Variability of Service Times and Throughput Efficiency Trade-Off in IEEE 802.11 DCF

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***Abstract*—First order metrics (throughput, average delay) of the IEEE 802.11 DCF MAC protocol have been extensively analyzed. In this work the probability distribution of the service time is derived and it is used to highlight the source of service time burstiness. It is proved that the variance of the service time is minimized by taking the same contention window value at each back off stage, under the constraint that the saturation throughput has a given value. A trade off between throughput efficiency and variance of the service times is computed and the binary exponential back off mechanism is found to be responsible of the bursty service process of IEEE 802.11 DCF. Implication of this result on the design and tuning of IEEE 802.11 DCF parameters is discussed.**

I. Introduction

Performance evaluation of a single-hop, Independent Basic Service Set (IBSS) IEEE 802.11 Distributed Coordination Function (DCF) has been largely focused on average, long-term metrics, like saturation throughput (e.g. see [1], [2], [3], [4], [5], [6], [7]), non saturated average throughput (e.g. [8], [9], [10]), delay analysis (e.g. [11], [12], [13], [14]), average throughput of long-lived TCP connections (e.g. [15], [16]) and short-lived TCP connections (e.g. [17]).

Analytical models, simulations and sometimes experimental test-beds have been used to analyze various extensions and improvement proposals on IEEE 802.11 DCF as well, e.g. exploitation of MIMO communications to enhance throughput of IEEE 802.11 DCF [18]; analysis of aggregation modes envisaged by IEEE 802.11n draft [19]; models for rate and/or power adjustment and control (e.g. see [20], [21], [22]); modification of backoff algorithms to smooth service burstiness to the benefit of VoIP traffic sources [23], [24]; adaptive TCP buffer sizing for IEEE 802.11e based WLANs [25].

The aim of this work is to set the problem of performance optimization of IEEE 802.11 DCF not only against first order metric (saturation throughput) but also second order ones (MAC frame service time variance). This new point of view sheds light on the impact of back off, specifically Binary Exponential Backoff (BEB), on traffic flow delay performance. It is shown that a trade off exists between robust throughput optimization and reduction of service time variability. It *is* actually possible to have jointly the best possible throughput and the least service time variance, but this working point is quite sensitive to knowledge of the number of active stations. Nevertheless, analysis points out that there is significant improvement margin in the design of CSMA/CA for IEEE 802.11 DCF, since it turns out that current parameter setting nearly attains the worst possible value of service time variance.

The rest of the paper is organized as follows. In Section II a review of previous contribution on the analysis of service time in IEEE 802.11 DCF is given. Section III outlines modeling assumptions and the embedded Markov chain analytical model is briefly recalled. Service times moment generating function and first two moments are derived in Section IV. Section V analyzes the trade-off between normalized saturation throughput and the variance of service times; numerical results are discussed as well. Conclusions are drawn in Section VI.

II. Related work

The service process of MAC MSDU is a point process defined by the sequence of times between two consecutive service completions. Service of a MAC frame is completed when it is successfully delivered to its destination or when it is discarded after the maximum number of transmission attempts has been reached, as envisaged by the IEEE 802.11 DCF standard [26]. A considerable amount of papers addresses the evaluation of service time metrics. In [28] the z-transform of the service time of a generic station in an isolated IBSS in saturation is derived and a short analysis of the impact of some parameters on the distribution is presented. A broader evaluation is carried out in [29], where tagged station is modeled as an M/G/1/K queue to evaluate MAC delay and throughput. In that context, the z-transform of the 802.11 DCF service time probability distribution is calculated under usual hypotheses, getting to the same result as [28], though with a discrete circuit analysis approach. Foh et alii [30] aim at defining a queuing model of the 802.11 DCF, by approximating the service time process with a phase-type probability distribution. To that end, they use elegant regenerative arguments to derive mean and variance of the service times; unfortunately, their model does not account properly for the BEB mechanism of 802.11 DCF, thus leading to the result that service time process is a quite smooth one. Exploitation of a G/G/1 queue model is pursued in [31] as well. Service time probability distribution z-transform is calculated here by allowing arbitrarily distributed MAC frame length, yet resulting in an excessively cumbersome expression. The contribution in [32] from same Authors yields very close results, in an abridged version. In [33] a Markov chain based model is defined to evaluate the probability distribution of the service times of a modified version of the IEEE 802.11 DCF, aimed at reducing service time jitter. The Authors show that their algorithmic modification is quite effective, however, their model is quite cumbersome, since it needs a state description of the number of stations in each back off stage.

A good contribution is given in [34]: Sakurai and Vu derive a (discrete time) model yielding the first two moments of the service times (called MAC access delay in their paper) and the whole generating function of the service time probability distribution. They prove the good accuracy of their model against simulations and use it to highlight the negative effect of BEB on service time variability. The derivation of the service time variance and probability distribution in this paper are closely related to theirs. Our work adds a small modeling detail, in that it accounts correctly for the statistical dependence of successive transmission attempts made by a tagged station for the *same* data frame. In [35] the service process is modeled from a system point of view (not a tagged station): guided by simulations, Authors propose a very simple Geometric distribution for the MAC frame inter-departure time. Yet another variant for the computation of the service time probability distribution generating function can be found in [36], with a collision probability value dependent on the back-off stage. The service time probability distribution in saturation and a study of its burstiness can be found in [42] as well. Back off counters are assumed to be geometrically distributed, so that service times can be studied by resorting to transient Markov chains. BEB is hinted at as a source of burstiness that could be at least partially mitigated by properly setting the value of the maximum contention window size.

