Modeling MNO and MVNO's Dynamic Interconnection Relations: Is Cooperative Content Investment Profitable for Both Providers?

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***Abstract*—We consider a Mobile Network Operator (MNO) who shares dynamically his limited resource spectrum with a Virtual Network Operator (MVNO) lacking the infrastructure. We start by introducing at each time period a three-level game: in the first step the MNO defines the wholesale access charge that the MVNO pays per traffic unit sent on his network and allocates his scarce resource between his own consumers and the MVNO; in a second step, both operators compete on their retail prices, the MNO discriminating between the market segments while the MVNO invests in contents to target niche markets or add value to her company; finally the consumers choose one of the providers' offers or none depending on their intrinsic preferences and of the opportunity cost values. The game admits a unique equilibrium. In a second part, a regulatory authority forces both providers to use cooperative content investment i.e., the MNO now shares the content investment costs with the MVNO; in exchange this latter shares her revenue with the MNO. The equilibrium is still uniquely defined at each time period. Besides we check numerically that it can increase both operators' revenues, incite the MVNO to invest more in contents and decrease her retail price; while the MNO should decrease his access charge.**

I. Introduction

Despite the absence of common definition, Mobile Virtual Network operators (MVNOs) are characterized as being mobile operators without their own infrastructure and government issued licenses. MVNOs buy spectrum and possibly also infrastructure from primary providers, referred to as Mobile Network operators (MNOs). A notable example of successful MVNO endeavor in the US is Virgin Mobile which has teamed with Sprint Nextel as its MNO and recently reached a subscriber basis of 4 million customers (cf Mutlu et al. [3]). However, up to date, MVNOs are not really powerfull in the retail market. Most contracts linking MNOs to MVNOs are too restrictive and prevent MVNOs from increasing their market shares on some foreign markets. The Arcep i.e., the regulatory authority in France, asked for more flexible contracts and a reduction of the length of commitment which prevents MVNOs from leaving their suppliers before five years and forbids simultaneous contract agreements between a MVNO and two or more MNOs. In the real world, voluntary relationships have developed between MNOs and MVNOs [8]. It signifies that profit pursuing MNOs have an economic interest in partnering with MVNOs, particularly those that have high brand reputations or can develop highly valued innovative services. The spontaneous emergence of these voluntary relationships is puzzling at first glance because, after all, MVNOs siphon off some sales from their host MNOs. However, Banerjee and Dippon [8] explain such behavior by the fact that MVNOs add value (principally through their brand reputations) in ways that mere resellers cannot, and therefore help to boost the profitability of MNOs under certain demand and sales conditions. But, their model incorporates neither the spectrum allocation problem which influences the consumer perception of the quality of service (QoS) nor the difficulty that the MVNOs encounter to define their optimal advertising / content investment levels. This latter point is fundamental; indeed many MVNOs have no prior connexion with the telecommunication industry. The latter type of MVNO may be for example, an airline, a major retailer, a sporting goods firm, a broadcasting or entertainment company, or a seller of popular beverages. The main idea is that such an entity attempts to leverage its popularity and brand appeal with certain segments of the population to cross-sell mobile telecommunication services. For instance, M6 Mobile a MVNO owned by the french TV channel M6, uses the Orange network and proposes to broadcast soccer games on her consumer mobile phones while Fnac Mobile gives advices about the latest expositions and live concerts. These MVNOs use the popularity that they have acquired in their former activities (via the TV channel for M6 or via her media shopping centers for Fnac) to seduce mobile phone users. Some other kinds of MVNOs might start by targeting specific niche markets. As an example, Sud Mobile offers are targeted at north african people living abroad. The aim of this article is to study whether the MNO has incentives to share the MVNO content investments. For instance, Orange might share the investments in soccer game broadcastings with M6 Mobile since it could increase their revenues due to the higher quality and diversity of the contents.

Our contribution is to model the MNO-MVNO relationship, the resource (spectrum) being scarce and considering retail price competition with consumer differentiation. Besides, we study the influence of the resource allocation strategy on both providers' revenue evolution. We check whether or not the MNO has incentives to price-discriminate between his market segments and which marketing strategy between foresighted, myopic or neutral is the most suitable for both operators under competition. The article can be seen as an extension of [9] where a MNO and a MVNO having fixed market shares, compete for the allocation of a scarce resource. Indeed, in the present article, the game is played dynamically; furthermore both providers compete on the consumers who have the ability to churn or to refuse all the offers.

In Sethi et al. model [2], a contract based on cooperative advertising is *optimal*, in the sense that the manufacturer and the retailer's profits are higher with such a contract than without, if the manufacturer is more powerful than the retailer, and it is not in the reverse case. Besides cooperative advertising is always optimal for a linear demand function, while it is optimal for iso-elasticity functions with high elasticity only. Coordinating the supply-chain improves the welfare of both actors since it forces the retailer to invest more in advertising and to lower his prices. To perform the system's coordination Sethi et al. [2] advise to use a combination of revenue-sharing and cooperative advertising. We extend Sethi et al. article [2] to a supply-chain made of a MNO (manufacturer), a MVNO (retailer) and consumers. The supply-chain is coordinated by a regulatory authority (unbiased decision-maker). Our approach differs from [2] since the MNO now has access to the consumers and shares his network between his own consumer traffic and the MVNO traffic. This complicates the analysis due to the inter-relations. However like in [2] and [9] the MVNO can invest in contents to compensate for her QoS degradation. We want to know if it is possible to regulate MNO-MVNO relationships using cooperative content investment and how it influences both operators' revenue evolution and pricing strategies as functions of their power relations.

Our article is organized as follows. In Section II we introduce the consumer model and define the QoS measure; in Section III we study the dynamic three-level game which occurs at each time period, between the MNO, the MVNO and the consumers since these latter have the opportunity at each time period to churn from one provider to the other or to not subscribe to any offer at all; finally in Section IV we introduce a regulatory authority who aims at coordinating the supply-chain made of the MNO, the MVNO and the consumers by forcing both providers to use cooperative content investment.

In all our article, for the sake of simplicity, the MNO will be associated with a *male* player, while the MVNO will be *female*[1].

II. THE CONSUMER MODEL

Two operators are in competition to provide access to the consumers: a MNO who owns a network and a MVNO who buys access to the MNO network. Their offers are distinct but substitutable i.e., a consumer can use one or the other to perform the same service. Due to the substitutability assumption, each consumer buys *at most* one offer.

