MUD Receiver Capacity Optimization for Wireless Ad Hoc Networks

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***Abstract-* In general the performance of ad hoc networks is limited by mobility, half-duplex operation, and possible collisions. One way to overcome these bottlenecks is to apply a Multi-User Detection (MUD) reception that can significantly increase the network throughput and improve quality of service. Recent technological advances allow implementation of a multi-user detection receiver in one software-defined radio chip and thus making it feasible to consider this technology for ad hoc networks. Nevertheless, the MUD receiver power consumption and complexity grow exponentially with its capacity defined as the maximum number of CDMA signals, originated at different nodes, which can be received simultaneously. Therefore the receiver capacity should be optimized to provide reasonable trade-off between the network performance and the MUD receiver cost and power consumption. We address this issue by presenting an approximate analysis of the network throughput as a function of MUD receiver capacity and other network parameters such as node density and offered traffic. The numerical analysis illustrates the gains in network throughput and performance with increasing receiver capacity. Based on this analysis we propose a framework for MUD receiver capacity optimization based on performance and cost utility functions.**

***Index Terms:* ad hoc, capacity, half-duplex, Multi-User Detection (MUD) reception, throughput.**

I. INTRODUCTION

Multi-hop mobile *ad hoc* networks have recently been the subject of extensive research due to their ubiquitous potential applications for military, emergency, neighbourhood, or conference networks to mention few. Nevertheless, the performance and bandwidth utilization of the proposed solutions are significantly inferior when compared with fixed wireless, or wired, networks. This is a significant drawback especially in view of growing need to serve multimedia applications also in *ad hoc* networks. Apart mobility, the main reasons for these limitations are half-duplex operation and possible collisions caused by hidden/exposed terminal problems. These characteristics can introduce long packet delays especially for multi-hop connections. Sophisticated adaptive directional antennas give some promise of performance improvement in the long term but, at the moment, this technology seems to be not developed enough to be considered in an unpredictable and variable *ad hoc* network environment. Therefore, in this paper, we focus on another technology that is more advanced and that can significantly improve both the performance and bandwidth utilization. The technology in question is Multi-User Detection (MUD). This solution allows parallel reception of several CDMA signals, from different nodes, with high efficiency by reducing mutual interference [1]. These features can increase significantly *ad hoc* network throughput and concurrently decrease packet delays, both characteristics being very important for multimedia application.

While MUD is known for a long time [2], only very recent technological advances allow integration of a CDMA MUD based receiver in one software-defined radio chip. This development allows considering application of this technology for multi-hop ad hoc networks. While there was quite a lot of publications on MUD based physical layer performance, the results on application of MUD technology to ad hoc networks, where Medium Access Control (MAC) and network performance are taken into account, appeared in the literature only recently. In [3] an asymptotic model is proposed for throughput evaluation of MUD based ad hoc networks with delay constraints. This throughput is compared with a system based on multi-user reception without MUD, showing that a threefold gain can be achieved. In [4], performance of MUD based ad hoc networks was analysed for a multicasting application. In this case, apart the MAC issue, the routing algorithm was also considered. Due to fundamentally different architecture of MUD based physical layer platforms, application of MUD in ad hoc networks requires novel approaches for higher layer protocols in order to take advantages of the new features. This is especially true for the MAC (medium access control) layer protocol and the scheduling algorithms as the solutions derived for "one transmission at a time in the neighbourhood" (CSMA/CA, IEEE 802.11) are not applicable for MUD based ad hoc networks. This issue, not treated in [3, 4], was addressed in [5, 6], where a MAC protocol for MUD based ad hoc networks is proposed and evaluated by means of simulation. The results show that apart significant increase of the network throughput, the QoS characteristics such as delay and packet losses are also improved.

The results presented in [3-6] assumed no limit on the MUD receiver capacity defined as the maximum number of CDMA signals, originated at different nodes, that can be received simultaneously. In this paper we derive an analytical model that takes into account such a limitation. This aspect is of importance since the complexity and power consumption of MUD receiver grows exponentially with the receiver capacity. The contributions of this paper can be divided into two parts. First we propose an approximate performance model for *ad hoc* networks with a MUD receiver where the receiver capacity is taken into account. Then we demonstrate how this model can be used for MUD receiver capacity optimization. While we did not find in the literature any reference related to optimization of MUD receiver capacity in *ad hoc* networks, on a conceptual level this issue is similar to optimizing the number of receivers and transmitters in other technologies. A good example is optimization of the number of tuneable transmitters and receivers for optical Wavelength Division Multiple Access (WDMA) technology that aims at providing a trade-off between the cost of transmitters/receivers and the network performance [7, 8].