With respect to previous work, the contribution of this paper is the identification of the optimum contention window size values and the ensuing investigation of the trade-off between throughput efficiency and service degradation in terms of delay jitter. This should hopefully clarify the trade-off behind design of key random access elements, like back off and BEB, and user perceived performance. As a matter of fact, variability of service times affects adversely queue performance of backlogged traffic inside stations (e.g. mean queue delays are proportional to the coefficient of variation of the service times). In view of support of real time and streaming services on WLANs, jitter of the service time is a problem as well. Moreover burstiness in the service process can degrade TCP performance due to ACK compression.

III. IEEE 802.11 DCF MODEL

We focus on an IBSS made up of n stations, possibly including an Access Point (AP), within full visibility of one another, so that carrier sense is fully functional. The model of 802.11 DCF is derived under the following assumptions:

- *Symmetry*: stations are statistically indistinguishable, i.e. traffic parameters (air bit rate, payload length probability distribution) and multiple access parameters (e.g. maximum retry limit) are the same.
- *Proximity*: every station is within the coverage area of all others, i.e. there are no hidden nodes.
- *Saturation*: stations always have packets to send.

Along with these we introduce a simplifying hypothesis:

- *Independence*: states of different stations are realization of independent random processes.

The independence hypothesis is essential to describe the system dynamics by using a low dimensionality Markov chain; its validity has been discussed from a theoretical viewpoint in [4][37] and is checked against simulations in our numerical results as well as in many other works, all showing that under traffic saturation assumption, independence based models work fine for first order metrics (e.g. throughput, mean delay) and even for the entire service time and back off counter probability distributions. A deep insight into the value and limits of independence assumption can be gained from [37][4].

Simulation results in this work are obtained by means of a matlab simulator that reproduces the 802.11 DCF system under the *Symmetry*, *Proximity* and *Saturation* hypotheses, but fully accounting for DCF procedures.

The procedures of IEEE 802.11 DCF are described in [26]. According to that version of CSMA/CA, time can be divided in slots from the point of view of a tagged station. The back off counter of the tagged station is decreased by one at the end of each slot, until it hits 0 and the channel is sensed idle for a DCF back off time slot[1]. Then, a transmission attempt starts. A slot is the time spanning from two consecutive back off counter decrements for a tagged station. Slot duration need not be constant obviously: it can be described as a random variable, whose distribution is found in Subsec. III-B.

The slot counting process is completely independent of the duration of the transmission attempt (either successful or not) thanks to the freezing of the back off counters and to the renewal of back off counters each time they are reset. Therefore, the description of the counting process is oblivious of details of transmission mode and data unit formats; on the contrary, evaluation of the throughput and of the service time probability distribution does depend on the actual transmission attempt duration, hence on the transmission mode/format and data frame length.

Back off process is sketched in subsection III-A, the statistical description of the transmission times, hereinafter referred to as *activity times* is introduced in subsection III-B; throughput results are summarized in subsection III-C.

A. Back-off process

According to IEEE 802.11 DCF CSMA/CA [26], a station with a backlogged packet attempts packet transmission up to a fixed number of times (maximum retry parameter), denoted as $m+1$ in the following. When m unsuccessful retransmissions have been experienced, after a first unsuccessful transmission, the packet is discarded. A station transmission attempt can

[1] It is here assumed that a station decrements the back off counter at the *beginning* of a time slot and it can start its transmission only at the *end* of the time slot with which its back off counter has reached zero. Accordingly, there is at least one back-off slot time after the IFS following each transmission attempt, either successful or not. This time slot is incorporated into the duration of the attempt, hence time slot count ranges from 0 to $W-1$, W being the current value of the contention window at the station.

start as soon as the back off counter hits 0: then the station starts an activity time, possibly colliding with other stations. As soon as the station ends up its activity time, it draws a new value of the back-off counter, uniformly distributed over a contention window $\{0, \ldots, W_i - 1\}$, whose size is $W_i = \min(CW_{min} 2^i, CW_{max})$ for $i = 0, \ldots, m$. The last values of this sequence need not be different, depending on the value of the MAC parameters CW_{min} and CW_{max}. In the following we only assume that the sequence of contention window values is non-decreasing with i for $i = 0, \ldots, m$.

Let us define consider the time points when the back off counter of the tagged station is decremented. The well known bi-dimensional Markov chain with state (I, J), with I = *back off stage* $\in \{0, 1, \ldots, m\}$ and J = *back off counter value* $\in \{0, 1, \ldots, W_I - 1\}$, can be defined over these time points as in [1], [2]. Let τ denote the probability of transmission of a station, i.e. the probability of the event 'Station starts transmission at the end of the current time slot', and let p be the conditional collision probability. From [1], [2] we have:

$$\tau = \frac{\sum_{k=0}^{m} p^k}{\sum_{k=0}^{m} p^k (W_k + 1)/2} \tag{1}$$

This result can be found without resorting to the embedded Markov chain, by means of an elegant regenerative argument (see [4] for details).