A. Definition of the consumer process

We consider two market segments but our model can be extended without difficulties to more than two. We let $\mathcal{N}$ be the fixed total number of consumers on the market: there are $\mathcal{N}_1$ on the segment 1 and $\mathcal{N}_2$ on the segment 2. At each time period t, $\mathcal{N}_{n1}(t)$ (resp. $\mathcal{N}_{n2}(t)$) is the number of consumers who subscribe to the MNO offer and belong to the market segment 1 (resp. to the market segment 2); while $\mathcal{N}_{v1}(t)$ (resp. $\mathcal{N}_{v2}(t)$) is the number of the MVNO consumers on the segment 1 (resp. on the segment 2).

The consumer takes his decisions independently of the others' choices at the same instant; however it depends on the perceived quality of service (QoS) i.e., on all the consumers' choices at the previous time period. On the market segment k $(k = 1, 2)$, the consumer process $X^{(k)}(t)$ can be in one of the three states: $\{n; v; o\}$. It takes the value n if the consumer on the segment k becomes MNO's consumer; v if he chooses the MVNO and o if he prefers to not subscribe to any offer.

The segment k consumer transition diagram is depicted in Figure 1. $\pi^{(k)}_{nn}(t)$ (resp. $\pi^{(k)}_{nv}(t)$ and $\pi^{(k)}_{no}(t)$) is the probability that conditionally to both operators' actions, the MNO consumer on the market segment k stays with the MNO at time period t (resp. churns to the MVNO and chooses to not subscribe to any offer). Similarly, for the MVNO $\pi^{(k)}_{vv}(t)$ (resp. $\pi^{(k)}_{vn}(t)$ and $\pi^{(k)}_{vo}(t)$) is the probability that conditionally to both operators' actions, the MVNO consumer on the market segment k stays with the MVNO at time period t (resp. churns to the MNO and chooses to not subscribe to any offer). And $\pi^{(k)}_{oo}(t)$ (resp. $\pi^{(k)}_{on}(t)$ and $\pi^{(k)}_{ov}(t)$) is the probability that conditionally to both operators' actions, the consumer on the market segment k who did not choose any offer at t does not change his mind at $t+1$ (resp. subscribes to the MNO and to the MVNO offer).

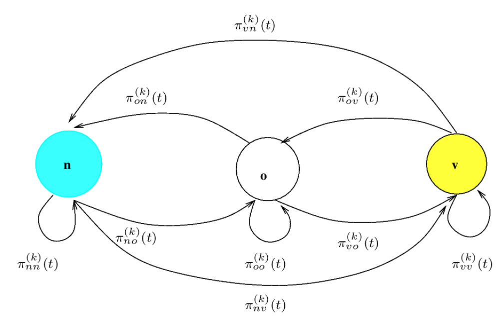

Fig. 1. Description of the states and transition probabilities for market segment k consumers.

B. Rate as a performance measure

At time period t, the MNO allocates his limited bandwith (spectrum) μ between his consumer segments and the MVNO's traffic according to the following sharing rule:

[1] This is a common assumption in Game theory.

- the MNO's consumers belonging to the market segment 1 receive $\xi_{n1}(t)\mu$;
- the MNO's consumers belonging to the market segment 2 receive $\xi_{n2}(t)\mu$;
- finally, the MVNO is allowed to send her traffic on a bandwith of $\xi_v(t)\mu$.

The sharing parameters satisfy the positivity and normality constraints: $\xi_{n1}(t),\ \xi_{n2}(t),\ \xi_v(t) \geq 0$ and $\xi_{n1}(t) + \xi_{n2}(t) + \xi_v(t) = 1$. The sharing mechanism is designed by the MNO (or the regulator) to maximize his profit under competition with the MVNO.

We assume that our measure of the perceived QoS is the *rate* i.e., the average bandwidth (time slots) per user. More precisely, the MNO capacity μ, can be associated with the total number of time-slots. At time period t, the MNO consumers belonging to the segment k can transmit their traffic on at most $\xi_{nk}(t)\mu$ time-slots while the MVNO can use $\xi_v(t)\mu$ time-slots. Each consumer has the opportunity to transmit his traffic on different time-slots *simultaneously*. The rate $\text{QoS}_k(t)$ then represents the number of time-slots that the segment k consumers can use to transmit their traffics and $\text{QoS}_v(t)$ contains the number of time-slots that each MVNO consumer can use. Under these assumptions, the QoS perceived by the MNO's consumers belonging to the market segment k $(k = 1, 2)$ or the maximal number of time-slots that they can use to transmit their traffic is $\text{QoS}_k(t) = \frac{\xi_{nk}(t)\mu}{\mathcal{N}_{nk}(t-1)}$. The MVNO do not have access to any private information about her consumers since she has no access to the MNO's home location registering stations (HLR) containing huge databases about consumers. Hence she cannot distinguish between her consumer segments; consequentely her perceived QoS once the MNO has re-allocated his bandwith or the maximal number of time-slots that they can use simultaneously for their traffic, is $\text{QoS}_v(t) = \frac{\xi_v(t)\mu}{\mathcal{N}_{v1}(t-1)+\mathcal{N}_{v2}(t-1)}$.

C. Definition of the consumers' opportunity costs

We assume that the MNO has determined his retail prices: $p_{n1}(t)$ for the segment 1 and $p_{n2}(t)$ for the segment 2; while $p_v(t)$ is the retail price fixed by the MVNO at time period t. To compensate for her QoS degradation and give the consumers some incentives to subscribe to her offer, the MVNO has the opportunity to invest a certain level $\theta(t)$ in contents (we can think for example, about soccer games, trendy exposition advices, cartoons, etc.).

The consumers valuate the *opportunity cost* [12] of consuming a unit of traffic (traffic rate, possibly content investment level and price) associated with each operator's access service. We assume that each consumer defines *a maximum admissible opportunity cost*[2]. The need to introduce a maximum admissible opportunity cost results from the following observation: a consumer will refuse to buy the offer or delay the purchasing process either if the offer retail price is too high, or if the QoS is not good enough, or if the content is not sufficiently diversified and rich compared to what he expects.

[2]Bernstein and Federgruen [4] introduce the idea of service level or fill rate defined as the fraction of demand that can be met from the existing inventory. Here we associate the inventory with the MNO's network; besides services can be seriously altered if the network is congested. Hence the maximum admissible opportunity cost extends the service level concept to the telecommunication settings.

We define the market segment k by assuming that the consumer maximum admissible opportunity cost is distributed according to a uniform density on the interval $[0; f_k]$ $(f_k > 0)$. The uniform density on the interval $[0; f_k]$ will be denoted $\mathcal{U}_{[0;f_k]} = \frac{1}{f_k}\mathbf{1}_{[0;f_k]}$. The use of a uniform density results from the fact that the operators have no prior information about the consumers' individual preferences. Since our model is dynamic, the consumer choice at time period t depends on his decision at the previous time step i.e., to which offer he has subscribed at time period $t-1$. Besides we introduce the parameters $\alpha_k \leq 0$ and $\beta_k \leq 0$ characterizing the segment k consumer sensitivity to the traffic rate and to the content investment level, respectively. These parameters are homogeneous on the market segment k.

