Performance Modeling of Threshold-Based Assembly Mechanism in Asynchronous Optical Packet-Switched Networks

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Abstract: The assembly delay and the optical packet utilization are two key performance measures of the optical packet assembly mechanisms in asynchronous optical packet-switched networks. The goal of this paper is to derive analytic equations of the assembly delay distribution and those of the optical packet utilization in the case of the short-range dependent (SRD) incoming traffic and the threshold-based assembly mechanism. Validating and analyzing them by simulations are conducted and the results show the theoretical traces and simulation results fit well.

Keywords: Assembly delay, Optical packet utilization, Threshold-based assembly mechanism, Asynchronous optical packet-switched networks.

1. INTRODUCTION

The ever-growing demands for transmission bandwidth, driven by the drastic increase of Internet users and the successive advent of diverse multimedia traffics in recent years, have been making the Internet backbone evolve quickly from an electronic network to an optical network [1, 2]. To establish the future intelligent and flexible optical network, huge works have been underway to bring the concept of packet switching into the optical domain. The asynchronous optical packet-switched (AOPS) network, deemed as a strong candidate for this optical network, has been investigated intensively because of its robustness, flexibility, ease...
and cheapness [3, 4]. Moreover, the bandwidth granularity of packet level in the AOPS networks is more suitable to support diverse TCP/IP-based applications and provide differentiated optical services. However, the outstanding shortcoming of AOPS networks is frequent contentions in the core nodes, which increase the packet loss probability (PLR) and deteriorate the network performance. So how to resolve contentions is always one of the key issues of interest.

As a promising solution to improve the network performance in terms of PLR, traffic shaping implemented by optical packet assembly mechanisms at the AOPS edge nodes is intensively investigated because it reduces the impact on the core node performance of the statistical features of the incoming traffic such as long-range dependence (LRD) and so on [5][6][7]. Concerns on optical packet assembly mechanisms are mainly the assembly delay and the payload utilization of optical packet [8][9]. Although most efforts are conducted by simulation, theoretical derivation of analytic models is another significant method to conduct performance evaluation of assembly mechanisms. Up to now, when the incoming IP traffic into the AOPS edges is only SRD or LRD, explicit analytic models based on the timer-based assembly mechanism have been obtained in [10] and [11], respectively. Those based on the fixed-time-min-length and max-time-min-max-length assembly mechanisms are done in [12]. Few efforts have been devoted to the threshold-based assembly mechanism, which is one of our objectives. Besides, validation and analysis of them are made in this paper.

2. PERFORMANCE ANALYSIS

2.1 Prerequisites

The AOPS network is composed of core nodes and edge nodes. The former mainly carry out the optical packet switching and routing. The latter, as legacy interfaces between IP electronic networks and AOPS core networks, perform functions of the MAC layer [13]. At the edge nodes assembly mechanisms are utilized to merge multiple IP packets together to generate one optical packet. Wherein the threshold-based assembly mechanism works as follows: when an IP packet arrives at its special assembly queue, an optical packet consisting of the existing IP packets in the assembly queue will be yielded if the length sum of the incoming IP packet and the filled capacity of the assembly queue exceeds the threshold $L_{\text{max}}$. Then this incoming IP packet will be delayed to the next optical packet payload. On the contrary, this incoming IP packet directly inserts the special assembly queue. The procedure continues back and forth.

Taking Poisson process $Y(t)$ to approximate the incoming SRD IP traffic, we calculate the probability of $k$ IP packets arriving in the interval $(0, t)$ as follows.

$$P\{Y(t) = k\} = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

where $\lambda$ denotes the average arrival rate of IP packets. Therefore, the probability density function (PDF) of the IP packets’ interarrival time $\text{IPT}$ is an exponential distribution with $E[\text{IPT}] = a$ and $1/a = \lambda$.

$$f(t) = \frac{1}{a} e^{-\frac{t}{a}}$$
We assume that the IP packet length $L$ conforms to an exponential distribution, too.

$$f(L) = \frac{1}{b} e^{-\frac{L}{b}}$$  \hspace{1cm} (3)

where $b$ is the expected value as well as the standard variance of $L$.

### 2.2 Assembly delay distribution

For the threshold-based assembly mechanism, the assembly delay $OPD$, defined as the time difference between the time of optical packet discarding the assembly queue and the time of the earliest IP packet in the optical packet arriving at its assembly queue, depends on not only the number of IP packets in one optical packet but also the interarrival time of IP packets. Thus we can obtain the distribution function of $OPD$ as follows.

$$F(t) = P(\text{OPD} \leq t)$$  \hspace{1cm} (4)

$$= \sum_{n=2}^{\infty} P\{n_i = n\} \cdot P\{\sum_{k=1}^{n-1} t_k \leq t | n_i = n\}$$

$$= \sum_{n=2}^{\infty} \left[ \frac{(L_{\text{max}})^n}{n!} \cdot e^{-\frac{L_{\text{max}}}{b}} \cdot P\left\{\sum_{k=1}^{n-1} t_k \leq t\right\} \right]$$

where $P\{n_i = n\}$ is the probability that $n$ IP packets exist in an assembly queue upon $L_{\text{max}}$ exceeding and $P\{\sum_{k=1}^{n-1} t_k \leq t | n_i = n\}$ is the probability that the sum of $n$ IPPT is less than and equal to $t$. According to the Poisson assumption, the latter meets Gamma distribution with parameters $(n-1, 1/a)$ [14].