It is $p = 1 - (1 - \tau)^{n-1}$ thanks to the Independence assumption. This equality and (1) make up a non linear equation system. It can be shown that a sufficient condition to guarantee that a unique fixed point exists in the symmetric scenario here considered is that the sequence $W_i, i = 0, 1, \ldots, m$ be non decreasing (see Theorem 5.1 in [39]).

B. Activity times

The system time evolution from the MAC point of view is depicted in Fig. 1. Activity times come after some idle back off slots have elapsed. An activity time comprises a transmission attempt, either successful or ending up in a collision if more than one station attempts transmitting a MAC Protocol Data Unit (MPDU). After that, a new backoff+activity cycle follows (Saturation assumption).

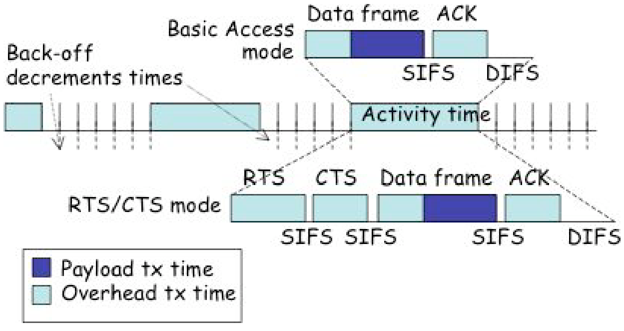

Fig. 1. Example of activity time with Basic Access mode and RTS/CTS mode; back off decrement times are highlighted as well.

In this section we focus on back off counter decrement times of a tagged station and on the time interval in between two generic successive back off decrements. This time interval can be modeled as sampled from a random variable, denoted as X. The interval X can reduce to a DCF back off time slot, of duration δ, or it can be $X = \delta + A$, where A is the random variable representing the duration of an activity time. In the following we assume the transmission channel is ideal, i.e. reception can be unsuccessful only in case of collision, i.e. (partially) overlapping transmissions. So, activity time can be either a successful reception or a collision; we add a subscript s or c to denote success or collision respectively.

Let Y denote the number of stations attempting transmission in a generic time slot; Y is a positive, integer-valued random variable, with $0 \le Y \le n$, n being the number of stations. Then, we have:

1) idle time, $X = \delta$, if $Y = 0$;
2) success, $X = \delta + A_s$, if $Y = 1$;
3) collision, $X = \delta + A_c$, if $Y \ge 2$.

The actual durations of success and collision times, namely A_s and A_c, depend on the access mode (Basic Access or RTS/CTS), on the utilized physical layer interface (b,a,g), on the length of the data frame payload. In general, A_s and A_c are the sum of two contributions:

1) a *constant* term accounting for the duration of any phase inside a transmission attempt, except of the time devoted to transmitting the data frame payload; we use $T_{oh,x}$ to denote such a time interval, with $x = s, c$ according to whether a success or a collision is referred to;
2) the random variable U accounting for the data frame payload time.

In the following we assume $T_{ackTO} = EIFS = SIFS + T_{ack} + DIFS$, which is a common setting for these parameters, where $ackTO$ denotes the MAC ACK time-out, EIFS is the extended IFS and T_{ack} is the transmission time of the MAC ACK frame. With this simplification, in case of *Basic Access (BA) mode* we get

$$\begin{cases} T_{oh,s} = T_{phy} + T_{MAChdr} + SIFS + T_{ack} + DIFS \\ T_{oh,c} = T_{phy} + T_{MAChdr} + T_{ackTO} = T_{oh,s} \\ A_s = T_{oh,s} + U_1 \\ A_c = T_{oh,c} + \max\{U_1, U_2, \ldots, U_Y\}, \qquad Y > 1 \end{cases} \tag{2}$$

where T_{phy} is the duration of the PLCP and physical layer preamble and header, T_{MAChdr} is the duration of the MAC layer header. In the rest of the paper, we set $T_{oh,s} = T_{oh,c} = T_{oh}$. The payload transmission time is $U = L/C$, with C being the air bit rate of the MAC interface and L the payload length; note that both terms can be described as random variables, to model variable packet length (L) and link adaptation mechanisms (C)[2].

[2] A substantial simplification of the probabilistic description of the overall system is obtained by the *Symmetry* assumption. Generalization are possible, yet notationally confusing and computationally quite hard; they are not needed here to get to the point, i.e. a quantify the trade-off between efficiency and service time variability.

The corresponding values with *RTS/CTS mode* are

$$\begin{cases} T_{oh,s} = T_{rts} + SIFS + T_{cts} + SIFS + T_{phy} + \\ \quad + T_{MAChdr} + SIFS + T_{ack} + DIFS \\ T_{oh,c} = T_{rts} + T_{ackTO} \\ A_s = T_{oh,s} + U_1 \\ A_c = T_{oh,c} \end{cases} \tag{3}$$

where T_{rts} and T_{cts} are the duration of the RTS and CTS frames respectively.

In the following we assume a general discrete distribution for U, with a finite number M of values, namely $a_1 \leq a_2 \leq \cdots \leq a_M$, with $\mathcal{P}(U = a_j) = q_j, j = 1, \ldots, M$. This a useful model, since only a finite number of rates are available in each standard specification of IEEE 802.11 physical layer and packet length probability distributions usually have most of their masses on a few values, all the other lengths having a negligible probability.