We analyse the network performance assuming Poisson model for distribution of number of mobile nodes in an area, using arguments similar to those in [9]. Then, to derive the network performance model we apply a uniform loading model on the MAC layer that results in a compact solution form that is independent from mobile density. In the derivation we use a truncated Poisson process model that is often used for performance analysis of other multiple-access TDMA and CDMA systems [10]. The resulting form of the solution allows getting important insight into the characteristics of the MAC optimal operating point that maximizes the throughput for given receiver capacity. In the second part of this paper, we use the performance model to formulate an optimization framework to find an optimal MUD receiver capacity. It is based on throughput, QoS, and cost utility functions that correspond to satisfaction (throughput and QoS utility) and dissatisfaction (cost utility) from the network design. The optimal receiver capacity corresponds to the maximum net-satisfaction. Validity of the MUD capacity dimensioning model is confirmed by comparison with simulation results presented in [6].

The remainder of the paper is organized as follows. Section II describes the system model and assumptions including calculation of the average number of receiving, transmitting, and idle nodes in the neighbourhood. A model for the receiver performance analysis is derived in Section III. In Section IV we use this model to calculate the network throughput and a QoS metric as a function of MUD receiver capacity and other system parameters. Optimizing the receiver capacity is discussed in Section V. The paper is concluded in Section VI.

II. SYSTEM MODEL AND ASSUMPTIONS

In this paper we follow the MAC structure proposed for MUD based ad hoc networks in [5, 6]. In this approach all data transmissions are synchronized by using periodic frame structure where all frames have the same duration. Each time frame consists of the signalling slot and the data transmission slot. In the signalling slot, each node exchanges information about its need to transmit packets with other nodes in its neighbourhood. Then, based on this exchange, the distributed scheduling algorithm decides which nodes become receivers and which nodes become transmitters in the following data slot. Note that some nodes can become idle if they do not have packets to send or if their packet destinations are selected to be transmitters in that data slot. Therefore, in any data slot, the mobile nodes are divided into three categories: transmitting nodes, receiving nodes, and idle nodes, as illustrated in Figure 1(a), where the arrows indicate the source and destination of the packet.

The configuration shown in Figure 1(a) corresponds to a particular data slot with j transmitters, k receivers, and d idle nodes in area a defined by radius r. Now let us consider the distribution of the number of mobiles in area a. We assume that $A >> a$ where A denotes the area of the considered ad hoc network with n mobile nodes. Let us also assume for the moment that $A \to \infty$ and $n \to \infty$, with constant ratio $\rho = n / A$, and that the nodes are placed randomly in area A. This case can be considered as a homogeneous Poisson point process with mobile intensity ρ. Then the probability that there are i nodes in the considered area a is given by

$$P\{D = i\} = \frac{\mu^i}{i!} e^{-\mu} \qquad (1)$$

where $\mu = \rho a = na / A = \rho \pi r^2$ is the average number of nodes in area a. In reality area A and number of mobiles n are limited so the question arises about the validity of the Poisson model. This issue was analysed in [9] where it was shown that in practice Eq. (1) is a good approximation if $n >> 1$ and $A >> a$ (with suggested values: $n \geq 1500 a / A$ and $A \geq 12.5 a$). This conclusion was verified in [11] by simulations. In the remainder of this paper we will use the presented Poisson model for analysing the distribution of different mobile categories in area a where radius r defines the maximum communication distance between any pair of nodes.

To analyze performance of the considered ad hoc network we need to evaluate the distributions and average numbers of transmitting, receiving, and idle nodes in area a. To do that we assume that in each frame a node becomes a transmitter or a receiver with independent transmitting and receiving probabilities p and q, respectively. The probability of becoming an idle node is s=1-p-q. In the following sub-sections we present models for each node category.