(i) **Case of the consumers who were MNO clients at time t-1**

At each time period t, each MNO's consumer on every market segment k, has to choose his provider: he can either *stay* with the MNO, *churn* to the MVNO or choose to *not subscribe to any contract* if all his opportunity costs are higher than his maximal admissible opportunity level [11]. To identify properly the most suitable provider using the available information, the segment k MNO consumers compute the opportunity costs associated to the MNO and to the MVNO: $c_{nn}^{(k)}(t) = p_{nk}(t) + \alpha_k \text{QoS}_k(t)$ and $c_{nv}^{(k)}(t) = p_v(t) + \beta_k\theta(t) - q_v$ respectively.

Since the segment k MNO's consumer has not experienced the MVNO's rate in the previous period, he estimates it as an average value: $q_v \geq 0$. In fact it would be unrealistic to neglect the MVNO rate since it impacts the opportunity cost that the consumers associate to the MVNO and it would mean that all the MNO consumers would stay with the MNO.

(ii) **Case of the consumers who were MVNO clients at time t-1**

Each MVNO consumer belonging to the market segment k also estimates the opportunity costs associated to the MVNO and to the MNO, to determine which issue is the most suitable to him. It takes the forms: $c_{vv}^{(k)}(t) = p_v(t) + \alpha_k \text{QoS}_v(t) + \beta_k\theta(t)$ for the MVNO and $c_{vn}^{(k)}(t) = p_{nk}(t) - q_1$ for the MNO. Since the MVNO consumer has not experienced the MNO QoS in the last time period and that it would be unrealistic to neglect it (for the same reasons as those evoked for the MNO consumers), the consumer uses an estimate of the MNO average traffic rate whose opposite value is: $q_1 \geq 0$.

(iii) **Case of the consumers who did not subscribe to any offer at time t-1**

Finally the consumers who did not subscribe to any contract at time period $t-1$ have the opportunity to select a provider at t; their opportunity costs for each provider are: $c_{ov}^{(k)}(t) = p_v(t) + \beta_k\theta(t) - q_v$ for the MVNO and $c_{on}^{(k)}(t) = p_{nk}(t) - q_1$ for the MNO.

Again since he has not experienced the operators' rates in the last period, the consumer uses average values: q_v and q_1.

The purchasing process description requires the introduction of the consumers' intrinsic utilities [12] for each operator, independent of the opportunity costs: U_n for the MNO and U_v for the MVNO. We assume that the consumers prefer the MNO to the MVNO since he represents a well-known trustfull brand whereas the MVNO is new on the market; thus $U_n \geq U_v$. Now, we let $s_i \sim \mathcal{U}_{[0;f_k]}$ be the maximal admissible opportunity cost for the consumer $i \in \mathcal{N}$ belonging to the market segment k

($k = 1, 2$). We distinguish between three cases depending on the consumer i choice at time period $t-1$.

• If the consumer i has subscribed to the MNO offer at time period $t-1$ then his utility for the MVNO at time period t is $u_{nv}(i,t) = 0$ if $s_i < c_{nv}^{(k)}(t)$; U_v if $s_i \geq c_{nv}^{(k)}(t)$ and for the MNO, it takes the form $u_{nn}(i,t) = 0$ if $s_i < c_{nn}^{(k)}(t)$; U_n if $s_i \geq c_{nn}^{(k)}(t)$.

Then $u_{nv}(i,t) > 0 \Leftrightarrow s_i \geq c_{nv}^{(k)}(t)$; $u_{nn}(i,t) > 0 \Leftrightarrow s_i \geq c_{nn}^{(k)}(t)$; $u_{nn}(i,t) > u_{nv}(i,t) \Leftrightarrow s_i \geq c_{nn}^{(k)}(t)$.

Thus, if $s_i \in [0; c_{nv}^{(k)}(t)[$ the consumer i does not choose any provider; if $s_i \in [c_{nv}^{(k)}(t); c_{nn}^{(k)}(t)[$ the consumer i subscribes to the MVNO offer; if $s_i \in [c_{nn}^{(k)}(t); f_k]$ the consumer i prefers the MNO offer.

• If the consumer i has subscribed to the MVNO offer at time period $t-1$ then his utility for the MVNO at time period t is $u_{vv}(i,t) = 0$ if $s_i < c_{vv}^{(k)}(t)$; U_v if $s_i \geq c_{vv}^{(k)}(t)$ and for the MNO, it takes the form $u_{vn}(i,t) = 0$ if $s_i < c_{vn}^{(k)}(t)$; U_n if $s_i \geq c_{vn}^{(k)}(t)$.

Then $u_{vv}(i,t) > 0 \Leftrightarrow s_i \geq c_{vv}^{(k)}(t)$; $u_{vn}(i,t) > 0 \Leftrightarrow s_i \geq c_{vn}^{(k)}(t)$; $u_{vn}(i,t) > u_{vv}(i,t) \Leftrightarrow s_i \geq c_{vn}^{(k)}(t)$.

Thus, if $s_i \in [0; c_{vv}^{(k)}(t)[$ the consumer i does not choose any provider; if $s_i \in [c_{vv}^{(k)}(t); c_{vn}^{(k)}(t)[$ the consumer i subscribes to the MVNO offer; if $s_i \in [c_{vn}^{(k)}(t); f_k]$ the consumer i prefers the MNO offer.

• If the consumer i has not subscribed to any offer at time period $t-1$ then his utility for the MVNO at time period t is $u_{ov}(i,t) = 0$ if $s_i < c_{ov}^{(k)}(t)$; U_v if $s_i \geq c_{ov}^{(k)}(t)$ and for the MNO, it takes the form $u_{on}(i,t) = 0$ if $s_i < c_{on}^{(k)}(t)$; U_n if $s_i \geq c_{on}^{(k)}(t)$..

Then $u_{ov}(i,t) > 0 \Leftrightarrow s_i \geq c_{ov}^{(k)}(t)$; $u_{on}(i,t) > 0 \Leftrightarrow s_i \geq c_{on}^{(k)}(t)$; $u_{on}(i,t) > u_{nv}(i,t) \Leftrightarrow s_i \geq c_{on}^{(k)}(t)$.

Thus, if $s_i \in [0; c_{ov}^{(k)}(t)[$ the consumer i does not choose any provider; if $s_i \in [c_{ov}^{(k)}(t); c_{on}^{(k)}(t)[$ the consumer i subscribes to the MVNO offer; if $s_i \in [c_{on}^{(k)}(t); f_k]$ the consumer i prefers the MNO offer.

