also one-to-one. Hence, there exists an inverse mapping assigning a unique SINR matrix $\boldsymbol{\Gamma}$ to every positive power vector. The domain of the mapping (3) is called *SINR feasibility region*, since the achievement of any $\boldsymbol{\Gamma}$ from within is possible with an admissible power vector $\mathbf{p} \in \mathcal{P}$. We omit the indication of power regions with P and $\hat{\mathbf{p}}$ in statements which hold regardless of power constraints.

Our interest is in parameters characterizing the link quality in terms of a desired QoS feature. We denote such link-QoS parameters by q_k and group them in the vector $\mathbf{q} = [q_1, q_2, \ldots, q_K]^T$. For every link $1 \leq k \leq K$ we assume a one-to-one continuous dependence $\Phi(q_k) = \gamma_k$. Due to bijectivity and continuity of Φ there exists an inverse mapping Ψ, such that for $1 \leq k \leq K$ holds $\Psi(\gamma_k) = q_k$. For monotone increasing Ψ the q_k increase with increasing SINR level and for monotone decreasing Ψ they decrease with the increment of SINR. We write the extensions of Φ and Ψ to matrix and vector valued functions as $\boldsymbol{\Phi}(\mathbf{q}) = \boldsymbol{\Gamma}$, and $\boldsymbol{\Psi}(\boldsymbol{\Gamma}) = \mathbf{q}$ respectively. Hence, the connection between the space of QoS parameter vectors and the space of power vectors can be written as a concatenated mapping

$$\mathbf{p} \longmapsto \boldsymbol{\Gamma} \overset{\boldsymbol{\Psi}}{\longmapsto} \mathbf{q}. \tag{4}$$

The inverse of this dependence is $\mathbf{q} \overset{\boldsymbol{\Phi}}{\longmapsto} \boldsymbol{\Gamma} \longmapsto \mathbf{p}$, which yields

$$\mathbf{p}(\mathbf{q}) = (\mathbf{I} - \boldsymbol{\Phi}(\mathbf{q})\mathbf{V})^{-1}\boldsymbol{\Phi}(\mathbf{q})\boldsymbol{\sigma}^2. \tag{5}$$

From (4) arises the notion of *QoS feasibility region*. In the remainder we call it simply QoS region and denote it as $\mathcal{Q}_P = \{\mathbf{q}(\mathbf{p}) : \mathbf{p} \in \mathcal{P}_P\}$ under sum-power constraint and $\mathcal{Q}_{\hat{\mathbf{p}}} = \{\mathbf{q}(\mathbf{p}) : \mathbf{p} \in \mathcal{P}_{\hat{\mathbf{p}}}\}$ under individual power constraints.

3 Convexity of the QoS Region

We first shortly address the issue of geometrical properties of the QoS region. Such properties have a crucial influence on the design of scheduling/transmission policies for uplink and downlink, see [1], [2] and references therein for detail analysis [1].

The following fundamental theorem is proven in [1] (in the context of the reference all q_k are considered as QoS requirements, but the statement is equivalent to the one below).

Theorem 1 *Suppose that $\boldsymbol{V}$ is irreducible and $\Phi(q) = \gamma$ is log-convex (a function is log-convex if its logarithm is convex). Then, the spectral radius $\rho(\boldsymbol{q})$ in a sum-power constrained network is also log-convex on $\mathcal{Q}_\infty \overset{\Delta}{=} \{\mathbf{q}(\mathbf{p}) : \mathbf{p} \in \mathbb{R}_+^K\}$.*

Theorem 1 allows for statements concerning convexity of the QoS regions of power constrained systems (see [1], [2]).

Corollary 1 *If Φ is log-convex the following holds,*
i.) the QoS region $\mathcal{Q}_P$ is a convex set,
ii.) the QoS region $\mathcal{Q}_{\hat{\mathbf{p}}}$ is a convex set.

[1] Note, that due to the definitions of the mappings (4), (5) and the QoS feasibility region the concern of this work is in concurrent transmission of several liks. The issue of time-sharing does not fall into the presented framework.

In [7] is shown that log-convexity of function Φ expressing the SINRs in terms of QoS parameters is a necessary and sufficient condition for $\mathcal{Q}_P$ to be convex in the case of an arbitrary, possibly variable, number of links. There exist numerous functions Φ satisfying the log-convexity requirement, which correspond to widely utilized QoS parameters. The examples are the slope of the bit-error rate curve over the SINR $q = \Psi(\gamma) = \frac{1}{\gamma}$ (i.e. $\gamma = \Phi(q) = \frac{1}{q}$), data-rate in high-power regime $q = \Psi(\gamma) = \log \gamma$ (i.e. $\gamma = \Phi(q) = \exp(q)$), etc. In what follows we implicitly use the above log-convexity assumption.