A. Distribution and average number of transmitting nodes

Eq. (1) defines the distribution of number of nodes i in an arbitrary area a of radius r. In a given frame, there will be j transmitting nodes out of these i nodes. Since we assumed that each node becomes a transmitter with independent probability p, the distribution of the number of transmitters for given i follows binomial distribution:

$$P\{X = j | D = i\} = \binom{i}{j} p^i (1-p)^{i-j} \qquad (2)$$

Combining Eqs. (1) and (2), the unconditional probability that j nodes are transmitting in area a is given by [12]

$$P\{X = j\} = e^{-\mu} \sum_{i=j}^{\infty} \frac{\mu^i}{i!} \binom{i}{j} p^j (1-p)^{i-j}$$

$$= e^{-\mu} p^j \mu^j \sum_{i=j}^{\infty} \frac{\mu^{i-j}}{i!} \frac{i!}{(i-j)!\, j!} (1-p)^{i-j}$$

$$= \frac{e^{-\mu} p^j \mu^j}{j!} \sum_{i=j}^{\infty} \frac{\mu^{i-j}}{(i-j)!} (1-p)^{i-j} = \frac{e^{-\mu} p^j \mu^j}{j!} e^{\mu(1-p)}$$

$$= \frac{(\mu p)^j}{j!} e^{-\mu p} \qquad 0 \le j < \infty . \qquad (3)$$

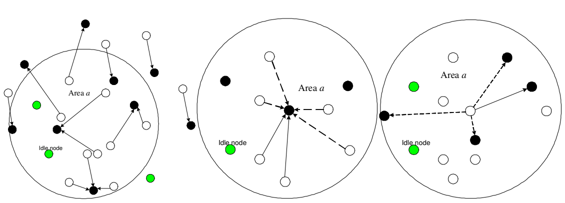
Eq. (3) proves that the distribution of transmitting nodes in area a is also a Poisson Point process and that the average number of transmitting nodes in area a is given by

$$N = \sum_{j=0}^{\infty} j \frac{(\mu p)^j}{j!} e^{-\mu p} = \mu p \qquad (4)$$

Now let us consider a case from the view point of a particular receiving node. In this case we are interested in distribution and average number of transmitters in the receiver neighbourhood defined by area a of radius r as illustrated in Figure 1(b). Note that the neighbouring nodes are distributed according to Poisson Point process and therefore the average number of transmitting nodes in the receiver neighbourhood a is given by

$$N^C = \mu^C p \qquad (5)$$

Where μ^C denotes average number of nodes in the neighbourhood of the receiver, excluding the receiver. It is interesting that due to the Poisson assumption we have $\mu^C = \mu$. This follows from the fact that in both cases the integration of mobile density is done over the same area a while the condition that in the centre there is a mobile does not change mobile density in the neighbourhood due to Poisson assumption.



(a) (b) (c)

Figure 1: (a) Transmitting, receiving and idle nodes. (b) Neighbourhood of a receiver. (c) Neighbourhood of a transmitter (in (b) and (c) arrows indicate possible transmissions among which the solid arrows indicate the transmissions offered in a given slot).

B. Distribution and average number of receiving nodes

Analogously to the distribution of transmitting nodes, the probability that k out of i nodes will be in receiving mode in area a is given by

$$P\{Y = k | D = j\} = \binom{i}{k} q^k (1-q)^{i-k} \qquad (6)$$

where q is the independent probability of becoming a receiver. Combining Eqs (1) and (6), the probability that k nodes are in receiving mode in area a is given by

$$P\{Y = k\} = \sum_{i=k}^{\infty} \frac{\mu^i}{i!} e^{-\mu} \binom{i}{k} q^k (1-q)^{i-k}$$

$$= \frac{(\mu q)^k}{k!} e^{-\mu q} \qquad 0 \le k < \infty \qquad (7)$$

Eq. (7) shows that the distribution of receiving nodes in area a is also a Poisson point process with the average number of receiving nodes in area a given by

$$M = \sum_{k=1}^{\infty} k \frac{(\mu q)^k}{k!} e^{-\mu q} = \mu q . \qquad (8)$$

Now let us consider a case from the view point of a particular transmitting node. In this case we are interested in distribution and average number of receivers in the transmitter neighbourhood defined by area a of radius r as illustrated in Figure 1(c). Then, using the same argument as in Subsection 2.1 for the receiver neighbourhood, the average number of receiving nodes in the transmitter neighbourhood a is given by [12]

$$M^C = \mu^C q \qquad (9)$$

where $\mu^C = \mu$.

C. Distribution and average number of idle nodes

Using the same procedure as in Subsections II.A and II.B, it can be shown that the distribution of the idle nodes in area a is also a Poisson point process with probability of being an idle node s=1-p-q. Then the average number of idle nodes in area a is [12]

$$L = \mu - N - M = \mu - \mu p - \mu q \qquad (10)$$

where $(p+q) \le 1$. If $(p+q) = 1$, consequently there is no idle nodes in the system.