Since the segment k consumers' maximum admissible opportunity cost is a random variable, their choices differ depending on their maximum admissible opportunity cost values.

III. Stackelberg game with feedback

We describe the three-level game which occurs between both providers and the consumers. Competition results from the fact that each operator wants to maximize his (her) revenue. Besides the MVNO defines a minimum expected profit level: $\mathcal{R}$. This is an exogeneous parameter of the model. Of course, powerful MVNOs would fix a higher minimum expected profit than smaller ones. Hence $\mathcal{R}$ can be seen as a first characteristic of the MVNO power in the game [1]. It means that if the MVNO's game output is lower than $\mathcal{R}$, she would refuse to cooperate i.e., to contract, with the MNO.

Description of the Stackelberg game at time period t

1) First level: game on capacity
(i) The MNO fixes the MVNO's wholesale access charge $w(t)$ such that it maximizes his utility and such that the MVNO cooperates.
(ii) The MNO allocates his bandwidth μ to maximize his long-term revenue: he saves $\xi_{n1}(t)\mu$ for his consumers belonging to the market segment 1, $\xi_{n2}(t)\mu$ for his consumers belonging to the market segment 2, and $\xi_v(t)\mu$ for the MVNO's traffic.
2) Second level: game on prices
The MNO determines his retail prices $p_{n1}(t)$ for the market segment 1 and $p_{n2}(t)$ for the market segment 2; *at the same time* the MVNO chooses her level of content investment $\theta(t)$ and then her retail price $p_v(t)$.
3) Third level: consumer allocation between both operators
The consumers choose the operator generating the highest utility or no offer if their opportunity costs are all higher than their maximum admissible opportunity costs.

In the last level of the game, the consumers either determine the highest utility provider or choose to not subscribe to any offer, considering that the operators have optimized their decisions in the previous levels. Similarly, in the top of the game, the MNO determines the contract parameters ($\xi_{n1}(t)$, $\xi_{n2}(t)$, $\xi_v(t)$ and $w(t)$) taking into account that both operators optimize their decisions (prices and content investment level) in the second level. It is possible for the MVNO to choose between two marketing strategies: she can either *target a specific market segment* by investing in specialized contents or she can choose to *increase her value*. Note that if the MVNO targets the market segment k, we just have to let $\beta_k < 0$ and $\beta_l = 0$ for $k, l \in \{1, 2\}$, $k \neq l$ since the market segment l will not be interested at all in the MVNO broadcasted contents. We will test in the numerical applications which marketing strategy is the most profitable for the MVNO on a long time scale.

A. Operators' utilities

The MNO's utility is the sum of the revenue generated from his consumers on each market segment and from the access charge payed by the MVNO to send some traffic on the MNO's network minus the MNO equipement cost C; it takes the form: $U_1(t) = \Big(p_{n1}(t)\mathcal{N}_{n1}(t) + p_{n2}(t)\mathcal{N}_{n2}(t)\Big) - C + w(t)\xi_v(t)\mu$. The MVNO determines the content investment level: $\theta(t)$ and the retail price for her consumers: $p_v(t)$ in the second level of the Stackelberg game. We let c_θ be the cost of a unit of content. The MVNO's utility is the sum of the revenue generated by her consumers minus the content investment and the contract costs: $U_v(t) = p_v(t)\Big(\mathcal{N}_{v1}(t) + \mathcal{N}_{v2}(t)\Big) - c_\theta\theta(t) - w(t)\xi_v(t)\mu$.

B. Consumer allocation between both operators

In the consumer model (cf Section II) the segment k consumer maximum admissible opportunity cost was distributed according to a uniform density on the interval $[0; f_k]$. Depending on the consumers' maximum admissible opportunity cost values on the interval $[0; f_k]$ and on the opportunity costs that they associate to each provider, we want to determine the number of consumers subscribing to each operator's offer at time period t. First, we observe that the consumers who do not choose any offer, should have the lowest maximum admissible opportunity costs. Then, we note than on the transition diagram depicted in Figure 1, the transition probabilities $\pi_{no}^{(k)}(t)$ and $\pi_{on}^{(k)}(t)$ at time period t should vanish. Indeed, if $\pi_{on}^{(k)}(t) \neq 0$ then $c_{on}^{(k)}(t) \leq s_i$ for all the smallest maximum admissible opportunity costs since the consumers who did not choose any offer at $t-1$ have the smallest maximum admissible opportunity costs. Similarly, if $\pi_{no}^{(k)}(t) \neq 0$ then $c_{nn}^{(k)}(t) > s_i$ and $c_{nv}^{(k)}(t) > s_i$ for all the highest maximum admissible opportunity costs since the consumers who prefer the MNO at time period $t-1$ have the highest maximum admissible opportunity costs. It would imply that there exists only two states: either the consumers would not subscribe to any offer, or they would choose the MNO. Hence a guarantee that the MVNO has some incentives to enter the market requires to set $\pi_{no}^{(k)}(t) = \pi_{on}^{(k)}(t) = 0,\ k = 1, 2$.

Consequentely, on the segment k, the frontier between the maximum admissible opportunity costs of the consumers choosing no provider and those choosing the MVNO is determined by $c_{vv}^{(k)}(t)$ or $c_{ov}^{(k)}(t)$; while the frontier between the maximum admissible opportunity costs of the consumers choosing the MVNO and those choosing the MNO is determined by $c_{nn}^{(k)}(t)$ or $c_{vn}^{(k)}(t)$. This sharing is pictured in Figure 2.

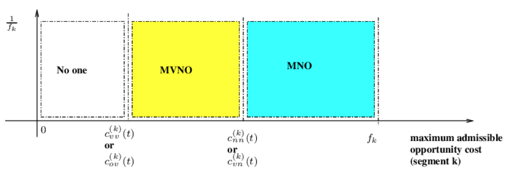

Fig. 2. Consumer sharing between the MNO and the MVNO depending on their utilities (segment k).

Depending on the consumer maximum admissible opportunity cost orders at the frontiers, we determine the number of consumers on the segment k at time period t for each operator, and distinguish between four cases [10].

C. A discounted profit maximization problem

The three-level Stackelberg game is played dynamically. Indeed each operator wants to maximize his (her) *long-term discounted* expected revenue. To include money depreciation and uncertainty on future revenues in our model, we introduce a discount factor for each operator. For example $\delta^T U(T)$ represents the revenue that the provider can expect to get at time period T provided he follows the recommendation (policy) up to that time. We let $\delta_n \in]0;1]$ be the discount factor for the MNO and $\delta_v \in]0;1]$, for the MVNO. The role of δ_n (resp. δ_v) is essential: if it is low (i.e., around 0) then the associated provider effectively behaves like a *myopic* agent and if it is high (i.e., around 1), like a *foresighted* provider (cf [7], [2]). We will see in the numerical applications that it is possible practically to determine which marketing strategy (between myopic, neutral or forsighted) is the most profitable for both operators depending on his (her) rival's one.

