since it cannot consider the positive covariance.](image)

The discussion in this section points out the directions of our approach to a complete network calculus, but more detailed investigation is still necessary.
7 Conclusion and Outlook

This paper presented the first steps towards a computationally feasible network calculus for end-to-end performance evaluation of bursty ON/OFF traffic. The approach relied on a description of the output process of each queue, which does not suffer from state-space increase: the output process is found to be better represented by a particular 1-Burst MAP extension than an MMPP, avoiding the performance impacting problem of empty bursts. In order to avoid state-space increase the ON period distribution was obtained from a moment fit of a Hyp-2 distribution to the Busy Period distribution of the previous queue. Although there are deviations of variance and correlation properties of the applied approximation as compared to the exact output process, the analysis of the performance behaviour in terms of average queue-lengths supports the suitability of the chosen approach.

More investigations are necessary to capture the impact of flow-aggregation, feedback loops, and correlation between subsequent queues, as pointed out in Section 6. In addition, earlier analysis of single-node queueing systems [1,15] indicates the potential deficiencies of steady-state analysis. Hence, a network calculus approach directed at transient analysis should be considered in the future.

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A Markov Process Generator of a two-node tandem queue

We consider a tandem queue with two exponential servers of rate $\nu_1$ and $\nu_2$ each with infinite buffer as depicted in Figure 1. The arrival process is an MMPP described by the generator matrix $Q$ and a diagonal matrix with corresponding Poisson rates $L$ as in earlier papers (in the special case of Poisson arrivals, $Q = 0$, $L = \lambda$). Furthermore $S := \text{dim}(Q)$ is the size of the state-space of the MMPP.

The state space of the tandem queue with MMPP arrivals can be described by the tuple $(i, n_1, n_2)$ where $i = 1, ..., S$ is the state of the MMPP arrival process, $n_1 = 0, 1, ...$ is the number of customers at Q1 and $n_2 = 0, 1, ...$ the number of customers at Q2. However, for enumeration purposes, we will use the slightly modified state-space $(i, N, n_1)$, where $N = n_1 + n_2$ is the overall number of customers in the tandem system, hence $n_1 = 0, ..., N$.

Using the state-space as above, the following transitions are possible:

- Internal transitions of the MMPP: $(i, N, n_1) \rightarrow (j, N, n_1)$
- Arrival at Q1 (generated by MMPP): $(i, N, n_1) \rightarrow (i, N + 1, n_1 + 1)$
- Arrival at Q2 (i.e. completed service at Q1): $(i, N, n_1) \rightarrow (i, N, n_1 - 1); N, n_1 \geq 1$
- Departure from Server 2: $(i, N, n_1) \rightarrow (i, N - 1, n_1); N > n_1$

Consequently, the generator matrix for the tandem system looks as follows:

$$Q = \begin{bmatrix}
Q - \Delta_0 & L & 0 \\
L_1 & D_1 & U_1 \\
L_2 & D_2 & U_2 \\
\vdots & \ddots & \ddots
\end{bmatrix},$$

which is a Quasi-Birth-Death (QBD) process with growing blocks for increasing $N$. The blocks correspond to states with $N = 0, N = 1, ...$, etc. while the individual matrix elements within a block (for given $N$) correspond to $n_1 = N, N - 1, ..., 0$. 

Using Kronecker Products, the block matrices in the QBD structure can be written down in compact form for block $n$, i.e. all states with $N = n$, $n > 0$:

$$
U_n = [I_{n+1} \otimes L, \text{zeros}(S \cdot (n+1), S)], \quad L_n = \begin{bmatrix}
\text{zeros}(S, S \cdot n) \\
I_n \otimes \{\nu_2 I\}
\end{bmatrix},
$$

$$
D_n = \begin{bmatrix}
Q - \Delta_1 & \nu_1 I \\
\ldots & \ldots & \ldots \\
Q - \Delta_{12} & \nu_1 I \\
Q - \Delta_2 & \nu_2 I
\end{bmatrix}, \quad \text{e.g., } D_1 = \begin{bmatrix}
Q - \Delta_1 & \nu_1 I \\
Q - \Delta_2
\end{bmatrix}.
$$

The matrices $\Delta_0, \Delta_1, \Delta_{12},$ and $\Delta_2$ are diagonal matrices chosen in order to have row sum zero, i.e. $\Delta_0 = L, \Delta_1 = L + \nu_1 I, \Delta_{12} = L + (\nu_1 + \nu_2) I,$ and $\Delta_2 = L + \nu_2 I.$ $I_n$ denotes the Identity matrix of size $n, I$ denotes the identity matrix of same dimension as the matrix $Q$.

### References