III. RECEIVER PERFORMANCE ANALYSIS

In order to obtain uniform loading of the network we have chosen a traffic model where each node generates the same level of traffic to all nodes in its neighbourhood with packet arrival rate λ. Therefore, for a given number of nodes in the i node neighbourhood, this node generates packets with the rate of λ. Now let us consider a situation in a particular frame from the perspective of a receiver as illustrated in Figure 1(b). Following equal load to each node assumption, the probability p_t that a transmitter will try to send a packet to the receiver is the same for all transmitters in the neighbourhood and is independent of the number of transmitters (j) in the neighbourhood. Therefore, the probability that l packets out of j transmitters are offered to the receiving node in given frame is

$$P\{Z = l | X = j\} = \binom{j}{l} (p_t)^l (1 - p_t)^{j-l} \tag{11}$$

Then, the unconditional probability that l packets are offered to the receiver is given by

$$P\{Z = l\} = \sum_{j=l}^{\infty} \frac{(\mu^C p)^j}{j!} e^{-\mu^C p} \binom{j}{l} (p_t)^l (1 - p_t)^{j-l}$$
$$= \frac{(\mu^C p\, p_t)^l}{l!} e^{-\mu^C p\, p_t} . \tag{12}$$

Then, the average number of the packets offered to the receiver is

$$V = \sum_{l=0}^{\infty} l \frac{(\mu^C p\, p_t)^l}{l!} e^{-\mu^C p\, p_t} = \mu^C p\, p_t . \tag{13}$$

Probability pt is a convenient parameter for defining the traffic load in the network. Note that under assumption that a transmitter can offer a packet to only one receiver in a frame, this probability has an upper bound being a function of the average number of receivers in the neighbourhood, M^C. Namely, the average number of packets offered by a transmitter cannot exceed one packet. This condition gives the bounding equation: $1 = p_t * M^C$ resulting in

$$\max\{p_t\} = 1/M^C = 1/\mu^C q \tag{14}$$

Since in this paper we want to optimize the capacity of a MUD receiver under a fully loaded network, we will use the maximum value of p_t in the remainder of this paper. Then, by using (14) and (9) in (13) we arrive at

$$P\{Z = l\} = \frac{\{(\mu^C p)/(\mu^C q)\}^l}{l!} e^{-\{(\mu^C p)/\{\mu^C q\}\}} = \frac{(p/q)^l}{l!} e^{-(p/q)} \tag{15}$$

$$V = \sum_{l=0}^{\infty} l \frac{(p/q)^l}{l!} e^{-(p/q)} = p/q . \tag{16}$$

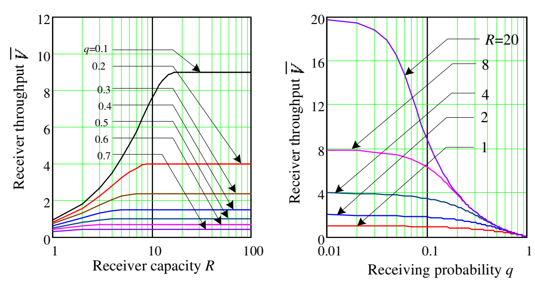
The last equation has simple intuitive explanation that the larger ratio of the number of transmitters to the number of receivers in the receiver neighbourhood, the more packets offered to the receiver.

Once the level of traffic offered to the receiver is established, we can analyse the MUD receiver performance as a function of its capacity R that defines maximum number of packets that can be received in one frame. Based on our assumptions the number of packets offered to the receiver can exceed the receiver capacity. In such a case only R offered packets will be accepted for transmission in the given slot while the rest of offered packets could be either delayed (with some maximum delay deadline) or cleared from the system. Note that since we analyse the system under the maximum offered load, the delay of rejected packets would not influence the system throughput. Therefore we do not consider the packet delay explicitly in our model. At the same time the rejection probability of offered packets (B) can be interpreted as the packet delay probability and therefore it constitutes an important QoS metric. Consequently in the remainder of the paper we concentrate on analysis of the packet throughput and the packet acceptance probability.