4 Optimization of Weighted Sum of QoS Parameters

We concentrate on the weighted sum of QoS measures

$$f_{\boldsymbol{\alpha}}(\mathbf{p}) = \boldsymbol{\alpha}^T \mathbf{q}(\mathbf{p}) = \sum_{k=1}^{K} \alpha_k q_k(\mathbf{p}) = \sum_{k=1}^{K} \alpha_k \Psi(\gamma_k(\mathbf{p})), \tag{6}$$

with $\boldsymbol{\alpha} \geq 0$, as the QoS optimization objective. From the network operator point of view (6) can be regarded as the simplest global revenue functional (respectively, a kind of global network QoS measure), with $\boldsymbol{\alpha}$ as the vector of pricing/priority factors. Objective (6) is linear (both convex and concave) on $\mathcal{Q}$. Due to this linearity and convexity of $\mathcal{Q}$ (under log-convexity of Φ) all $\mathbf{q}$ at the boundary of $\mathcal{Q}$ can be parametrized by the weight vectors $\boldsymbol{\alpha}$ in the following sense [8]. The QoS vector $\mathbf{q}^{\text{opt}}$ generated by the power allocation optimizing (6) corresponds to the vector at the boundary of $\mathcal{Q}$, at which the hyperplane with normal vector $\boldsymbol{\alpha}$ supports $\mathcal{Q}$. Vector $\mathbf{q}^{\text{opt}}$ is called *Pareto-optimal* for $\boldsymbol{\alpha}$ [8]. Although given interpretation in terms of QoS vectors is nice due to convexity of the QoS region, the actual optimization of (6) over $\mathcal{P}$ is in general nonconvex. However, efficient solving is still possible due to the following statement, proven in [5] (without loss of generality in the remainder we assume decreasingness of Ψ and hence optimization in form of minimization).

Theorem 2 *If $\Phi = \Psi^{-1}$ is log-convex, then the problem*

$$\min_{\boldsymbol{p} \in \mathcal{P}} f_{\boldsymbol{\alpha}}(\boldsymbol{p}) = \min_{\boldsymbol{p} \in \mathcal{P}} \sum_{k=1}^{K} \alpha_k \Psi(\gamma_k(\boldsymbol{p})) \tag{7}$$

has a connected set of minimizers and hence, the KKT conditions are necessary and sufficient for the optimum.

Furthermore, as was shown in [5], with the use of additional variables

$$g_k \stackrel{\Delta}{=} \alpha_k \left. \frac{d\Psi}{d\gamma} \right|_{\gamma=\gamma_k} \frac{\gamma_k}{p_k}, \tag{8}$$

$1 \leq k \leq K$ (grouped in vector $\mathbf{g}$) we are able to write the set of KKT conditions in an illustrative matrix form. The KKT condition set takes the form

$$\begin{cases} \mathbf{p} = (\mathbf{I} - \boldsymbol{\Gamma}\mathbf{V})^{-1} \boldsymbol{\Gamma} \boldsymbol{\sigma}^2 \\ \mathbf{g} = -(\mathbf{I} - \mathbf{V}^T \boldsymbol{\Gamma})^{-1} \mathbf{c}, \end{cases} \tag{9}$$

with vector $\mathbf{c}$ dependent on power constraint type and dual optimization variables. The first matrix equation in (9) represents the correspondence between the power vector and the SINR matrix (corresponding one-to-one to the QoS vector). The second equation connects further the vector of weights with the powers and SINRs, since $\mathbf{g} = diag(\frac{d\Psi}{d\gamma_k}\frac{\gamma_k}{p_k})\boldsymbol{\alpha}$.

5 Convex Restatement of the QoS Problem

The QoS optimization problem (7) was shown in Theorem 2 to have a connected set of minimizers. Although this fact indicates good optimization-theoretic behaviour of (7), an obvious arising question is: which requirements have to be satisfied by the function $\Phi = \Psi^{-1}$ (besides log-convexity) in order to render the QoS optimization convex? The convex restatement of the optimization problem is extremely desirable in terms of convergence properties of the applied iterative techniques. Without going into detail we refer here to [8]. The question of convex restatement can be answered by the following statement, the first part of which is proven in [9] and the second part following immediately.