The variable A_s has the same probability distribution as U, except its values are shifted by a fixed amount. Only the probability distribution of A_c in case of BA mode needs some calculations. By the *Independence* assumption, we have that the number Y of stations attempting transmission simultaneously has a binomial probability distribution with parameter (n, τ). Then, we let $V_r = \max\{U_1, \ldots, U_r\}$ and it is easily verified that $\mathcal{P}(V_r \leq a_j) = Q_j^r$ for $j = 1, \ldots, M$, with $Q_j = \sum_{i=1}^{j} q_i$. We set $Q_0 = 0$ for ease of notation; so, $\mathcal{P}(V_r = a_j) = Q_j^r - Q_{j-1}^r$ for $j = 1, \ldots, M$.

In case of BA mode, according to eq. (2), X is a discrete random variable taking $M+1$ values, namely

$$\begin{cases} \mathcal{P}(X = \delta) = (1-\tau)^n \\ \mathcal{P}(X = \delta + T_{oh} + a_j) = \sum_{r=1}^{n} \mathcal{P}(V_Y = a_j | Y = r) \pi_{n,r}(\tau) \\ \quad = (1 - \tau + \tau Q_j)^n - (1 - \tau + \tau Q_{j-1})^n, \quad 1 \leq j \leq M \end{cases}$$

where $\pi_{n.r}(\tau) = [n!/[(n-r)!r!]\tau^r(1-\tau)^{n-r}$. In case of RTS/CTS mode, the random variable X can take $M+2$ values according to eq. (3), so:

$$\begin{cases} \mathcal{P}(X = \delta) = (1-\tau)^n \\ \mathcal{P}(X = \delta + T_{oh,s} + a_j) = n\tau(1-\tau)^{n-1} q_j \quad 1 \leq j \leq M \\ \mathcal{P}(X = \delta + T_{oh,c}) = 1 - (1-\tau)^n - n\tau(1-\tau)^{n-1} \end{cases}$$

Hence, the moments of X are easily derived.

C. Saturation throughput

The normalized, long-term average saturation throughput ρ can be found as the ratio of the probability of a successful slot and the mean slot duration:

$$\begin{aligned} \rho &= \frac{n\tau(1-\tau)^{n-1} \mathrm{E}[U_1]}{\mathrm{E}[X]} \\ &= \frac{n\tau(1-\tau)^{n-1} \mathrm{E}[U_1]}{\delta p_e + (\delta + \mathrm{E}[A_s]) p_s + (\delta + \mathrm{E}[A_c]) p_c} \\ &= \frac{\mathrm{E}[U_1] p_s}{\delta + T_{oh,s} p_s + \mathrm{E}[U_1] p_s + \mathrm{E}[A_c] p_c} \end{aligned} \tag{4}$$

TABLE I
IEEE 802.11 DCF PARAMETERS NUMERICAL VALUES

Parameter	Value	Parameter	Value
Backoff slot, δ	20 μs	RTS length	20 bytes
SIFS	10 μs	CTS length	14 bytes
DIFS	SIFS+2δ	ACK length	14 bytes
EIFS	364 μs	PHY overhead	192 μs
Basic bit rate	1 Mbps	MAC header	34 bytes
Air bit rate	11 Mbps	Payload length, L	1500 bytes
CW_{min}	31	max retry, m	7
CW_{max}	1023	ACK time-out	364 μs

with $p_e = (1-\tau)^n$, $p_s = n\tau(1-\tau)^{n-1}$ and $p_c = 1 - p_e - p_s$. For the RTS/CTS mode, $A_c = T_{oh,c}$ is a constant, so ρ has exactly the same value with variable or constant payload transmission time. This invariant property is essentially tied to RTS/CTS mode being similar to collision detection: MAC data frame is aborted early in case of collision detected on the preliminary RTS message. For the *Basic Access mode* (BA), A_c is a random variable, which takes value $T_{oh} + a_j$ with probability $[(1-\tau+\tau Q_j)^n - (1-\tau+\tau Q_{j-1})^n]/[1-(1-\tau)^n]$ for $j = 1, \ldots, M$.

In the special case of constant payload transmission time (same for all stations), the well known result is recovered:

$$\rho = \frac{n\tau(1-\tau)^{n-1} a_1}{\delta + (T_{oh} + a_1)\left[1 - (1-\tau)^n\right]}$$

The average throughput results for the *Basic Mode* are depicted in Fig. 2, by varying the number of competing stations n, for different values of CW_{max}. As far as regards other system parameters, we used standard IEEE 802.11b setting; the values used for DCF parameters are shown in Tab. I. It is apparent that throughput degradation resulting from $CW_{max} = 127$ or $CW_{max} = 255$ is almost negligible with respect to the standard case $CW_{max} = 1023$.

IV. 802.11 DCF SERVICE TIME CHARACTERIZATION

Let t_k denote the k-th back-off decrement time of the tagged station; it occurs either after an idle back off slot or after an activity time followed by a back off slot time. At each transmission attempt, either a frame is successfully delivered, or a collision occurs. In the former case, a frame has been served, i.e. we have a service completion epoch. In the collision case, frame delivery is attempted again after a back off time, except for those frames whose maximum number of attempts has been exhausted (max_retry parameter). For those frames service is complete as well, although ending up with a failure. An example of the service time structure, as a sequence of *attempt cycles* is depicted in Fig. 3.