For the MNO, the discounted profit maximization problem takes the form: $\max_{W;\Xi} \max_{\pi_{n1},\pi_{n2}} \sum_{t=0}^{T} \delta_n^t \mathbf{E}\Big[U_1(t)\Big]$ where $\pi_{nk} = \Big(p_{nk}(0)\ \ p_{nk}(1)\ \ ...\ \ p_{nk}(T)\Big)$ contains the retail prices on the segment k maximizing the MNO discounted long-term revenue, $W = \Big(w(0)\ \ w(1)\ \ ...\ \ w(T)\Big)$ is the sequence of wholesale access charges and $\Xi = \Big(\xi_{n1}(0), \xi_{n2}(0)\ ...\ \xi_{n1}(T), \xi_{n2}(T)\Big)$ is the resource allocation mechanism maximizing the MNO long-term discounted revenue.

We proceed similarly for the MVNO; her discounted profit maximization problem can be written: $\max_{\pi_v} \sum_{t=0}^{T} \delta_v^t \mathbf{E}\Big[U_v(t)\Big]$ where $\pi_v = \Big(p_v(0), \theta(0)\ \ \ p_v(1), \theta(1)\ \ \ ...\ \ \ p_v(T), \theta(T)\Big)$ contains the MVNO retail prices and content investment level maximizing her discounted long-term revenue. The expectations are taken with respect to both operators' chosen strategies (i.e., optimal prices and content investment sequences).

Solving these discounted maximization problems is equivalent to solve by *backward recursion*, dynamic programming equations [7]. Thus, at each time period t, it requires the introduction of value vectors: $V_1^t = \big(V_1^t(n)\ V_1^t(v)\ V_1^t(o)\big)^{tr} \in \mathbb{R}^3$ for the segment 1 and $V_2^t = \big(V_2^t(n)\ V_2^t(v)\ V_2^t(o)\big)^{tr} \in \mathbb{R}^3$ for the segment 2; where tr is the transpose operator. Since the consumer process state space is of size 3: $\{n; v; o\}$, the value vectors are three-dimensional. Besides we assume that the initial allocation of the consumers ($\mathcal{N}_{nk}(0)$ and $\mathcal{N}_{vk}(0)$, $k = 1, 2$) is known. The backward recursion of dynamic programming applied to our three-level game with feedback can be described as follows [7]

1) Step 1. (Initiation) we set $V_1^{-1}(s) = 0$, $V_2^{-1}(s) = 0\ \ \forall s \in \{n; v; o\}$ and define the MNO and the MVNO optimization problems; for the MNO: $\max_{w \geq 0, \xi_{n1}, \xi_{n2} \geq 0} \max_{p_{n1}, p_{n2}} \Big[p_{n1}\mathcal{N}_{n1}(T) + p_{n2}\mathcal{N}_{n2}(T) - C + w\xi_v\mu\Big]$ under the constraints that $U_v(T) \geq \mathcal{R},\ p_{n1},\ p_{n2} \geq 0,\ \xi_{n1} + \xi_{n2} \leq 1,\ \xi_v = 1 - \xi_{n1} - \xi_{n2}$; and for the MVNO: $\max_{p_v \geq 0, \theta \geq 0} \Big[p_v\Big(\mathcal{N}_{v1}(T) + \mathcal{N}_{v2}(T)\Big) - w\xi_v\mu - c_\theta\theta\Big]$.
   We recall that $\mathcal{N}_{nk}(T)$ is a function of p_{nk} and ξ_{nk}, while $\mathcal{N}_{vk}(T)$ is a function of p_{nk}, p_v, θ, ξ_{nk}, ξ_v; with $k = 1, 2$.
   We update the value functions on the segment k: for the MNO $V_k^0(n) = p_{nk}(T)\mathcal{N}_{nk}(T) + \gamma_k\Big(w(T)\xi_v(T)\mu - C\Big)$ and for the MVNO $V_k^0(v) = p_v(T)\Big(\mathcal{N}_{v1}(T) + \mathcal{N}_{v2}(T)\Big) - \gamma_k\Big(w(T)\xi_v(T)\mu + c_\theta\theta(T)\Big)$ such that $\gamma_1 + \gamma_2 = 1,\ \gamma_1 \geq 0,\ \gamma_2 \geq 0$. For the MNO we choose $\gamma_1 = \frac{\mathcal{N}_{n1}(t)}{\mathcal{N}_{n1}(t)+\mathcal{N}_{n2}(t)}$ on the segment 1 and for the MVNO, we set: $\gamma_1 = \frac{\mathcal{N}_{v1}(t)}{\mathcal{N}_{v1}(t)+\mathcal{N}_{v2}(t)}$.
2) Step 2. (Recursion) For each t=1,2,...,T the MNO optimization problem is: $\max_{w,\xi_{n1},\xi_{n2}} \max_{p_{n1},p_{n2}} \Big[p_{n1}\mathcal{N}_{n1}(T-t) + p_{n2}\mathcal{N}_{n2}(T-t) - C + w\xi_v\mu\Big] + \delta_n\Big(\pi_{nn}^{(1)}(T-t)V_1^{t-1}(n) + \pi_{nn}^{(2)}(T-t)V_2^{t-1}(n)\Big)$ under the constraints that $U_v(T-t) \geq \mathcal{R},\ p_{n1},\ p_{n2} \geq 0,\ w \geq 0,\ \xi_{n1},\ \xi_{n2} \geq 0,\ \xi_{n1} + \xi_{n2} \leq 1,\ \xi_v = 1 - \xi_{n1} - \xi_{n2}$.
   And for the MVNO, it takes the form: $\max_{p_v \geq 0, \theta \geq 0} \Big[p_v\Big(\mathcal{N}_{v1}(T-t) + \mathcal{N}_{v2}(T-t)\Big) - w\xi_v\mu - c_\theta\theta\Big] + \delta_v\Big(\pi_{vv}^{(1)}(T-t)V_1^{t-1}(v) + \pi_{vv}^{(2)}(T-t)V_2^{t-1}(v)\Big)$.
   We update the value functions on the segment k ($k = 1, 2$): for the MNO $V_k^t(n) = p_{nk}(T-t)\mathcal{N}_{nk}(T-t) + \gamma_k\Big(w(T-t)\xi_v(T-t)\mu - C\Big) + \delta_n\pi_{nn}^{(k)}(T-t)V_k^{t-1}(n)$ and for the MVNO $V_k^t(v) = p_v(T-t)\Big(\mathcal{N}_{v1}(T-t) + \mathcal{N}_{v2}(T-t)\Big) - \gamma_k\Big(w(T-$

$t)\xi_v(T-t)\mu + c_\theta\theta(T-t)\Big) + \delta_v \pi_{vv}^{(k)}(T-t)V_k^{t-1}(v)$
such that $\gamma_1 + \gamma_2 = 1,\ \gamma_1 \geq 0,\ \gamma_2 \geq 0$.