Theorem 3 *The function $\Psi(e^x)$ is convex if and only if $\Phi = \Psi^{-1}$ is log-convex. Hence, the restatement of problem (7) in the form $\min_{\mathbf{x}\in\mathcal{X}} f_{\boldsymbol{\alpha}}(\exp(\mathbf{x}))$, with $\mathcal{X} = \{\mathbf{x} = \log(\mathbf{p}) : \mathbf{p} \in \mathcal{P}\}$, is convex.*

In other words, the QoS optimization problem can be made convex under modification of the concatenated mapping (4) to the form $\mathbf{x} \stackrel{\exp}{\longmapsto} \mathbf{p} \longmapsto \boldsymbol{\Gamma} \stackrel{\Psi}{\longmapsto} \mathbf{q}$.

6 Iterative QoS Optimization

The general algorithmic optimization theory is well developed, see e.g. [8]. However, specialized algorithms utilizing the special problem structure can be advantageous in terms of convergence rate or computational complexity. Examples of such designs in terms of power control and QoS optimization can be found e.g. in [4], [10] and references therein. A specialized algorithm optimizing (7) for power-unconstrained networks is proposed in [4]. Our design uses elements of the design in [4] and is applicable to power-constrained networks.

6.1 Sum-Power Constrained Networks

Rewriting (2) the sum-power constraint can be expressed by

$$\mathbf{1}^T\boldsymbol{\Gamma}\mathbf{V}\mathbf{p} + \mathbf{1}^T\boldsymbol{\Gamma}\boldsymbol{\sigma}^2 = \mathbf{1}^T\mathbf{p} \leq P. \tag{10}$$

Obviously, the above holds with equality at the optimum of the QoS problem (tight constraint) [5]. The parameters $\mathbf{V}$ and $\boldsymbol{\sigma}^2$ can be arbitrarily scaled in order to arrive at the problem equivalent to (7) with constraint $P = 1$. (2) and (10) can now be written in joint matrix equation form

$$\begin{bmatrix} \boldsymbol{\Gamma}\mathbf{V} & \boldsymbol{\Gamma}\boldsymbol{\sigma}^2 \\ \mathbf{1}^T\boldsymbol{\Gamma}\mathbf{V} & \mathbf{1}^T\boldsymbol{\Gamma}\boldsymbol{\sigma}^2 \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix} \triangleq \mathbf{X}(\boldsymbol{\Gamma}) \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix}. \tag{11}$$

Lemma 1 $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}(\mathbf{p})$ *solves (11) if and only if it solves*

$$\rho(\mathbf{X}(\boldsymbol{\Gamma})) = 1. \tag{12}$$

Proof. Matrix equation (11) has the form of eigenvalue equation, with $\lambda(\mathbf{X}(\boldsymbol{\Gamma})) = \rho(\mathbf{X}(\boldsymbol{\Gamma})) = 1$. Since $\mathbf{X}(\boldsymbol{\Gamma})$ is irreducible under irreducibility of $\mathbf{V}$ it follows from Perron-Frobenius Theory, that the eigenvector corresponding to the spectral radius is nonnegative and unique. Equivalently, from (12) equation (11) follows with uniqueness of the right Perron-Frobenius eigenvector due to irreducibility of $\mathbf{X}(\boldsymbol{\Gamma})$ [6].

From nonnegativity of the right Perron-Frobenius eigenvector of $\mathbf{X}(\boldsymbol{\Gamma})$ we have that the admissibility condition is implied by (12) [6]. With bijections (4), (5) and Lemma 1 follows that (12) characterizes the boundary[2] of the power region $\mathbf{bd}(\mathcal{P}_P)$, the boundary of the SINR region $\{\boldsymbol{\Gamma}(\mathbf{p}) : \mathbf{p} \in \mathbf{bd}(\mathcal{P}_P)\}$, and the boundary of the QoS region $\mathbf{bd}(\mathcal{Q}_P)$. Due to these one-to-one correspondences, equivalently to (7) we can consider the problem $\min_{\mathbf{q}\in\mathbf{bd}(\mathcal{Q}_P)} \sum_{k=1}^{K} \alpha_k q_k$ or $\min_{\mathbf{q}:\rho(\mathbf{X}(\boldsymbol{\Phi}(\mathbf{q})))=1} \sum_{k=1}^{K} \alpha_k q_k$. The associated KKT condition corresponding to zeroing of the derivative of the Lagrangean is $\alpha_k + \nu \frac{\partial}{\partial q_k}\rho(\mathbf{X}(\boldsymbol{\Phi}(\mathbf{q}))) = 0, 1 \leq k \leq K$, which is equivalent to $\boldsymbol{\alpha} = -\nu\nabla_q \rho(\mathbf{X}(\boldsymbol{\Phi}(\mathbf{q})))$, where $\nu \geq 0$ is the dual variable associated with the sum-power constraint. Hence, $\boldsymbol{\alpha}$ and $\nabla_q(\mathbf{X})$ are antiparallel at the minimum. Under log-convex Φ the considered problem is convex and satisfies Slaters constraint qualifications, so that the antiparallelism condition and $\rho(\mathbf{X}(\boldsymbol{\Phi}(\mathbf{q}))) = 1$ are necessary and sufficient conditions for the minimum.