Let $t_k^{(s)}$ be the service completion epochs (either with success or failure) as seen by a tagged station. The sequence $\{t_k^{(s)}\}$ is obtained by sampling the full sequence $\{t_k\}$. The k-th service time for the tagged station is denoted as Θ_k. Under *Saturation* assumption we have $\Theta_k = t_k^{(s)} - t_{k-1}^{(s)}$. At steady

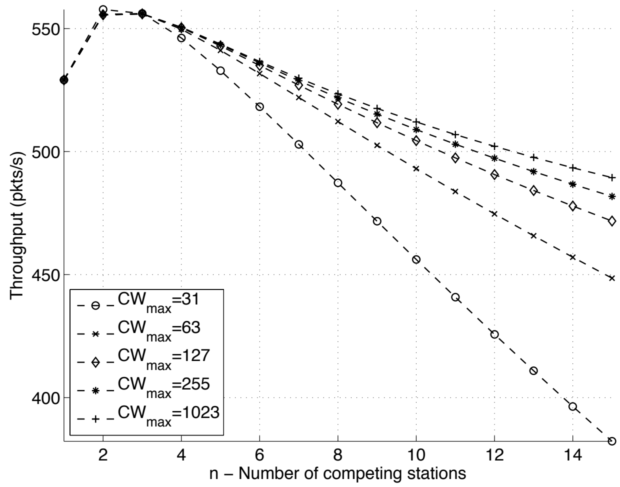

Fig. 2. Throughput varying the number of competing stations n for $m = 7$ and different values of CW_{max}.

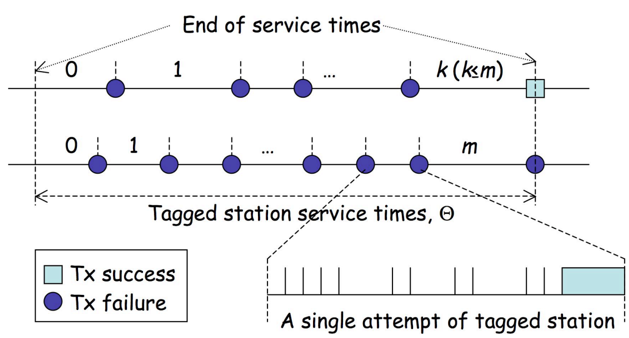

Fig. 3. Tagged station service time structure.

state, we have $\Theta_k \sim \Theta$, $\forall k$. In the following we derive the Laplace transform of the probability density function of Θ.

Unlike collision probability, MAC frame discard probability and other metrics related only to the back off counting process, service times *do depend* on the time spent in activity intervals, hence on MAC frame lengths and transmission formats. By referring to Fig. 3, a service time Θ is the sum of a number of attempts of the tagged station. Each attempt is made up of

- a number of slots, during which the tagged station stays idle and some other station(s) possibly attempt transmission (activity times conditional on the tagged station being idle);
- a final activity time involving the tagged station.

Let $\tilde{X}$ denote the random variable associated to the duration of such generalized slots where the tagged station stays idle and up to $n-1$ other station compete. It has the same probability distribution as X in Sec. III-B with $n-1$ stations.

We introduce the notation

$$Z_j \equiv \sum_{i=1}^{\nu_j} \tilde{X}_i^{(j)}, \qquad S_k(U) \equiv \sum_{j=0}^{k-1} \max\{U, V^{(j)}\} \tag{5}$$

for $j, k = 0, \dots, m$; ν_j is the number of back-off slots waited for at stage j, hence it is a uniformly distributed random variable over $[0, W_j - 1]$; $\tilde{X}_i^{(j)}$ is the duration of the i-th generalized slot during the j-th back-off stage; U is the random variable denoting the duration of the payload transmission of the frame under service in the tagged station; finally, the variables $V^{(j)}$ are the duration of the payload transmission of the stations colliding with the tagged one in the first unsuccessful $k-1$ attempt cycles, i.e. $V^{(j)} = \max\{U_1, \dots, U_{Y^{(j)}}\}$ given $1 \le Y^{(j)} \le n-1$. We are using the convention that $\sum_{j=0}^{-1} \equiv 0$.

In the BA case, the service time, conditional on the overall service requiring k attempt cycles, is

$$\begin{cases} \Theta(k) = (k+1)T_{oh} + U + S_k(U) + \sum_{j=0}^{k} Z_j & 0 \le k \le m \\ \Theta(m+1) = (m+1)T_{oh} + S_m(U) + \sum_{j=0}^{m} Z_j & k = m+1 \end{cases} \tag{6}$$

In the RTS/CTS case, we have

$$\begin{cases} \Theta(k) = kT_{oh,c} + T_{oh,s} + U + \sum_{j=0}^{k} Z_j & k = 0, \dots, m \\ \Theta(m+1) = (m+1)T_{oh,c} + \sum_{j=0}^{m} Z_j & k = m+1 \end{cases} \tag{7}$$