Under our assumptions the distribution of number packets accepted by the receiver in a frame will follow truncated Poisson distribution. Since in general we are interested in maximizing the network throughput, the important characteristic of the receiver is the average number of packets received in a frame which is given by

$$\overline{V} = (p/q)\left\{1 - \frac{\frac{(p/q)^R}{R!}}{\sum_{m=0}^{R} \frac{(p/q)^m}{m!}}\right\} \tag{17}$$

The nice feature of this solution is that it is scalable with the network node density μ as it depends only on the ratio of transmitting and receiving probabilities and the transmitter capacity. In fact it also does not depend on the probability of being an idle node s. Nevertheless, to simplify the numerical result presentation, in the reminder of this paper we assume that the probability of being an idle node is zero (s=0) and therefore p+q=1. This assumption influences the total network throughput discussed in Section 4. For s=0 the total network throughput is maximized which is consistent with our objective and using the maximum value of p_t.



(a) (b)

Figure 2: Receiver throughput as a function of receiver capacity and receiving probability.

Figure 2 shows a sample of numerical results, obtained from Eq. (17), where average receiver throughput is presented as a function of receiver capacity R, Figure 2(a), and receiver probability q, Figure 2(b). As could be expected the throughput increases with the receiver capacity increase until a certain point from which the throughput saturates. The point of saturation depends on receiving probability q with higher throughputs achieved for smaller values of q that correspond to a larger number of neighbouring transmitters and therefore larger number of offered packets.

As mentioned before the receiver QoS metric of interest is probability of packet acceptance that can be defined as

$$E = \frac{\overline{V}}{V} = \left\{ 1 - \frac{\dfrac{(p/q)^R}{R!}}{\displaystyle\sum_{m=0}^{R} \frac{(p/q)^m}{m!}} \right\}. \qquad (18)$$

The numerical results show that the fraction of accepted packets is quite low for receiver capacity R=1 but it increases relatively fast with the receiver capacity increase to achieve its maximum value of one for a reasonable value of R. This very important result shows that an application of a MUD receiver can greatly improve the performance of ad hoc networks with multimedia applications. The results also indicate that the fraction of accepted packets is decreasing with receiving probability decrease and that is a logical consequence of increased offered packet load.

IV. Network performance

In this section we present a network performance analysis. In Subsection IV.A, we define first the network throughput as the average number of transmitted packets in the area a. Then the receiver performance model from Section 3 is used to calculate the network throughput as a function of receiver capacity R and receiving probability q. In Subsection IV.B, we indicate that for given receiver capacity there is an optimal operating point that maximizes the network throughput with respect to the receiving probability.

A. *Network throughput*

In Section 3 we derived the receiver performance model that gives the average number of accepted packets per receiver and corresponding packet acceptance probability. In order to optimize the network throughput we need to define a metric that is not conditioned on the node state. One possibility is to use the average number of successfully transmitted packets per frame per node, w. Nevertheless, to facilitate presentation and analysis of the results, first we will use a metric S defined as the average number of successfully transmitted packets in an arbitrary area a; the two metrics are related since $S = \mu w$. The area throughput S can be calculated based on the receiver throughput $\overline{V}$, given by Eq. (17), and M, the average number of receivers in area a given by Eq. (8). By combining the two equations, the area throughput is given by

$$S = M\overline{V} = \mu p \left\{ 1 - \frac{\dfrac{(p/q)^R}{R!}}{\displaystyle\sum_{m=0}^{R} \frac{(p/q)^m}{m!}} \right\}. \qquad (19)$$

Figure 3(a) shows the area throughput as a function of receiver capacity R for different receiving probabilities q. Similarly to the results presented for receiver throughput in Figure 2(a), the area throughput increases with the receiver capacity increase until certain point from which the throughput saturates. The point of saturation depends on q with higher saturation throughput achieved for smaller values of q and for larger receiver capacity. The difference with Figure 2(a) is that in the region of smaller receiver capacity, the area throughput first increases with increase of receiving probability. Only from a certain value of q this trend changes to the opposite direction and the q value of the reversal depends on the receiver capacity. This effect can be explained by the fact that Eq. (19) consists of two parts corresponding to the offered packet rate and to the success probability, respectively. Both parts have opposite behaviour with respect to receiving probability q. For example, with low q value the offered traffic becomes high but the packet success probability becomes low. Since the success probability depends also on the receiver capacity, which of the two parts becomes dominant depends also on R. This observation indicates that for a given receiver capacity there is an optimal value of the receiving probability that maximizes the area throughput. Since the area throughput is proportional to the network throughput, in the remainder of this paper we will use these two terms interchangeably.