The design of our algorithm bases on this fact. Input parameters of the algorithm are $\boldsymbol{\alpha}$, $\mathbf{V}$, $\boldsymbol{\sigma}^2$, Ψ (with log-convex inverse) and starting power vector $\mathbf{p}(0)$ satisfying (10) for $P = 1$ with equality. The step-size value is denoted by s. We define $\nabla_q \rho(\mathbf{X}(n)) \triangleq \nabla_q \rho(\mathbf{X}(\boldsymbol{\Gamma}(\mathbf{q}(n))))$. The algorithm iterations are as follows.

While not $(\alpha \parallel -\nabla_q \rho(\mathbf{X}(n-1)))$ do

1. given $\mathbf{p}(n-1)$ compute $\boldsymbol{\Gamma}(n)$ from (2)
2. $\mathbf{q}(n) = \boldsymbol{\Psi}(\boldsymbol{\Gamma}(n))$
3. $\mathbf{q}^*(n) = \mathbf{q}(n) + s\left[-\boldsymbol{\alpha} + \frac{\boldsymbol{\alpha}\circ\nabla_q \rho(\mathbf{X}(n))}{\|\nabla_q \rho(\mathbf{X}(n))\|_2^2}\nabla_q \rho(\mathbf{X}(n))\right]$
4. $\boldsymbol{\Gamma}^*(n) = \boldsymbol{\Phi}(\mathbf{q}^*(n))$
5. $\mathbf{p}^*(n) = (\mathbf{I} - \boldsymbol{\Gamma}^*(n)\mathbf{V})^{-1}\,\boldsymbol{\Gamma}^*(n)\boldsymbol{\sigma}^2$
6. $\mathbf{p}(n) = \frac{1}{\|\mathbf{p}^*(n)\|_1}\mathbf{p}^*(n)$
7. $n \to n+1$,

with $\parallel$ denoting parallelism of vectors. For the gradient computation in step 3) we have with (5)

$$\frac{\partial}{\partial q_k}\rho(\mathbf{X}(\mathbf{q})) = \Phi'(q)|_{q=q_k}\frac{\partial}{\partial \gamma_k}\rho(\mathbf{X}(\boldsymbol{\Gamma})) = \Phi'(q)|_{q=q_k}\mathbf{r}^H\frac{\partial}{\partial \gamma_k}\mathbf{X}(\boldsymbol{\Gamma})\mathbf{r}, \quad 1 \leq k \leq K, \tag{13}$$

where $\mathbf{r}$ is the right Perron-Frobenius eigenvector of $\mathbf{X}$. After a simple but lengthy calculation it can be shown that $\frac{\partial}{\partial \gamma_k}\rho(\mathbf{X}(\boldsymbol{\Gamma})) = (r_k + r_{K+1})(\sum_{j=1\;,\;j\neq k}^{K} V_{kj}r_j + r_{K+1}\sigma_k^2), 1 \leq k \leq K$. To show the convergenece of the algorithm it is convenient to write $\mathcal{Q}_P = \mathbf{epi} f_k$,

[2] Boundary of a region in the context of this work is assumed to be only the part of the geometric boundary which corresponds to the set of power vectors satisfying the sum-constraint inequality with equality, or satisfying at least one of the individual constraint inequalities with equality.