3) Step 3. We construct the providers' strategies: $\Big\{(w(t), \xi_{n1}(t), \xi_{n2}(t); p_{n1}(t), p_{n2}(t))\Big\}_{t=0,\ldots,T}$ for the MNO and $\Big\{p_v(t), \theta(t)\Big\}_{t=0,\ldots,T}$ for the MVNO.

We assume that the parameters q_1 and q_v are chosen so that the relationships between the opportunity costs at the frontiers remain identical between both market segments.

Theorem 1. *The three-level Stackelberg game admits a unique equilibrium at each time period. In particular, both providers' retail prices and content investment level form a unique Nash equilibrium in the second level of the game.*

Proof of Theorem 1. The proof can be found in [10].

Numerical illustrations. The exogeneous model parameters are defined as $\mu = 10$, $C = 5$, $c_\theta = 3$, $\delta_n = 0.7$, $\delta_v = 0.3$, $f_1 = 5$, $f_2 = 7$, $\alpha_1 = -0.7$, $\alpha_2 = -0.9$, $\beta_1 = -0.3$, $\beta_2 = -0.83$, $\mathcal{N}_1 = 100$, $\mathcal{N}_2 = 100$, $\gamma_1 = 0.7$ and $T = 100$. The initial consumer allocation[3] is $\mathcal{N}_{n1}(0) = 70$, $\mathcal{N}_{v1}(0) = 10$, $\mathcal{N}_{n2}(0) = 40$ and $\mathcal{N}_{v2}(0) = 60$. In Figure 3 we have plotted both operators' optimized values at each time period. The MVNO value always remains positive and is in general lower than the MNO long-term profit. Furthermore, the MNO and the MVNO market shares are around 43% and 20% respectively i.e., the global penetration rate is of 63%. The optimal prices to choose for the MNO at time period T, on each market segment are: $p_{n1}^*(T) = 40$ and $p_{n2}^*(T) = 40$ i.e., he does not price-discriminate between his segments. The MVNO optimal price is $p_v^*(T) = 30$ which is stricly lower than the MNO retail prices and the optimal content investment level is 25. The MNO chooses $w^*(T) = 3$ as access charge. The market share allocation between both providers is rather stable. However, the resource sharing mechanism being updated at each time period, both providers' values evolve over time.

In Figure 4, we observe that the MVNO has no incentives to increase too much her minimum expected profit, $\mathcal{R}$, since it would make her long-term revenue decrease.

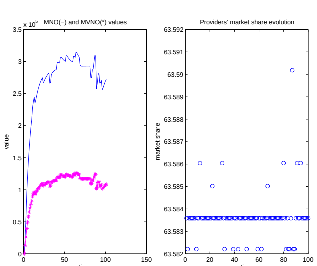

Fig. 3. Both operators' value and market share evolutions.

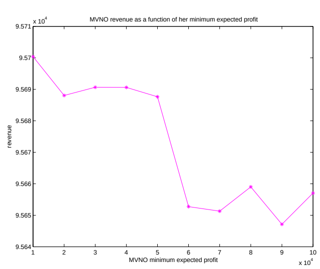

Fig. 4. MVNO long-term revenue evaluated at $T = 100$ as a function of her minimum expected revenue level.

[3]We assume that the segment 2 consumers are young people addicted to trendy mobile phone broadcasted contents; while the segment 1 represents older people more reluctant to new mobile technologies.

TABLE I
INFLUENCE OF BOTH OPERATORS' MARKETING STRATEGIES ON THEIR VALUES ($\times 10^4$).

δ_n \ δ_v	0.1 (**myopic**)	0.5 (**neutral**)	0.8 (**foresighted**)
0.1 (**myopic**)	(4.84;4.28)	(8.86;7.88)	(0;0)
0.5 (**neutral**)	(3.41;3.55)	(8.72;7.70)	(153.50;67.86)
0.8 (**foresighted**)	(0;0)	(0;2.70)	(43.62;38.53)

The discount factor $\delta_n \in]0;1]$ for the MNO and $\delta_v \in]0;1]$ for the MVNO, is fundamental. Indeed if it is low (i.e., around 0) the firm behaves like a *myopic* agent and if it is high (i.e., around 1) like a *foresighted* one. At time period $T = 100$, we evaluate both operators' values in Table I. In each cell, the left number coincides with the MNO value at time period T and the right one, with the MVNO value at T. There are two Nash equilibria: $\delta_v = 0.1; \delta_n = 0.5$ which implies that $V_1^*(T) = 8.86.10^4$ and $V_v^*(T) = 7.88.10^4$; and $\delta_v = 0.5; \delta_n = 0.8$ which generates values such as $V_1^*(T) = 153.5.10^4$ and $V_v^*(T) = 67.86.10^4$. Hence the second Nash equilibrium is Pareto superior to the first i.e., both operators' values increase if they leave the first Nash equilibrium for the second. Hence, the operators whould prefer the second Nash equilibrium. Practically, it means that the MNO should use a foresighted marketing strategy whereas the MVNO had better to remain rather neutral.

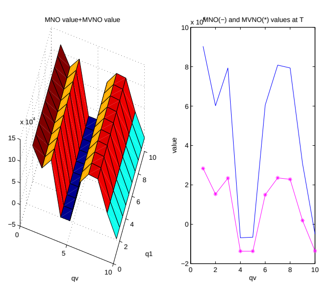

Fig. 5. Influence of the consumers' mean rates (q_1, q_v) on both providers' long-term profits.

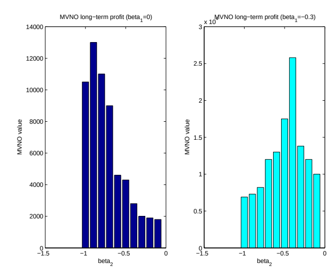

Fig. 6. Comparison of targeting ($\beta_1 = 0$) and value-adding strategies ($\beta_1 = -0.3$) for the MVNO.

We test the influence of the choice of the mean rate parameters (q_1, q_v) on the providers' values at $T = 100$ (cf Figure 5). We observe that in this example, the providers' values are independent of q_1; but they depend on the model exogeneous parameters and on q_v. Besides, the provider values admit distinct maxima in q_v.