The Laplace transform $f_\Theta(s) \equiv \mathrm{E}[e^{-s\Theta}]$ of the service time pdf in the RTS/CTS case is

$$\begin{aligned} f_\Theta(s) &= \sum_{k=0}^{m} (1-p)p^k \mathrm{E}[e^{-s\Theta(k)}] + p^{m+1}\mathrm{E}[e^{-s\Theta(m+1)}] \\ &= \sum_{\ell=1}^{M} q_\ell e^{-sa_\ell} \sum_{k=0}^{m} (1-p)p^k e^{-s(T_{oh,s}+kT_{oh,c})}\varphi_k(s) \\ &\quad + p^{m+1} e^{-s(m+1)T_{oh,c}} \varphi_m(s) \end{aligned} \tag{8}$$

with

$$\varphi_k(s) = \prod_{i=0}^{k} \sum_{r=0}^{W_i-1} \frac{1}{W_i} \mathrm{E}[e^{-s\tilde{X}}]^r = \prod_{i=0}^{k} \frac{1-\kappa(s)^{W_i}}{W_i\,[1-\kappa(s)]} \tag{9}$$

where $\kappa(s) \equiv \mathrm{E}[e^{-s\tilde{X}}]$ can be found as in subsection III-B, by replacing n with $n-1$.

In case of BA mode, a very similar derivation leads to:

$$\begin{aligned} f_\Theta(s) &= \sum_{k=0}^{m} (1-p)p^k e^{-s(k+1)T_{oh}} \sum_{\ell=1}^{M} q_\ell e^{-sa_\ell} \varphi_k(s) g_\ell^k(s) \\ &\quad + p^{m+1} e^{-s(m+1)T_{oh}} \varphi_m(s) g_\ell^{m+1}(s) \end{aligned} \tag{10}$$

with $g_\ell(s) = \sum_{j=1}^{M} e^{-s\max\{a_\ell, a_j\}}(\psi_j - \psi_{j-1})$ and $\psi_j = (1-\tau+\tau Q_j)^{n-1}/(1-(1-\tau)^{n-1})$ for $j = 1, \dots, M$.

Inversion of the Laplace transform can be accomplished as described in [38]. Moments can be found by derivation, or they can be computed directly. As a matter of example, the mean service time is tied to the saturation throughput of the tagged station, i.e. $\mathrm{E}[\Theta] = (1-p^{m+1})\mathrm{E}[U_1]/\rho_1$, with $\rho_1 = \rho/n$ and ρ given by eq. (4). We let $Var(A) = \sigma_A^2$ denote the variance of the random variable A. Then, $Var(\Theta) = \mathrm{E}[\Theta^2] - \mathrm{E}[\Theta]^2$, so it suffices to calculate the second moment of Θ, i.e.

$$\mathrm{E}[\Theta^2] = v + \sum_{k=0}^{m} (1-p)p^k \mathrm{E}[\Theta(k)]^2 + p^{m+1}\mathrm{E}[\Theta(m+1)]^2 + \sigma_{\tilde{X}}^2 \sum_{k=0}^{m} p^k \frac{W_k - 1}{2} + \mathrm{E}[\tilde{X}]^2 \sum_{k=0}^{m} p^k \frac{W_k^2 - 1}{6} \tag{11}$$

with $v = (1-p^{m+1})\sigma_U^2$ in case of RTS/CTS mode and $v = \sum_{k=0}^{m}(1-p)p^k Var(U + S_k(U)) + p^{m+1}Var(S_m(U))$ in case of BA mode. Calculation of these variances is straightforward, given the Independence assumption. Also we have:

$$\begin{aligned} \mathrm{E}[\Theta(k)] &= k(T_{oh,c} + a) + T_{oh,s} + \mathrm{E}[U] + \mathrm{E}[\tilde{X}] \sum_{j=0}^{k} \frac{W_j - 1}{2} \\ \mathrm{E}[\Theta(m+1)] &= (m+1)(T_{oh,c} + a) + \mathrm{E}[\tilde{X}] \sum_{j=0}^{m} \frac{W_j - 1}{2} \end{aligned} \tag{12}$$

where $k = 0, \ldots, m$ and $a = \mathrm{E}[\max\{U, V\}]$ in case of BA mode, $a = 0$ in case of RTS/CTS mode. The random variable V is defined as $V = \max\{U_1, \ldots, U_Y\}$ given $1 \le Y \le n-1$.

V. Minimization of service time variance and trade-off with throughput

We can pose an inverse problem, i.e. to find the optimum contention window sizes so as to minimize the variance of the service time under the constraint of a given value for the normalized saturation station throughput ρ.

The value of ρ as a function of τ has the bell shape typical of CSMA-like protocols. There is an optimal value τ^* that yields the maximum throughput $\rho(\tau^*) \equiv \rho^*$ and the function $\rho(\tau)$ is strictly increasing for $\tau \in [0, \tau^*]$ from 0 up to ρ^* and strictly decreasing for $\tau \in [\tau^*, 1]$ from ρ^* down to 0 asymptotically as $\tau \to \infty$. In the following we consider the two intervals separately, so that we can deal with $\rho(\tau)$ as uniquely invertible under the restriction $\tau \in [0, \tau^*]$ (or $\tau \in [\tau^*, 1]$).

Requiring that ρ takes a given value yields a given value of τ, hence also p and $\mathrm{E}[\Theta]$ are fixed; therefore minimizing σ_Θ^2 is the same as minimizing $\mathrm{E}[\Theta^2]$. We will refer to variance minimization and use eq. (11) in the following.