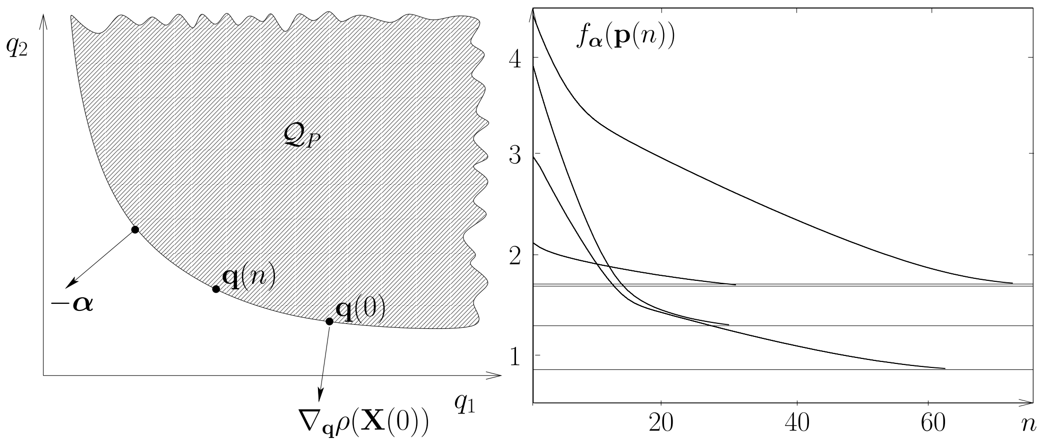

Fig. 1. Left hand side: The geometry behind the algorithm for sum-power constrained networks in an exemplary two-link case. Right hand side: Convergence of the algorithm for sum-power constrained networks for $\Psi(\gamma) = 1/\gamma$ under variation of link number (K=8,...,11) and system parameters (uniform normalized distribution of all σ_k^2 and V_{kj}).

with $\mathbf{epi}f$ denoting the epigraph of function f [8]. The function $f_k : \mathbb{R}^{K-1} \to \mathbb{R}$ expresses the k-th QoS parameter in terms of other $K-1$, i.e. $f_k(\mathbf{q}^{(k)}) \stackrel{\Delta}{=} q_k(\mathbf{q}^{(k)})$, with $\mathbf{q}^{(k)} = [q_1, \ldots, q_{k-1}, q_{k+1}, \ldots, q_K]^T$. Function f_k cannot be expressed in closed form, but we only utilize its convexity and the convexity of its epigraph for any $1 \le k \le K$, which is assured due to log-convexity of Φ. Given $\mathbf{q}(0) \stackrel{\Delta}{=} \mathbf{q}(\mathbf{p}(0))$ choose k such that $f_k(\mathbf{q}_{\text{opt}}^{(k)}) < f_k(\mathbf{q}^{(k)}(0))$. Since gradients are orthogonal to the corresponding level sets and we already have that $\{\mathbf{q} : \rho(\mathbf{X}(\boldsymbol{\Phi}(\mathbf{q}))) = 1\} = \mathbf{bd}(\mathcal{Q}_P)$, it follows that $\nabla_q\rho(\mathbf{X})$ are normal vectors to the manifold $f_k(\mathbf{q}^{(k)})$. It can be now recognized that the term $\Delta\mathbf{q}(n) \stackrel{\Delta}{=} \left[-\boldsymbol{\alpha} + \frac{\boldsymbol{\alpha}\circ\nabla_q\rho(\mathbf{X}(n))}{\|\nabla_q\rho(\mathbf{X}(n))\|_2^2}\nabla_q\rho(\mathbf{X}(n))\right]$ occuring in step 3) represents the vector tangential to the manifold f_k at $\mathbf{q}^{(k)}(n)$. This is illustrated in Fig.6.1 for an exemplary two-link case. With the epigraph representation we can express the normal vectors as scaled vectors of partial derivatives $\nabla_q\rho(\mathbf{X}(\boldsymbol{\Phi}(\mathbf{q}))) = c_1\left[-\frac{\partial q_k}{\partial q_1}, \ldots, -\frac{\partial q_k}{\partial q_{k-1}}, 1, -\frac{\partial q_k}{\partial q_{k+1}}, \ldots, -\frac{\partial q_k}{\partial q_K}\right]^T$, with c_1 as suitable scaling factor. With this it can be further seen that given some k, $1 \le k \le K$,

$$[\Delta\mathbf{q}(n)]_i = \begin{cases} -\frac{\partial q_k}{\partial q_i}\Big|_{\mathbf{q}_{\text{opt}}} + \frac{\partial q_k}{\partial q_i}(n)\frac{\sum_{j=1}^K \frac{\partial q_k}{\partial q_j}\Big|_{\mathbf{q}_{\text{opt}}}\frac{\partial q_k}{\partial q_j}(n)}{\sum_{j=1}^K\left(\frac{\partial q_k}{\partial q_j}\Big|_{\mathbf{q}_{\text{opt}}}\right)^2}, & 1 \le i \le K, i \ne k \\ 1 - \frac{\sum_{j=1}^K \frac{\partial q_k}{\partial q_j}\Big|_{\mathbf{q}_{\text{opt}}}\frac{\partial q_k}{\partial q_j}(n)}{\sum_{j=1}^K\left(\frac{\partial q_k}{\partial q_j}\Big|_{\mathbf{q}_{\text{opt}}}\right)^2}, & i = k. \end{cases} \quad (14)$$