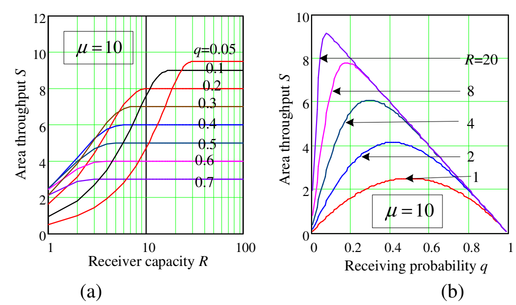


Figure 3: Area throughput S as a function of receiver capacity R and receiving probability q.

B. *Optimal operating point*

Based on the observations from the previous subsection, we present in Figure 3(b) the area throughput as a function of receiving probability q for different receiver capacities R. The results are quite revealing and show that for a given receiver capacity the function is not monotonic and a maximum throughput is achieved for a certain receiving probability as suggested in Subsection IV.A. In general, the larger the receiver capacity the smaller the receiving probability for which the maximum throughput is achieved. This feature indicates that the maxima represent optimal operating points that maximize the area throughput. As a consequence, for given receiver capacity R, the distributed MAC algorithm should achieve the optimal operating point by realizing the corresponding receiving probability q that defines the ratio between the average number of receivers and transmitters.

V. Optimizing the MUD receiver capacity

In this section we present a framework and related models for MUD receiver capacity optimization. First, in Subsection V.A we calculate the relation between the maximum throughput and receiver capacity. This relation is used to define a utility function from serving the packets in the network. To find optimal receiver capacity we compare this utility function with a network cost function that has a component associated with the MUD receiver capacity. This component grows exponentially with receiver capacity due to the growing complexity and power consumption of the receiver. The intersection of these two functions indicates optimal receiver capacity. Moreover in Subsection V.B we present dependence of the packet acceptance probability on the receiver capacity. This important performance relation is used as an additional factor in the receiver capacity optimization.

A. *Receiver capacity optimization*

To optimize receiver capacity one would like to have a function relating maximum area throughput *S* to receiver capacity *R*. To obtain this function we can first differentiate Eq. (19) with respect to *q*. This can be done by setting *p* =1-*q* in Eq. (19) to arrive at

$$S=\mu(1-q)\left\{1-\frac{\dfrac{\left(\frac{1-q}{q}\right)^{R}}{R!}}{\displaystyle\sum_{m=0}^{R}\frac{\left(\frac{1-q}{q}\right)^{m}}{m!}}\right\}. \qquad (20)$$

Differentiating Eq. (20) with respect to *q* gives us

$$\frac{dS}{dq}=\mu(1-q)\left\{-\frac{\displaystyle\sum_{m=0}^{R}\frac{\left(\frac{1-q}{q}\right)^{m}}{m!}R\frac{\left(\frac{1-q}{q}\right)^{R-1}}{R!}\left(-\frac{1}{q^{2}}\right)-\frac{\left(\frac{1-q}{q}\right)^{R}}{R!}\left(-\frac{1}{q^{2}}\right)\sum_{m=0}^{R-1}\frac{\left(\frac{1-q}{q}\right)^{m}}{m!}}{\left\{\displaystyle\sum_{m=0}^{R}\frac{\left(\frac{1-q}{q}\right)^{m}}{m!}\right\}^{2}}\right\}$$

$$-\mu\left\{1-\frac{\dfrac{\left(\frac{1-q}{q}\right)^{R}}{R!}}{\displaystyle\sum_{m=0}^{R}\frac{\left(\frac{1-q}{q}\right)^{m}}{m!}}\right\}$$

Then by equating it to zero ($\frac{dS}{dq}=0$) we obtain

$$\frac{1-q}{q^{2}}\frac{\left(\frac{1-q}{q}\right)^{R-1}}{(R-1)!}\left[\sum_{m=0}^{R}\frac{\left(\frac{1-q}{q}\right)^{m}}{m!}-\frac{\left(\frac{1-q}{q}\right)}{R}\sum_{m=0}^{R-1}\frac{\left(\frac{1-q}{q}\right)^{m}}{m!}\right]$$

$$=\sum_{m=0}^{R}\frac{\left(\frac{1-q}{q}\right)^{m}}{m!}\sum_{m=0}^{R-1}\frac{\left(\frac{1-q}{q}\right)^{m}}{m!}. \qquad (21)$$