If $\beta_1 = 0$ i.e., the segment 1 consumers are not interested at all in the contents that the MVNO delivers, then the MVNO should target her service to the segment 2 consumers exclusively. We observe in Figure 6 (left) that the MVNO long-term revenue grows with the segment 2 consumers' interest in the broadcasted contents. Besides the MVNO maximal value in $T = 100$ is 1.29×10^4; it is reached in $\beta_1 = 0;\ \beta_2 = -0.9$. But the value-adding strategy (cf right part of Figure 6) is more profitable than the targeting one; all the more that the content / advertising preference differentiation between the segments is small: indeed the MVNO long-term profit in $T = 100$ is higher than the MNO one for $\beta_1 = -0.3$ and $\beta_2 \in [-0.3; -0.6]$. Besides its maximum is 2.53×10^4.

IV. Is cooperative content investment profitable for both providers?

Investing in contents / advertising is fundamental for the MVNOs' survivals [8]. The advertising aims at increasing the MVNO popularity. It uses media like TV channels, radios, Internet, to spread adverts often targeted at specific market niches (for example the youths, the foreigners, the britons, etc.). The goal is then to *boast the MVNO brand image*. Furthermore providers (like MVNOs) often broadcast specific contents (for example soccer games) targeted at specific users. It aims at reaching specific market niches. In the numerical illustration introduced in Subsection III-C, the operators' total market share represents only 63% of the total market. Hence, we want to know whether or not it is possible to increase this market share, and if this would also increase both providers' revenues. A solution appears in [2]. Sethi et al. prove that introducing cooperative advertising between a manufacturer and a retailer i.e., when the manufacturer share partially or totally the retailer advertising costs, can increase both operators' long-term revenues depending on the operators' power relations. Besides it forces the retailer to invest more in advertising and to lower its prices which increases the consumers' welfare. Consequentely, we identify the MNO, the MVNO and the consumers with a supply-chain [6] supervised by a regulatory authority (unbiased decision-maker), and check if the introduction of cooperative content investment can increase the supply-chain utility i.e., the social-welfare.

A. Definition of the actors' utilities in the supply-chain

The cooperative content investment contract between the MNO and the MVNO can be described as follows. At time period t, the MNO shares the costs of the MVNO's content / advertising investment with the MVNO according to a parameter $\rho(t) \in [0; 1]$. In exchange the MVNO shares the revenue generated by her consumers with the MNO according to a parameter $\Phi(t) \in [0; 1]$ [2]. It is the regulatory authority (unbiased decision-maker) who fixes the contract parameters. He aims at determining the parameters $\Phi(t)$ and $\rho(t)$, *coordinating* the supply-chain i.e., such that it maximizes the social-welfare and such that no provider deviation can increase his (her) profit. Both operators' utilities take now the forms: $U_1(t) = \Big(p_{n1}(t)\mathcal{N}_{n1}(t) + p_{n2}(t)\mathcal{N}_{n2}(t)\Big) + \Phi(t)\Big(p_v(t)(\mathcal{N}_{v1}(t)+\mathcal{N}_{v2}(t))\Big) - \rho(t)c_\theta\theta(t) - C + w(t)\xi_v(t)\mu$ for the MNO and $U_v(t) = \Big(1-\Phi(t)\Big)\Big(p_v(t)(\mathcal{N}_{v1}(t) + \mathcal{N}_{v2}(t))\Big) - \Big(1-\rho(t)\Big)c_\theta\theta(t) - w(t)\xi_v(t)\mu$ for the MVNO. The supply-chain's utility is the sum of the providers' utilities: $U(t) = U_1(t) + U_v(t) = \Big[p_{n1}(t)\mathcal{N}_{n1}(t) + p_{n2}(t)\mathcal{N}_{n2}(t)\Big] + \Big[p_v(t)(\mathcal{N}_{v1}(t) + \mathcal{N}_{v2}(t))\Big] - c_\theta\theta(t) - C$.

It coincides with the system social-welfare. We start by assuming that the supply-chain's utility U is linear in the MNO's utility U_1 i.e., there exists $\beta \geq 1$ and $\gamma \in \mathbb{R}$ such that $U = \beta U_1 + \gamma$[4]. β and γ characterize the providers *power* in the supply-chain. These parameters are estimated *a priori* by the regulatory authority.

Proposition 2. *The parameters coordinating the supply-chain are:* $\Phi^*(t) = \frac{1}{\beta}\Big[1 + (1-\beta)\frac{p_{n1}(t)\mathcal{N}_{n1}(t)+p_{n2}(t)\mathcal{N}_{n2}(t)}{p_v(t)\Big(\mathcal{N}_{v1}(t)+\mathcal{N}_{v2}(t)\Big)}\Big]$; *and* $\rho^*(t) = \frac{1}{\beta c_\theta\theta(t)}\Big[\gamma + c_\theta\theta(t) + \beta w(t)\xi_v(t)\mu + (1-\beta)C\Big]$.

Proof of Proposition 2. It can be found in [10].

We introduce $\xi^*_{nk}(t)$ $k = 1, 2$ ($\xi^*_v(t) = 1 - \xi^*_{n1}(t) - \xi^*_{n2}(t)$), the bandwidth sharing parameters maximizing the MNO value at time period t, and $\xi^0_{nk}(t)$ $k = 1, 2$ ($\xi^0_v(t) = 1 - \xi^0_{n1}(t) - \xi^0_{n2}(t)$), the allocation maximizing the supply-chain utility. Useful parameters to analyze the cooperative content investment performances are $\mathcal{S}(t) = \frac{U_1\Big(\xi^*_{n1}(t),\xi^*_{n2}(t),w^*(t)\Big)}{U\Big(\xi^0_{n1}(t),\xi^0_{n2}(t)\Big)}$ which represents the MNO share of the supply-chain optimal profit [1]. At time period t the contract attractivness for the MNO increases with $\mathcal{S}(t)$; $\eta(t) = \frac{U\Big(\xi^*_{n1}(t),\xi^*_{n2}(t)\Big)}{U\Big(\xi^0_{n1}(t),\xi^0_{n2}(t)\Big)}$, the contract efficiency [1], measuring the ratio between the supply-chain utility (while the MNO behaves selfishly) and the supply-chain utility under coordination.

B. Dynamic cooperative content investment

We transpose the recursive procedure obtained in Subsection III-C to the case where both operators are coordinated by a contract based on cooperative advertising / content investment. In particular we want to characterize (i) how both operators' revenues evolve with time and (ii) how the power parameters (β, γ) influence the dynamic revenue evolutions.