Due to $f_k(\mathbf{q}_{\text{opt}}^{(k)}) < f_k(\mathbf{q}^{(k)}(0))$ and convexity of f_k follows, that for $1 \le i \le K$, $i \ne k$ we have $\left|\frac{\partial q_k}{\partial q_i}\Big|_{\text{opt}}\right| < \left|\frac{\partial q_k}{\partial q_i}(0)\right|$ and hence $[\Delta\mathbf{q}(n)]_k < 0$. This shows that the search direction from step 3) in the algorithm is descending with respect to $q_k = f_k(\mathbf{q}^{(k)})$. If s is now sufficiently small, it holds approximately $q_k^*(n) = f_k(\mathbf{q}^{(k)*}(n)) = f_k(\mathbf{q}^{(k)}(n+1))$, $n \in \mathbb{N}$. In other words, the new vector obtained in step 3) can be regarded to (nearly) pertain to $\mathbf{bd}(\mathcal{Q}_P)$ under a sufficiently small step of shift tangential to $\mathbf{bd}(\mathcal{Q}_P)$. In such case the algorithm generates a decreasing sequence $\{q_k(n)\}_{n\in\mathbb{N}}$. With convexity of f_k this implies that the sequences $\{\frac{\partial q_k}{\partial q_i}\}_n$, $1 \le i \le K$, $i \ne k$ are also decreasing. From (14) it follows now easily, that $\{\Delta\mathbf{q}(n)\}_{n\in\mathbb{N}}$ converges to zero, which means further that with $n \to \infty$, $\nabla_q \rho(\mathbf{X}(n)) \to \nabla_q \rho(\mathbf{X}_{\mathrm{opt}}) \parallel \boldsymbol{\alpha}$. Summarizing we can state the following.

Theorem 4 *For log-convex Φ the presented algorithm for sum-power constrained networks converges monotonically to the optimum of problem (7).*

We can interpret the algorithm as a descending walk on $\mathbf{bd}(\mathcal{Q}_P)$ towards the optimum of (7). Since in reality the step-size is not evanescent, the vectors obtained in step 3) lie slightly outside $\mathcal{Q}_P$. Then, the steps 4)-6) come to their own and provide a mapping (indirectly, in the power domain $\mathcal{P}_P$) of the vector back on $\mathbf{bd}(\mathcal{Q}_P)$. By standard proof techniques of optimization theory one can show, that under non-evanescent but sufficiently small step-sizes the good convergence bahaviour is retained [8]. The mapping in step 6) can be conducted alternatively in the domain of SINR instead of $\mathcal{P}_P$, which however does not lead to significant changes in convergence behaviour. Exemplary convergence results in Fig. 6.1 illustrate the convergence under parameter variations. The convergence rate can be improved by the adaptation of the step size, which is however a topic for separate consideration. Moreover, significant improvement of the convergence rate can be obtained by the application of the machinery of the presented algorithm directly to the equivalent convex problem formulation from Theorem 3.

6.2 Networks with Individual Power Constraints

Under individual power constraints the problem of interest can be stated as $\min_{\mathbf{q}\in\mathbf{bd}(\mathcal{Q}_{\hat{\mathbf{p}}})} \sum_{k=1}^K \alpha_k q_k$, where $\mathbf{bd}(\mathcal{Q}_{\hat{\mathbf{p}}})$ corresponds one-to-one to the set of $\mathbf{p}(\mathbf{q})$ satisfying $0 \le \mathbf{p}(\mathbf{q}) \le \hat{\mathbf{p}}$ and such that there exists at least one j, $1 \le j \le K$ such that $p_j(\mathbf{q}) = \hat{p}_j$. Unfortunately, one can show that the optimality conditions under individual power constraints can not be interpreted as the (anti-) parallelism of two vectors.