Since τ and p are given, eq. (1) implies that

$$\sum_{k=0}^{m} p^k \frac{W_k - 1}{2} = \frac{1-\tau}{\tau} \frac{1-p^{m+1}}{1-p} \tag{13}$$

Then, the optimization problem can be stated as: choose contention window sizes $W_k, k = 0, \ldots, m$, so as to minimize σ_Θ^2 under constraint (13). It is easy ti check from eqs. (11) and (12) that σ_Θ^2 is a quadratic form in the variables W_k. By accounting for the constraint (13), minimizing σ_Θ^2 is equivalent to minimizing the function

$$f(\mathbf{W}) \equiv const + \sum_{k=0}^{m} (1-p)p^k \mathrm{E}[\Theta(k)]^2 + p^{m+1}\mathrm{E}[\Theta(m+1)]^2 + \mathrm{E}[\tilde{X}]^2 \sum_{k=0}^{m} p^k \frac{W_k^2 - 1}{6} \tag{14}$$

where *const* does not depend on the contention window sizes, i.e. the variable vector $\mathbf{W} \equiv [W_0, W_1, \ldots, W_m]$.

We introduce the further constraint that contention windows must form a non-decreasing sequence: $W_{k+1} \ge W_k, k = 0, \ldots, m-1$, motivated by Theorem 5.1 of [39] and by the argument in [5], where it is shown that non decreasing W_k's make saturation throughput value insensitive to the specific back off probability distribution. Then, we can prove that the minimum of σ_Θ^2 is achieved for equally sized contention windows, whose common value is set by means of the constraint (13). This is a hint that binary exponential back off can be harmful, i.e. for the same average long-term trhoughput it inflates unduly the service time variation.

It is easy to see that $W_0 \le W_1 \le \cdots \le W_m$ implies $\mathrm{E}[\Theta(k)] \ge k(T_{oh,c} + a) + T_{oh,s} + \mathrm{E}[U] + (k+1)\mathrm{E}[\tilde{X}](W_0 - 1)/2, k = 0, \ldots, m$ and analogously for $k = m+1$. To simplify notation, we let $x_k \equiv (W_k - 1)/2, k = 0, \ldots, m$. Then, with a slight abuse of notation, we get

$$\begin{aligned} f(\mathbf{x}) &\ge \sum_{k=0}^{m} (1-p)p^k [kb(x_0) + T_{oh,s} + \mathrm{E}[U] + \mathrm{E}[\tilde{X}]x_0]^2 + \\ &+ p^{m+1}(m+1)^2 b(x_0)^2 + \frac{2}{3}\mathrm{E}[\tilde{X}]^2 x_0(x_0+1) \sum_{k=0}^{m} p^k + const \\ &\equiv \beta_0 + \beta_1 x_0 + \beta_2 x_0^2 \end{aligned}$$

with $b(x_0) = T_{oh,c} + a + \mathrm{E}[\tilde{X}]x_0$. This lower bound can actually be achieved by choosing all contention windows equal to a same value, which is then set by the constraint (13), i.e. it must be $W_k = 2/\tau - 1, k = 0, \ldots, m$, with τ given according to the chosen value of ρ.

The argument above shows that *the service time variance is minimized by setting all contention windows to a same value, under the joint constraint that the saturation throughput be assigned a given value and that the contention window sizes form a non decreasing sequence*. This result sheds a new light on the binary exponential back-off mechanism, pointing out that it hinders the variability of the service times, while gaining nothing from the throughput side, *provided one can set the contention window value correctly*.

To exploit this result, we should set the contention windows all equal to a same value, $2/\tau - 1$, where τ is set according to the desired throughput. The critical point lies here: the smaller we try to maintain the variance of the service time, the more sensitive the throughput becomes to variation of τ. The optimal value of τ, i.e. the value τ^* maximizing the saturation throughput, can be derived by the expression of the normalized saturation throughput ρ. A good approximation is $\tau^* \approx \gamma n^{-\alpha}$, with $\alpha \approx 1$ for typical parameter values of DCF. The optimal value of the contention window, both maximizing saturation throughput and minimizing service time variance, is therefore $W^* = 2/\tau^* - 1 \approx (2/\gamma)n - 1$, i.e. it grows (almost) proportionally with n.

Many algorithms have been devised to estimate n and hence adapt the random access parameters of CSMA/CA (e.g. see [40], [41]), showing that accurate and fast estimation of n, i.e. of the number of stations actually contending for the medium, is possible in infrastructured WLANs.

Given a value of ρ, hence of τ and p, we have shown that the optimum values for the contention window sizes that minimize service time variance under constraint (13), are $W_k = 2/\tau - 1, k = 0, \ldots, m$. If instead we use the standard values of the contention windows, denoted with W_k^0 in the following, then we can still assign the normalized throughout a desired value ρ by scaling W_k^0 so as to match eq. (13). The scale factor ζ must be set to $\zeta = (2/\tau - 1)\sum_{k=0}^m p^k / \sum_{k=0}^m p^k W_k^0$.

The first approach guarantees contention windows are greater than 1. The latter does not. Also, resulting contention window values in both approaches can be non integer. This can be dealt with by assuming that each time a new value of the back off counter is drawn which must be uniformly distributed over $[0, W-1]$ with $\lceil W \rceil \neq \lfloor W \rfloor$, first the contention window size is rounded randomly, i.e. it is set to $\lfloor W \rfloor$ with probability $(\lceil W \rceil - W)/(\lceil W \rceil - \lfloor W \rfloor)$, otherwise it is set to $\lceil W \rceil$.