$$P\left\{\sum_{k=1}^{n-1} t_k \leq t\right\} = \sum_{k=n-1}^{\infty} \frac{\left(\frac{t}{a}\right)^k}{k!} \cdot e^{-\frac{t}{a}}$$  \hspace{1cm} (5)

So we obtain the $OPD$ PDF by differentiating (4):

$$f(t) = \frac{1}{a} \cdot e^{-\frac{L_{\text{max}}}{b}} \cdot e^{-\frac{t}{a}} \cdot \sum_{n=2}^{\infty} \frac{\left(\frac{L_{\text{max}}}{b}\right)^{n-2}}{n!(n-2)!} \cdot \left(\frac{t}{a}\right)^n$$  \hspace{1cm} (6)

### 2.3 Optical packet utilization

The actual length of multiple IP packets contained in the optical packet by $L_{\text{max}}$, a constant for a concrete threshold-based assembly algorithm, defines the optical packet utilization. Therefore, the PDF of the optical packet utilization is equal to that of the optical packet length. Then, we shall analyze the latter for simplification.

1) $0 < l \leq L_{\text{max}}$
Let $A$ denote the event of an optical packet transmission and let $B_x$ indicate the case that the size of an assembly queue is $x$, $x \in (0, L_{\text{max}})$. So $P(A|B_x)$ denotes the transmission probability of an optical packet with the length of $x$. If an IP packet with the length of $l_{\text{next}}$ arrives at an assembly queue with the capacity of $x$, the condition of an optical packet generation is that $x + l_{\text{next}} \geq L_{\text{max}}$. We can obtain the conditional probability $P(A|B_x)$ as follows.

$$P(A|B_x) = P(l_{\text{next}} \geq L_{\text{max}} - x)$$

$$= e^{\frac{L_{\text{max}} - x}{b}}$$

(7)

Just as the distribution function of $\text{OPD}$ shown by equation (4), the probability that an assembly queue has a capacity of $x$ is:

$$P(B_x) = \sum_{n=1}^{\infty} P(n_i = n) \sum_{k=1}^{n} I_k = x$$

$$= 1 \cdot e^{\frac{-2x}{b}} \sum_{n=1}^{\infty} \frac{\left(\frac{x}{b}\right)^n \left(\frac{x}{b}\right)^{n-1}}{n!(n-1)!}$$

(8)

According to the full probability formula, the probability of an optical packet transmission is:

$$P(A) = \int_0^{L_{\text{max}}} P(A|B_x) \cdot P(B_x) dx$$

(9)

Finally, we can obtain the PDF of the optical packet length by Bayes formula.

$$f(x) = \frac{1}{b} \cdot e^{\frac{-L_{\text{max}}+x}{b}} \cdot I_1\left(\frac{2x}{b}\right)$$

$$\int_0^{L_{\text{max}}} \frac{1}{b} \cdot e^{\frac{-L_{\text{max}}+x}{b}} \cdot I_1\left(\frac{2x}{b}\right) dx$$

(11)

where $I_n(z)$ is the modified Bessel function of the first kind.

2) $l > L_{\text{max}}$

The ideal case, which is impossible in the actual system, is a complementary set of case 1). An optical packet only holds one IP packet larger than $L_{\text{max}}$. Namely, the length $x$ of optical packet equals the length $l$ of the IP packet. So we can approximately obtain the following equation.

$$f(x) \approx \frac{1}{b} e^{\frac{-x}{b}} \to 0$$

(12)

3. VALIDATION

In order to validate the aforementioned theoretical models, we conduct simulations. Therein,
we assign $\lambda$ to be 1000 packets/sec and $L_{\text{max}}$ to be 3000 bytes. The IP packet size $L$ and the interarrival time $IPT$ follow negative exponential distribution with $E[L] = 395$ bytes and $E[IPT] = 1\text{ms}$, respectively. Moreover, the maximal size of IP packets isn’t limited to look into the second case for the optical packet utilization. During simulations, the threshold value $L_{\text{max}}$ substitutes for $\infty$ and $\sum$ with the step 1 takes the place of $\int$. 

Fig. 1. The probability density function of assembly delay

Fig. 1 illustrates the assembly delay PDF, which evidently demonstrates that the simulation trace fits the theory trace well. Obviously, the simulation and theory traces can be divided into two parts: the ascending and the descending. Therefore, the maximal probability density exists, the abscissa of which is about 0.005 second in the simulation scenario. Moreover, the trace trend on the whole is like Gaussian curve, as is also in line with the simulation results in other published works [12]. Fig. 1(b), a close-up of the assembly delay PDF, shows the feeble difference between the simulation trace and the theory one, which results from finite sample data. Meanwhile, it shows more clearly that the simulation trace closely centers on the theory trace. These verify (5) better.

Fig. 2. The probability density function of optical packet utilization
Fig. 2 shows the simulation trace and theory one of the optical packet utilization PDF. Albeit the simulation trace fluctuates around the theory one when optical packet utilization is less than 1, both of them have the same trend as exponential curve. Once optical packet utilization overtakes 1, the probability density decays abruptly and approaches zero. This phenomenon, which can’t occur in the actual system, manifests the equation (12).

4. CONCLUSIONS

In this paper, we have established explicit analytic models of the assembly delay distribution and the packet utilization distribution on the base of the threshold-based assembly mechanism operating on the AOPS edge nodes in the case of the SRD incoming traffic. Their PDF traces take on the trend of the Gaussian curve and the exponential one on the whole, respectively. Moreover, the simulation results closely center on the theory values, which successfully validates our derivative equations.

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