Therefore, let us define an *extended* power region $\mathcal{P}^*_{\hat{\mathbf{p}}} = \{\mathbf{p} : -\prod_{k=1}^K (\hat{p}_k - p_k) \le 0\}$, where obviously $\mathcal{P}_{\hat{\mathbf{p}}} \subset \mathcal{P}^*_{\hat{\mathbf{p}}}$. The regions $\{\boldsymbol{\Gamma}(\mathbf{p}) : \mathbf{p} \in \mathcal{P}^*_{\hat{\mathbf{p}}}\}$ and $\mathcal{Q}^*_{\hat{\mathbf{p}}} \stackrel{\Delta}{=} \{\mathbf{q}(\mathbf{p}) : \mathbf{p} \in \mathcal{P}^*_{\hat{\mathbf{p}}}\}$ arise automatically. For the modified problem of optimization over $\mathbf{bd}(\mathcal{P}^*_{\hat{\mathbf{p}}})$ one can show, that the KKT condition corresponding to zeroing of the derivative of the Lagrangean takes the form $\boldsymbol{\alpha} = \nu \nabla_q \prod_{k=1}^K (\hat{p}_k - p_k(n-1))$ and hence corresponds again to parallelism condition. This allows for the use of the algorithm mechanism for sum-power constraint also under individual power constraints.

The input parameters of the algorithm are $\boldsymbol{\alpha}$, $\mathbf{V}$, $\boldsymbol{\sigma}^2$, Ψ (with log-convex inverse), $\hat{\mathbf{p}}$ and starting vector $\mathbf{p}(0) \in \mathbf{bd}(\mathcal{P}_{\hat{\mathbf{p}}})$. The step-size is s. The iterations can be stated as follows (for notational simplicity we write $\mathbf{p}(n) = \mathbf{p}(\mathbf{q}(n))$).

While not $(\alpha \parallel \nabla_q \prod_{k=1}^K (\hat{p}_k - p_k(n-1)))$ do

1. given $\mathbf{p}(n-1)$ compute $\boldsymbol{\Gamma}(n)$ from (2)
2. $\mathbf{q}(n) = \boldsymbol{\Psi}(\boldsymbol{\Gamma}(n))$

3. $\mathbf{q}^*(n) = \mathbf{q}(n) + s\Big[-\boldsymbol{\alpha} + \frac{\boldsymbol{\alpha}\circ\nabla_q \prod_{k=1}^K(\hat{p}_k - p_k(n))}{\|\nabla_q \prod_{k=1}^K(\hat{p}_k - p_k(n))\|_2^2}\nabla_q \prod_{k=1}^K(\hat{p}_k - p_k(n))\Big]$
4. $\boldsymbol{\Gamma}^*(n) = \boldsymbol{\Phi}(\mathbf{q}^*(n))$
5. $\mathbf{p}^*(n) = (\mathbf{I} - \boldsymbol{\Gamma}^*(n)\mathbf{V})^{-1}\,\boldsymbol{\Gamma}^*(n)\boldsymbol{\sigma}^2$
6. $\mathbf{p}(n) = \mathbf{p}^*(n)\min_k \frac{\hat{p}_k}{p_k^*(n)}$
7. $n \to n+1$

In computing the components of $\nabla_{\mathbf{q}} \prod_k(\hat{p}_k - p_k)$ in step 3) we use the property $\frac{\partial}{\partial x_k}\mathbf{A}^{-1}(\mathbf{x}) = -\mathbf{A}^{-1}(\mathbf{x})\frac{\partial}{\partial x_k}\mathbf{A}(\mathbf{x})\mathbf{A}^{-1}(\mathbf{x})$. This yields for $1 \le j, k \le K$

$$\frac{\partial p_k(\mathbf{q})}{\partial q_j} = \frac{\partial}{\partial q_j}\mathbf{e}_k^T(\mathbf{I} - \boldsymbol{\Gamma}(\mathbf{q})\mathbf{V})^{-1}\boldsymbol{\Gamma}(\mathbf{q})\boldsymbol{\sigma}^2 = -\mathbf{e}_k^T(\boldsymbol{\Gamma}(\mathbf{q})^{-1} - \mathbf{V})^{-1}\frac{\partial}{\partial q_j}\boldsymbol{\Gamma}(\mathbf{q})^{-1}(\boldsymbol{\Gamma}(\mathbf{q})^{-1} - \mathbf{V})^{-1}\boldsymbol{\sigma}^2. \tag{15}$$

Using the chain rule for product derivation we yield for the gradient

$$\nabla_{\mathbf{q}}\prod_k(\hat{p}_k - p_k(\mathbf{q})) = \sum_{k=1}^K -\mathbf{e}_k^T(\boldsymbol{\Gamma}(\mathbf{q})^{-1} - \mathbf{V})^{-1}\frac{\partial}{\partial q_k}\boldsymbol{\Gamma}(\mathbf{q})^{-1}(\boldsymbol{\Gamma}(\mathbf{q})^{-1} - \mathbf{V})^{-1}\boldsymbol{\sigma}^2\prod_{\substack{j=1\\ j\neq k}}^K(\hat{p}_j - p_j(\mathbf{q})). \tag{16}$$

From the fact that $\boldsymbol{\Gamma}^{-1} = \mathrm{diag}(\frac{1}{\gamma_1}, \ldots, \frac{1}{\gamma_K})$ follows that the only one nonzero element of $\frac{\partial}{\partial q_j}\boldsymbol{\Gamma}(\mathbf{q})^{-1}$ is its jj-th element $\frac{\partial}{\partial q_j}\frac{1}{\gamma_j} = -\frac{1}{\gamma_j^2}\Phi'(q)|_{q=q_j}$.