By solving this equation within the desired range numerically, we can obtain the optimal values of receiving probability q as a function of receiver capacity *R*. These optimal values of receiving probabilities allow to calculate, based on Eq. (20), the maximum area throughput $S_{\max}$ as a function of receiver capacity *R* for different node densities, as illustrated in Figure 4. Note that the curve for $\mu=10$ from Figure 4 corresponds to the upper bound of all curves in Figure 3(b). As expected the maximum area throughput increases with the increase of receiver capacity and node density. Figure 4 also shows that the asymptotic limit for the maximum throughput is defined by the node density. This follows from the form of Eq. (20) in which the part associated with offered traffic approaches 1 for $q\rightarrow 0$ and the part associated with the acceptance probability also approaches 1 for $R\rightarrow\infty$.

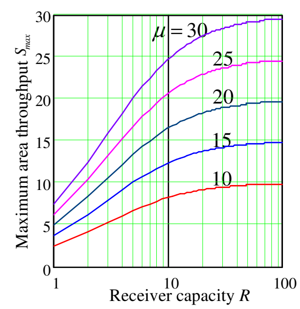


Figure 4: Maximum area throughput $S_{\max}$ vs. receiver capacity *R*.

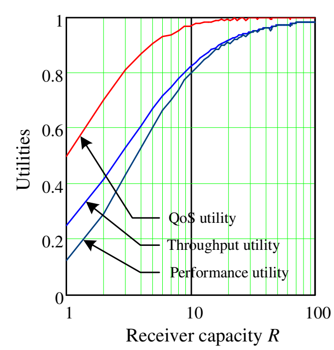


Figure 5. Three utility functions as a function of receiver capacity for optimal receiving probabilities.

In general the optimal capacity of a receiver is a function of the area throughput, QoS performance, and the network cost. To facilitate optimization analysis let us decompose it into two parts. First we consider the area throughput and the network cost. Then, in subsection V.B, the QoS performance factor will be added. To consider the first part of the receiver capacity optimization, it is useful to create a throughput utility function, $U(R)$, that corresponds to the level of satisfaction from the area throughput and the network cost utility function, $C(R)$, that corresponds to the level of dissatisfaction from the network cost. Then the optimal operating point would correspond to the receiver capacity that gives maximum positive difference between the level of satisfaction and dissatisfaction.

Since each node has a MUD receiver, it is convenient to consider the area throughput and the network cost in units per node. In this case, the node throughput is given by

$$w(R)=\left(1-q_{opt}\right)\left\{1-\frac{\dfrac{\left(\dfrac{1-q_{opt}}{q_{opt}}\right)^{R}}{R!}}{\sum_{m=0}^{R}\dfrac{\left(\dfrac{1-q_{opt}}{q_{opt}}\right)^{m}}{m!}}\right\}. \tag{22}$$

Note that the node throughput is independent of the node density which facilitates the receiver capacity optimization. Then we assume that the throughput utility function is proportional to the node throughput and use *U(R)=w(R)* as illustrated in Figure 5. We also assume that the node cost has a constant component equal to the cost of a node with a receiver of capacity one and a component proportional to the cost of a MUD receiver that grows exponentially with receiver capacity due to increased complexity and power consumption. Then, assuming that the dissatisfaction from the cost is proportional to the cost we consider the following cost utility function form

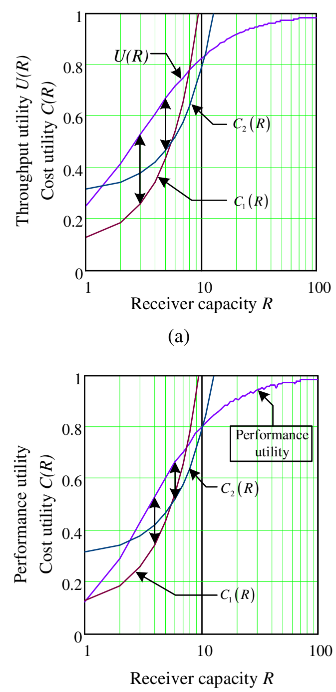
$$C(R)=\alpha+\beta R^{\gamma}. \tag{23}$$

Figure 6(a) shows the throughput utility function that is compared with two examples of the cost utility function: $C_1(R)$ with α=0.1, β=0.03, γ=1.5, and $C_2(R)$ with α=0.3, β=0.015, γ=1.5. Obviously the cost utility function parameters α, β, and γ depend on many factors and will change in time so the given examples are for illustration purposes only.