Next we evaluate the trade-off between σ_Θ and ρ. Default numerical values of parameters are given in Table I; they are consistent with IEEE 802.11b standard using DSSS PHY. Figure 4 depicts the trade-off between the coefficient of variation of the service times ($C_\Theta \equiv \sigma_\Theta/\mathrm{E}[\Theta]$) and the normalized throughput ρ. This is computed for $n = 10$ stations and MAC data frame payload length $L = 1000$ bytes.

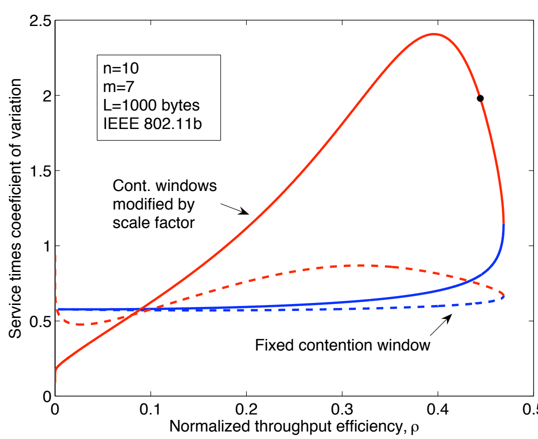

Fig. 4. Coefficient of variation of the service time vs. normalized throughput efficiency ρ for $n = 10$ stations. Blue curves: $\tau \leq \tau^*$; red curves: $\tau \geq \tau^*$.

Two cases are considered: i) contention windows are scaled by a factor α as given above; ii) contention windows are all set equal to a same value. These arrangements are made to satisfy eq. (13) for the value of τ corresponding to the abscissa ρ. For each given ρ there are actually two values of τ, one smaller and one larger of the value τ^*. The blue part of the curves is obtained by taking the smaller τ, i.e. by inverting $\rho(\tau)$ for $\tau \in [0, \tau^*]$, whereas the red part is obtained by inverting $\rho(\tau)$ for $\tau \in [\tau^*, 1]$. With the considered numerical values of the parameters, it is $\tau^* = 0.0172$ and $\rho^* = 0.4686$. The asterisk marked in the graph is the trade-off values corresponding to the standard values of the contention windows (in other words, $\zeta = 1$). For the standard values, $\tau = 0.0373$ and $\rho = 0.4443$.

The interesting point is that the binary exponential back off (solid curve) entails a less favourable trade-off than the constant contention window setting (dashed curve). Specifically, the working point of the standard 802.11 DCF achieves close to optimum throughput, though at the price of a rather high variability of service times. This can be substantailly reduced by setting the system to work at values of τ slighlty less than τ^*, so that a small throughput penalty is paid with respect to the optimum value, yet the coefficient of variation of the service times is kept around 1. A much nicer trade-off is obtained by simply removing the exponentail back-off, where it is seen that service time variability is not so sensitive as the value of ρ, hence of τ, is varied. However, a significant throughput loss with respect to optimum can be easily incurred in this case if a value of τ different from τ^* is chosen.

Figure 5 shows the same trade-off as Fig. 4, except $n = 20$. It can be seen that the scaled congestion window curve exhibits an even worse trade-off than it was in the previous case. Also, the working point of the IEEE 802.11b is the worst possible as for the service times variability. Many other numerical results (not shown for the sake of space) including parameter setting as in IEEE 802.11g, show similar values of trade-off and hence point out at same conclusions.

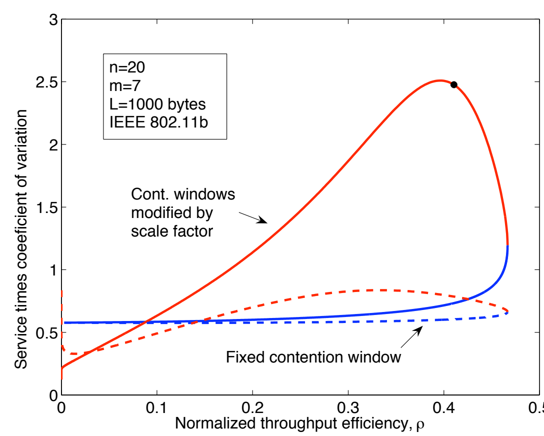

Fig. 5. Coefficient of variation of the service time vs. normalized throughput efficiency ρ for $n = 20$ stations. Blue curves: $\tau \leq \tau^*$; red curves: $\tau \geq \tau^*$.

VI. Final Remarks

The by now classic embedded Markov chain model of a IEEE 802.11 single cell is exploited in this work to evaluate the Laplace transform of the service time probability distribution. The well known throughput and service time results are generalized to the case of generally variable payload transmission times, which is important to included link adaptation mechanism provided by the physical interfaces of 802.11b/g. This is done both for RTS/CTS and Basic Access modes.

The key contribution of this work is to use the analytic expression of the service time variance to show that it can be minimized *without loss of throughput*, by choosing a constant contention window size for all re-transmission stages of DCF.

This is a major hint that BEB affects service time variability adversely: numerical results highlight that there is margin for improvement with respect to IEEE 802.11 DCF standard setting of contention window sizes.

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