Despite utilization of the extended power region the algorithm provides convergence to the optimum. This is due to step 6), where the power vector $\mathbf{p}^*(n)$ generated after the search step is projected onto $\mathbf{bd}(\mathcal{P}_{\hat{\mathbf{p}}})$. The convergence analysis analogous to the sum-power constrained case applies also to the above algorithm. The difference consists in the behaviour on subregions of $\mathcal{Q}_{\hat{\mathbf{p}}}$ corresponding to *edges* of the polyhedral region $\mathcal{P}_{\hat{\mathbf{p}}}$. For $K > 1$ edges of $\mathcal{P}_{\hat{\mathbf{p}}}$ are characterized by tightness of more than one power constraint, i.e. $p_k = \hat{p}_k$ for $k \in \mathcal{A} \subseteq \{1, 2, \ldots, K\}$, $Card(\mathcal{A}) > 1$. From (16) follows, that the gradient becomes zero on edges of $\mathcal{P}_{\hat{\mathbf{p}}}$. Consequently, if the minimizer of problem (7) pertains to any edge of $\mathcal{P}_{\hat{\mathbf{p}}}$, then the optimality requirement ($\alpha \parallel \nabla_q \prod_{k=1}^K(\hat{p}_k - p_k(\mathbf{q}))$ cannot be verified. However, a simple argument shows that this does not prevent the algorithm from finding the solution $\mathbf{p}_{\text{opt}} = \mathbf{p}(\mathbf{q}_{\text{opt}})$ with arbitrary accuracy ϵ. Assume that $\mathbf{p}(\mathbf{q}_{\text{opt}})$ pertains to any edge and that the required accuracy ϵ characterizes an ϵ-neighbourhood $O_\epsilon(\mathbf{q}_{\text{opt}})$. Then, we can always choose a sufficiently small stepsize $s(\epsilon)$ and there exists $N \in \mathbb{N}$, such that for all $n > N$ $\mathbf{q}(n) \in (O_\epsilon(\mathbf{q}_{\text{opt}}) \cap \mathbf{bd}(\mathcal{Q}_{\hat{\mathbf{p}}}))$. This means, that after entering $O_\epsilon(\mathbf{q}_{\text{opt}})$ in n-th iteration the exact solution can be possibly crossed arbitrary many times, but after every n-th step $\mathbf{q}(n)$ remains in $O_\epsilon(\mathbf{q}_{\text{opt}})$ due to the choice of appropriate $s(\epsilon)$. Hence, we conclude the analysis with the following statement.

Theorem 5 *Assume Φ to be log-convex and $\mathbf{q}_{opt} = \mathbf{q}(\mathbf{p}_{opt})$ to be the QoS vector achieved by the minimizer of (7). Then, for any accuracy ϵ and sufficiently small $s = s(\epsilon)$ the presented algorithm for networks with individual power constraints generates vectors $\mathbf{q}(n)$ remaining within the ϵ-neghbourhood $O_\epsilon(\mathbf{q}_{opt})$ for $n > N$, with some $N \in \mathbb{N}$.*

The convergence bahaviour is quantitatively similar to this of algorithm for sum-power constrained networks and can be improved in the same way as in the case of sum-power constrained networks.

7 Conclusions

In this work we analyzed the QoS optimization problem for elastic traffic utilizing a global multi-link QoS objective of weighted sum of QoS parameters. We stated the condition for convexity of the region of feasible QoS parameters, which requires the SINR-QoS dependence be log-convex. Under this conditions we proved that the studied optimization problem, although not convex, is efficiently solvable. Basing on the sufficiency of the KKT conditions we characterized the optimum in the form of a matrix equation system. Further, we showed that the problem can be restated in convex form, desirable in terms of efficiency of the iterative optimization methods. Finally, we introduced two algorithms conducting QoS optimization under sum- and individual power constraints. We proved convergence of the algorithms and provided an illustrative interpretation of the iterative search. The optimization of the convergence bahaviour of the algorithms, e.g. by the design of a suitable step-size control, was not in focus of this work and is a topic for separate consideration.

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