The discounted profit maximization problem for the supply-chain coordinated via a contract based on cooperative advertising is described below. At time step t, going backward, it takes the following form for the MNO: $\max_{p_{n1},p_{n2}} S(T-t)U + \delta_n\Big[\pi^{(1)}_{nn}(T-t)V_1^{t-1}(n) + \pi^{(2)}_{nn}(T-t)V_2^{t-1}(n)\Big]$ with $p_{n1},\ p_{n2} \geq 0$ and where a regulatory authority (unbiased decision-maker) has previously determined the resource sharing mechanism $\xi^0_{n1}(T-t) \geq 0$, $\xi^0_{n2}(T-t) \geq 0$ with $\xi^0_{n1}(T-t) + \xi^0_{n2}(T-t) \leq 1$ and $\xi^0_v(T-t) = 1 - \xi^0_{n1}(T-t) - \xi^0_{n2}(T-t)$, maximizing the social-welfare U. Then, using the definition of the MNO share of the supply-chain profit ($S(T-t)$); it is equivalent for the MNO to solve: $\max_{\xi_{n1},\xi_{n2},w} \max_{p_{n1},p_{n2}} U_1 + \delta_n\Big[\pi^{(1)}_{nn}(T-t)V_1^{t-1}(n) + \pi^{(2)}_{nn}(T-t)V_2^{t-1}(n)\Big]$ under the constraints that $U_v(T-t) \geq \mathcal{R}$, $p_{n1},\ p_{n2} \geq 0$, $w \geq 0$, $\xi_{n1} + \xi_{n2} \leq 1$, $\xi_{n1},\ \xi_{n2} \geq 0$, $\xi_v = 1 - \xi_{n1} - \xi_{n2}$.

The resource sharing mechanism solution of this problem is denoted $\xi^*_{n1}(T-t)$, $\xi^*_{n2}(T-t)$, $\xi^*_v(T-t)$ while the optimal access charge for the MVNO is $w^*(T-t)$. For the

[4] This is a classical assumption in Supply-chain theory [1].

MVNO, we need to solve: $\max_{p_v \geq 0, \theta \geq 0} \Big(1 - S(T-t)\Big) U + \delta_v \Big[\pi_{vv}^{(1)}(T-t) V_1^{t-1}(v) + \pi_{vv}^{(2)}(T-t) V_2^{t-1}\Big]$. This is equivalent to solve: $\max_{p_v \geq 0, \theta \geq 0} U_v \Big(\xi_{n1}^*(T-t), \xi_{n2}^*(T-t), w^*(T-t)\Big) + \delta_v \Big[\pi_{vv}^{(1)}(T-t) V_1^{t-1}(v) + \pi_{vv}^{(2)}(T-t) V_2^{t-1}\Big]$ where $\xi_{n1}^*(T-t)$, $\xi_{n2}^*(T-t)$, $w^*(T-t)$ are determined by the MNO.

Theorem 3. *At each time period t, the game with cooperative content investment admits a unique equilibrium. In particular, both providers' retail prices and content / advertising investment level form a unique Nash equilibrium.*

Proof of Theorem 3. The proof can be found in [10].

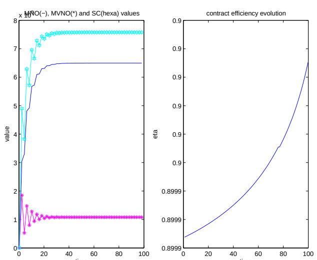

Fig. 7. Providers long-term revenue and efficiency evolutions.

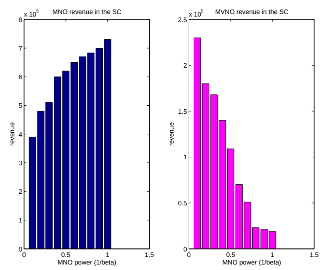

Fig. 8. Providers long-term revenues picked at $T = 100$ as functions of the MNO power.

Numerical illustrations. The exogeneous model parameters are chosen as in the numerical illustration provided in Subsection III-C. The power parameters are estimated initially by the regulatory authority: $\frac{1}{\beta} = 0.6$ and $\gamma = 0.3$. We aim to compare the results obtained in Subsection III-C where both providers use a contract based on a wholesale access charge to the case where the MNO introduces cooperative content investment. Under coordination the MNO discriminates between his market segments, since $p_{n1}^{co}(T) = 30$ and $p_{n2}^{co}(T) = 40$ while the MVNO lowers her retail price: $p_v^{co}(T) = 20$. Besides the MVNO advertising / content investment level is higher than in the case without cooperative content investment: $\theta^{co}(T) = 35$ and the MNO access charge is lower: $w^{co}(T) = 2$. Both providers long-term revenues are higher (cf Figure 7): $U_1^{co}(T) = 6.5.10^5$ for the MNO and $U_v^{co}(T) = 1.2.10^5$ for the MVNO (it was $U_1^*(T) = 2.54.10^5$ for the MNO and $U_v^*(T) = 1.08.10^5$ for the MVNO, without cooperative content investment in Figure 3). Furthermore, the total market share is around 83% while it was only 63% without cooperative content investment. We note in Figure 7, that the cooperative content investment efficiency increases slightly over time, reaching the value 0.9 i.e., the MNO incentive to use cooperative content investment increases when time runs. The linearity assumption on the supply-chain utility enables us to characterize very simply the MNO power. Indeed $U_1 = \frac{1}{\beta} U - \frac{\gamma}{\beta}$. We then assume that $\gamma = 0.3$, and test the influence of the parameter $\frac{1}{\beta} \in]0;1]$ on both operators' long-term profits. In Figure 8 we observe that the MNO long-term revenue under coordination increases with the parameter $\frac{1}{\beta}$.

TABLE II
INFLUENCE OF COOPERATIVE CONTENT INVESTMENT ON THE ACTORS OF OUR EXAMPLE.

Impact of coop. content investment
(i) the MNO price-discriminates between his market segments;
(ii) he decreases his access charge to his network;
(iii) the MVNO decreases her retail price;
(iv) she invests more in content / advertising;
(v) the market penetration increases;
(vi) both operators' long-term revenues are higher than with a simple wholesale access charge.

Hence the MNO has more incentives to coordinate when he is powerful. We sum up the benefits of the cooperative content investment for the operators and the consumers in Table II, for this example.

V. CONCLUSIONS

We have modeled the dynamic MNO-MVNO relationships. The introduction of a regulatory authority who forces both operators to coordinate via a cooperative content investment contract, increases both providers' long-term revenues, makes the wholesale access charge to the MNO network and the MVNO retail price decrease, while this latter invests more in content / advertising. Our work can be extended by considering two MVNOs, who dynamically bargain their accesses with two distinct MNOs. However, these latter hide their resource allocation strategies and both MVNOs have uncertainties about their rival resource requirement. Considering various learning strategies [5], a challenge will be to determine whether the game ultimately reaches an equilibrium and how the power relations between MVNOs impact their learning rates.

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