In the considered examples, the optimal receiver capacities, corresponding to the maximum net-satisfaction levels, are R_1=3 and R_2=5. Note that in the second example of the cost utility function, the receiver capacity equal to one is not acceptable since the net-satisfaction level is negative. This example illustrates that the MUD receiver allows applying more expensive nodes due to the gains in the network throughput.

B. *QoS performance factor*

As shown in Section 3 the packet acceptance probability is an important QoS metric that corresponds to the packet delay probability. Note that in general the acceptance probability is a function of receiving probability and the receiver capacity. Therefore this QoS performance characteristic can also influence the level of satisfaction from the network operation when optimizing the receiver capacity. Based on results from Section III, the packet acceptance probability is growing with the increase of the receiver capacity which further adds to the advantages of the MUD receiver application in *ad hoc* networks. To illustrate this aspect let us introduce a new network performance utility function that represents satisfaction from both throughput and QoS level. For this illustration we assume that the satisfaction from the QoS is proportional to the packet acceptance probability but in general this may be a non-lineal relation. Then, we define the performance utility function as a product of the throughput and QoS utility functions. Figure 5 depicts the throughput utility, packet acceptance probability (corresponding to packet delay probability) and performance utility as a function of receiver capacity.



(a)

(b)

Figure 6: Comparison of cost utility functions with (a) throughput utility function (b) performance utility function, where optimal receiver capacities are indicated by vertical arrows corresponding to maximum net-satisfaction.

The influence of taking into account the QoS utility is illustrated in Figure 6(b) for the same examples of cost utility functions that were used in Subsection 5.1. With performance utility function the optimal receiver capacities are now increased from 3 to 4 for $C_1(R)$ and from 5 to 6 for $C_2(R)$. This result illustrates the two main advantages of a MUD receiver: increased throughput and improved QoS, both being very important when considering multimedia applications in *ad hoc* networks.

Validity of our MUD capacity dimensioning model is confirmed by comparison with simulation results presented in [6]. Namely, Figure 6a (in our paper) indicates that for receiving probability *q*=0.4 the throughput is close to maximum for receiver capacity R=4 and reaches maximum for R=6. In reference [6], Figure 4b (top curve) indicates also that for M=R=4 the throughput is close to maximum and for M=R=6 it reaches maximum. Moreover Figure 6b in [6] shows that the receiving probability is close to 0.4. These results indicate that although we used in our model several approximations compared to the system studied in [6], so the exact throughput values are different, but the model reflects well the throughput saturation with increase of MUD capacity for given receiving probability.

VI. CONCLUSIONS

Application of multi-user detection in *ad hoc* networks brings promise of increased network throughput and performance improvements that can allow implementation of multimedia applications. At the same time the complexity and power consumption of a MUD receiver grow exponentially with its capacity. In this paper we address this issue by first developing an approximate performance model and then proposing a framework for optimization of the receiver capacity.

The performance model assumes a Poisson distribution of the number of nodes in the neighbourhood and under this assumption the receiver performance is independent of node density. Using this model we illustrated the throughput and QoS gains achieved by applying a MUD receiver, as a function of the receiver capacity. Also several interesting features of the system were revealed. In particular we showed that for given receiver capacity there is an optimal operating point that maximizes the network throughput. The optimal operating point is defined by average ratio of number of receivers to the number of transmitters in the neighbourhood and its value depends on the receiver capacity. This feature indicates that the medium access protocol should realize this ratio in order to maximize the network throughput.

Based on the performance model, we proposed a framework for the MUD receiver capacity optimization. It is based on performance and cost utility functions that depend on the receiver capacity. The performance utility corresponds to satisfaction from the achieved throughput and QoS levels while the cost utility corresponds to dissatisfaction from the network cost. The difference between the performance and cost utilities defines the net-satisfaction for given receiver capacity and the optimal receiver capacity should maximize this metric. We showed that for some cases it may be not practical to implement an *ad hoc* network without the MUD receiver. In summary, the proposed approximate performance model provides valuable insight into the behaviour of MUD based *ad hoc* networks and allows MUD receiver capacity optimization. Validity of the MUD capacity dimensioning model was confirmed by comparison with simulation results presented in [6].

In this work we did not consider the interference and power factors by assuming that a node can receive a signal from any R nodes in its neighbourhood. However a study of this issue is a topic of interest for further investigation. Another topic worth consideration is a case with two priorities for frames corresponding to real time and elastic traffic.

VII. REFERENCES

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